

A Decoupled and Linear Framework for Global Outlier Rejection over Planar Pose Graph

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Abstract—We propose a robust framework for planar pose graph optimization contaminated by loop closure outliers. Our framework rejects outliers by first decoupling the robust PGO problem wrapped by a *Truncated Least Squares* kernel into two subproblems. Then, the framework introduces a linear angle representation to rewrite the first subproblem that is originally formulated in rotation matrices. The framework is configured with the *Graduated Non-Convexity* (GNC) algorithm to solve the two non-convex subproblems in succession without initial guesses. Thanks to the linearity property of the angle representation, our framework requires only a linear solver to optimally solve the optimization problems encountered in GNC. We extensively validate the proposed framework, named *DEGNC-LAF* (*DEcoupled Graduated Non-Convexity with Linear Angle Formulation*) in planar PGO benchmarks. It turns out that it runs significantly (sometimes up to over 30 times) faster than the standard and general-purpose GNC while resulting in high-quality estimates.

I. INTRODUCTION

Pose Graph Optimization (PGO) is a fundamental estimation engine for Simultaneous Localization and Mapping (SLAM) in many robotic applications. PGO aims to find poses of the robot that best explain the obtained noisy measurements. The solution space of the poses to be estimated in PGO lies in the special Euclidean group, which makes solving this problem untrivial.

Despite the challenges of PGO, recently proposed techniques (e.g., *SE-Sync* [1]) can solve PGO optimally under mild preconditions. However, a portion of loop closure measurements are outliers in practice due to the inevitable perceptual aliasing. In the presence of outliers, even the optimal solution of PGO can still significantly deviate from the ground-truth. Therefore, it is common to use robust PGO techniques [2]–[6] to gain resilience against outliers.

However, the state-of-the-art methods for robust PGO are either *local* [4], implying an over-dependence on a reliable initial guess, or consume heavy computational resources [7], [8] and thus are far from being able to take on a broader range of tasks flexibly. Therefore, it is desirable to develop *global*¹ outlier rejection techniques that often succeed in eliminating the misinformation of outliers, while performing considerable efficiency.

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¹We identify a method as global if it requires no initial guess or its result is independent of the initial guess.

In this paper, we show that we can fill the abovementioned gaps in the *planar* PGO setup. Specifically, by decoupling rotation and translation estimates in planar PGO and adopting an angle setting for planar rotation to substitute rotation matrices, we construct two internally linear subproblems wrapped by the *Truncated Least Squares* (TLS) kernel. We adopt the standard *Graduated Non-Convexity* (GNC) [8], [9] algorithm to solve these two non-convex subproblems. Thanks to the linearity properties of the two subproblems, our proposed framework requires only linear solvers.

Contribution. Our main contribution is to propose a novel framework to globally reject outliers over planar pose graph with considerable efficiency. The corresponding algorithm, named *DEcoupled Graduated Non-Convexity with Linear Angle Formulation* (*DEGNC-LAF*), can (i) perform global outlier rejection while showing superior efficiency to general-purpose global methods (e.g., [7], [8]), and (ii) withstand outlier rates that may occur in real-world tasks and thus results in high-quality estimates.

In addition, we also propose and validate that the first subproblem in a linear angle setting converges GNC with better accuracy and robustness (and much faster) than the one based on rotation matrix, which results in a significant performance enhancement of the complete estimation algorithm (see *Remark 1* and Section VII).

We extensively validate the proposed approach in standard PGO benchmarks. The result illustrates that *DEGNC-LAF* is significantly faster than the standard and general-purpose GNC algorithm [8] while usually outperforming or matching GNC and local robust PGO techniques [4], [5] in terms of accuracy and robustness.

II. RELATED WORK

Ideas for designing robust PGO algorithms can be divided into two main categories: (i) reformulating to robustify the original PGO problem and solving the reformulated problem, and (ii) mining empirical or statistical evidence of outliers and filtering them out accordingly.

A. Reformulating to Gain Robustness against Outliers

The existing robust PGO algorithms reformulating to robustify the original problem mainly follow three ways: (i) introducing *switchable variables*, (ii) adopting the *Consensus Maximization* paradigm [10], [11] and (iii) adopting the *M-estimation* paradigm [12], [13].

1) *Switchable variable*: The formulations proposed in [2]–[4], [14] introduce switchable variables for untrusted

edges in the pose graph. For continuous optimization problems with switchable variables (e.g., [2]–[4]), standard nonlinear optimization techniques can be used to solve them. Some off-the-shelf libraries, such as ceres-solver [15] and g2o [16], provide implementations of these techniques. By contrast, the switchable variables in [14] are binary (i.e., discrete), so an alternating minimization framework proposed by the authors was used to solve the problem. Unfortunately, the solving techniques mentioned above are all local, requiring reliable initial guesses.

2) *Consensus Maximization*: Consensus Maximization (CM) looks for an estimate that maximizes the number of measurements with errors under a prescribed threshold. RANSAC [17] is a popular heuristic for CM and requires no initial guess, but the performance of RANSAC is indeterminate and its runtime grows exponentially with increasing outlier rates. Tzoumas et al. [7] proposed ADAPT, a general-purpose heuristic algorithm to solve *Minimally Trimmed Squares* (MTS) estimation which shares commonalities with CM [18].

3) *M-estimation*: M-estimation robustifies the original cost function of a problem via wrapping it in a robust kernel. Local nonlinear optimization is an efficient way to solve robust structure from motion (SFM) [19], [20] and robust PGO [21] formulated as M-estimation problems. To get rid of the dependence of robust kernel-wrapped PGO problems on reliable initial guesses, Carlone and Calafiore [22] proposed convex relaxations for the l_1 -norm and Huber kernels. *Graduated Non-Convexity* (GNC) has been used in robust estimation problems [9], [23] as a heuristic to globally solve the M-estimation problem. More recently, Yang et al. [8] combined modern global non-minimal solvers (e.g., SE-Sync) with Black-Rangarajan duality [9] for M-estimation, which benefits our work (see Section V).

B. Other Models/Evidence for Outliers

Latif et al. [5] proposed the *Realizing, Reversing, and Recovering* (RRR) algorithm to reject outliers based on the residuals of local PGO. Mangelson et al. [6] proposed *Pairwise Consistency Maximization* (PCM) to select a set of inliers, which checks pairwise consistency for each two loop closures and infers the set of inliers from a graphical representation of pairwise consistency. Indelman et al. [24] modeled the problem of finding loop closure outliers as a *maximum a posteriori estimation* problem. The authors inferred the formulated problem via *expectation maximization*, to which a similar idea can be found in [25].

III. DECOUPLED ROBUST PLANAR PGO

A. Coupled and Rotation Matrix-based Planar PGO

Planar PGO solves 2D poses (R_i, t_i) sampled along the robot trajectory, where $R_i \in \text{SO}(2)$ and $t_i \in \mathbb{R}^2$. Typically, we anchor the first pose and assume that the noisy measurements $(\tilde{R}_{ij}, \tilde{t}_{ij})$ obtained by the robot are sampled from the following probabilistic generative model [1]:

$$\tilde{R}_{ij} = R_{ij}R_{ij}^\epsilon, \quad R_{ij}^\epsilon \sim \text{Langevin}(I_d, \kappa_{ij}), \quad (1a)$$

$$\tilde{t}_{ij} = t_{ij} + t_{ij}^\epsilon, \quad t_{ij}^\epsilon \sim \mathcal{N}(0, \tau_{ij}^{-1}I_d), \quad (1b)$$

where for all pairwise poses (i, j) , (t_{ij}, R_{ij}) is the true but latent value of relative poses.

Given the noisy measurements and substituted into the probability model of Langevin [26] and Gaussian distribution appearing in (1a) and (1b), the (coupled and rotation matrix-based) PGO is modeled as a *maximum likelihood estimation* problem:

$$(\hat{R}, \hat{t}) = \arg \min_{\substack{R_i \in \text{SO}(2) \\ t_i \in \mathbb{R}^2}} \sum_{(i,j)} \left\{ \begin{array}{l} \kappa_{ij} \| R_j - R_i \tilde{R}_{ij} \|_F^2 \\ + \tau_{ij} \| t_j - t_i - R_i \tilde{t}_{ij} \|_2^2 \end{array} \right\}, \quad (2)$$

where (\hat{R}, \hat{t}) is a compact representation of the solution to problem (2).

Problem (2) is currently proposed to be tightly relaxed as a (convex) *semidefinite programming* under mild preconditions [1], and thus can be solved optimally. However, in real-world tasks, a fraction of the loop closure measurements are outliers. The original formulation (2) is sensitive to these outliers, so it is common to apply robust kernels to wrap the original problem to gain resilience against the outliers.

B. Robust Planar PGO with Truncated Least Squares Kernel

To limit as much as possible the sensitivity of planar PGO to outliers, we adopt a TLS-based scheme to reformulate the planar PGO problem (2), which leads to the *Coupled and Rotation Matrix-Based Truncated Least Squares Planar PGO (TLS-PGO)* problem:

$$\min_{\substack{R_i \in \text{SO}(2) \\ t_i \in \mathbb{R}^2}} \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij}^{pgo})^2 + \sum_{(i,j) \in \mathcal{E}_{lc}} \rho_c(r_{ij}^{pgo}), \quad (3)$$

where

$$\rho_c(r) = \min(r^2, c^2) \quad (4)$$

is the TLS kernel, \mathcal{E}_{od} and \mathcal{E}_{lc} are the sets consisting of odometry and loop closure respectively, and

$$r_{ij}^{pgo} = \sqrt{\kappa_{ij} \| R_j - R_i \tilde{R}_{ij} \|_F^2 + \tau_{ij} \| t_j - t_i - R_i \tilde{t}_{ij} \|_2^2}. \quad (5)$$

In problem (3), we only apply robust kernels to loop closure measurements, since we can typically trust odometry measurements as inliers.

C. Decoupled Robust Planar PGO

In this subsection, we propose a decoupled formulation for robust planar PGO. We decouple problem (3) into two subproblems that are solved successively. The subproblem to be solved first is the *Truncated Least Squares Rotation Matrix-Based Planar Rotation Averaging (TLS-RA)* problem:

$$\min_{R_i \in \text{SO}(2)} \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij}^{ra})^2 + \sum_{(i,j) \in \mathcal{E}_{lc}} \rho_{c_1}(r_{ij}^{ra}), \quad (6)$$

where

$$r_{ij}^{ra} = \sqrt{\kappa_{ij} \| R_j - R_i \tilde{R}_{ij} \|_F^2}. \quad (7)$$

Once problem (5) is solved, the rotation R_i will be fixed in the second subproblem. Thereby, we have the second

subproblem, the *Truncated Least Squares Rotation Matrix-Based Planar Translation Averaging (TLS-TA)* problem:

$$\min_{t_i \in \mathbb{R}^2} \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij}^{ta})^2 + \sum_{(i,j) \in \mathcal{E}_{ic}} \rho_{c_2} (r_{ij}^{ta}), \quad (8)$$

where

$$r_{ij}^{ta} = \sqrt{\tau_{ij} \| t_j - t_i - \hat{R}_i \tilde{t}_{ij} \|_2^2}, \quad (9)$$

where \hat{R}_i is the rotation estimate determined by solving problem (6) and remains constant in problem (8).

We note that the estimate obtained by successively optimally solving subproblem (6) and subproblem (8) is not equivalent to the one obtained by optimally solving problem (3). But an important fact is that empirically, the optimal rotation estimate from solving problem (6) is close to the one from solving problem (3). This can be explained by the fact that (i) in the absence of outliers, these two solutions are usually close to each other, as shown in Table 1, and (ii) if problems (3) and (6) are solved optimally in the presence of outliers, the solutions will be close to those in the absence of outliers due to the robustness of TLS kernel, and thus are close to each other according to (i). Therefore, robust rotation estimation does not undergo significant distortion due to decoupling. This implies the fact that translation estimates obtained from optimally solving problem (8) are consistent with those obtained from optimally solving problem (3).

IV. ROBUST PLANAR ROTATION ESTIMATION WITH ANGLE-BASED LINEAR FORMULATION

Although rotation matrices can extensively represent planar and spatial rotations without singularity, the planar rotation has another direct *angle* representation that is less mentioned today but enables the decoupled framework to work better. In this section we show how to use this angle setting to construct a problem to replace the problem (6) in the decoupled framework.

A. Linear Angular Domain for Planar Rotation

We use the robot's orientation angle θ_i instead of the rotation matrix to represent the rotational component of the robot's pose, so that the probabilistic generative model of the rotation measurements is replaced from Eq. (1a) with the following model:

$$\tilde{\theta}_{ij} = \langle \theta_j - \theta_i + \theta_{ij}^\epsilon \rangle = \theta_j - \theta_i + k_{ij} 2\pi + \theta_{ij}^\epsilon, \quad (10)$$

$$\theta_{ij}^\epsilon \sim \mathcal{N}(0, \kappa_{ij}^{-1} I_d),$$

where $\langle \cdot \rangle : \mathbb{R} \rightarrow (-\pi, +\pi]$ is the 2D *geodesic distance* and $k_{ij} \in \mathbb{Z}$ is called a *regularization variable* that regularizes the angle measurements in the interval $(-\pi, +\pi]$.

We now consider the rotation averaging problem in this orientation angle setting when no robust kernel is applied:

$$\min_{\substack{\theta_i \in \mathbb{R} \\ k_{ij} \in \mathbb{Z}}} \sum_{(i,j)} \kappa_{ij} \| \theta_j - \theta_i + k_{ij} 2\pi - \tilde{\theta}_{ij} \|_2^2. \quad (11)$$

We can clearly see that if the regularization variable k_{ij} can be determined as a priori and fixed, we can obtain a *linear*

TABLE I

AVERAGE ROTATION ERROR (ARE) BETWEEN OPTIMAL SOLUTIONS TO PGO AND ROTATION AVERAGING ON STANDARD DATASETS

Dataset (no outliers)	ARE (deg)
city10000	0.6971
intel	0.9215
kitti_00	0.6453
kitti_02	0.3268
kitti_05	0.1494
manhattan	0.5306
csail	0.0631

estimation problem from (11), which will make its robust version much less difficult to solve (see *Remark 1*).

We adopt the approximate method proposed in [27] to estimate the regularization variable k_{ij} without solving the integer-mixed problem (11). k_{ij} can be determined based on the fact that in the absence of noise, the measurements $\tilde{\theta}_{ij}$ along each *cycle* in the pose graph have to sum-up to zero, of which a sufficient and necessary condition is that the measurements along each cycle in the *cycle basis* [28] of the pose graph sum-up to zero. According to this fact, the integer k_{ij} can be closed-form solved from a linear system. In the presence of noise, k_{ij} solved from the linear system will no longer be an integer, but the corresponding regularization variable can be rounded to the integer closest to it. This rounding scheme is perfectly accurate in the majority of real-world scenarios [27], so we can convincingly adopt it to determine k_{ij} as a priori, thus converting problem (10) into a linear estimation.

B. Robust Rotation Averaging in Linear Angle Setting

By adopting the linear angle setting shown in (11), we can replace problem (6) with the *Truncated Least Squares Angle-Based Planar Rotation Averaging (TLS-ARA)* problem:

$$\min_{\theta_i \in \mathbb{R}} \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij}^{ara})^2 + \sum_{(i,j) \in \mathcal{E}_{ic}} \rho_{c_1} (r_{ij}^{ara}), \quad (12)$$

where

$$r_{ij}^{ara} = \sqrt{\kappa_{ij} \| \theta_j - \theta_i + k_{ij} 2\pi - \tilde{\theta}_{ij} \|_2^2}. \quad (13)$$

Although both problem (12) and problem (6) are non-convex and difficult to solve directly, intuitively, problem (12) will be easier to solve because it is a robust *linear* estimation problem, whereas problem (6) is still a nonlinear problem in the absence of the robust kernel.

The following section demonstrates how to solve problems (3), (6), (8), (12) using a general GNC framework.

V. SOLVING ROBUST PGO VIA GRADUATED NON-CONVEXITY

Firstly, we note that problems (3), (6), (8), (12) share the uniform form:

$$\min \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij})^2 + \sum_{(i,j) \in \mathcal{E}_{ic}} \rho_c (r_{ij}), \quad (14)$$

which can be equivalently rewritten as follows:

$$\min \left\{ \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij})^2 + \sum_{(i,j) \in \mathcal{E}_{lc}} \min_{w_{ij} \in \{0,1\}} [w_{ij} r_{ij}^2 + (1 - w_{ij}) c^2] \right\}. \quad (15)$$

Problem (15) allows the use of (i) *Alternating Optimization* (AM), (ii) *Semidefinite Programming* (SDP) [29] and (iii) *Graduated Non-Convexity* (GNC) [8] to solve it. However, while AM is efficient, it requires an initial guess and can easily fall into local minima. By contrast, SDP can provide optimality certificates, but is very slow to solve at this stage. Therefore, we adopt GNC, which requires no initial guesses and can effectively avoid local minima while performing good efficiency, to solve the problem.

Applying a *GNC function* [9] controlled by a parameter μ to approximate the TLS kernel, GNC uses a *continuation-type* algorithm [30] to optimize the following problem instead of problem (15):

$$\min \left\{ \sum_{(i,j) \in \mathcal{E}_{od}} (r_{ij})^2 + \sum_{(i,j) \in \mathcal{E}_{lc}} \min_{w_{ij} \in [0,1]} \left[w_{ij} r_{ij}^2 + \frac{\mu(1 - w_{ij})}{\mu + w_{ij}} c^2 \right] \right\}, \quad (16)$$

for which the parameter μ is gradually updated by multiplying a continuation factor f at a time. Intuitively, updating μ implies adjusting the convexity of the kernel that will gradually revert to TLS as μ increases.

GNC is initialized by solving a globally solvable problem, i.e., the optimization problem in the form of (16) with μ tending to zero and all w_{ij} being 1, shown as follows:

$$\min \sum_{(i,j)} (r_{ij})^2, \quad (17)$$

which can be solved optimally with global PGO techniques [1] with r_{ij} specified as r_{ij}^{pgo} (5), global rotation averaging techniques [1], [31] with r_{ij} specified as r_{ij}^{ra} (7) and linear solver with r_{ij} specified as r_{ij}^{la} (9) and r_{ij}^{ra} (11).

The solution to problem (17) will be used as the initial guess for subsequent optimizations in GNC. For each update of μ (called one iteration), GNC alternately optimizes two subproblems of problem (15): the first subproblem optimizes w_{ij} , where r_{ij} is determined by the solution of the second subproblem in the previous iteration, for which a closed-form solution exists (cf. [8]); the second subproblem optimizes r_{ij} , where w_{ij} is determined by the first subproblem in the current iteration. The second subproblem can be considered a weighted version of problem (17) and thus can be solved globally with the same tools. GNC will keep iterating until all w_{ij} become binaries. We can identify a measurement as an outlier if its corresponding w_{ij} converges to 0.

VI. DECOUPLED GRADUATED NON-CONVEXITY WITH LINEAR ANGLE FORMULATION

By introducing the angle setting and GNC algorithm, we proposed a decoupled and linear framework, named *DEcoupled Graduated Non-Convexity with Linear Angle Formulation* (DEGNC-LAF), to globally reject outliers over planar pose graph, the pipeline of which is shown as follows:

- 1) Computing regularization variables k_{ij} in Eq. (13)
- 2) Using GNC with k_{ij} to solve TLS-ARA (problem (12)) to obtain the rotation estimate \hat{R}_i
- 3) Using GNC with \hat{R}_i to solve TLS-TA (problem (8)) and rejecting outliers according to the resulting w_{ij} (w_{ij} corresponding to outliers will converges to zero)
- 4) Using SE-Sync with inliers to finally estimate rotations and translations $(\hat{\mathbf{R}}, \hat{\mathbf{t}})$

The pseudocode is summarized in Algorithm 1.

We note that instead of taking the solutions to TLS-ARA and TLS-TA as the output estimates, we use the rotation estimates of TLS-ARA to serve for solving TLS-TA and reject outliers based on the resulting w_{ij} of TLS-TA. We end up optimizing poses in coupled PGO using SE-Sync. Such an approach will result in more accurate estimates.

Remark 1 (Benefits of Decoupling and Linear Formulation): Linear formulation (9) and (13) enables the initialization problem (17) and the second subproblem of (15) in the alternating optimization of GNC linearly solvable, thus ensuring that optimal solutions can be obtained. In contrast, in the iterations of solving the problem (3) and problem (7), GNC needs to repeatedly call nonlinear global solvers which require preconditions to get the global optimal solution. As a result, the solvers may fail to get the optimal solution in the first few iterations, which increases the risk of GNC falling to a local minima. In addition, linear systems can be solved exceptionally efficiently and do not require the computation of optimality certificates which is rather time-consuming.

Algorithm 1: DEcoupled Graduated Non-Convexity with Linear Angle Formulation (DEGNC-LAF)

Input: reduced incidence matrix \mathbf{A} (topology of the pose graph), measurements set $\{(\tilde{\theta}_{ij}, \tilde{t}_{ij})\}$, noise level sets $\{\kappa_{ij}\}$ and $\{\tau_{ij}\}$, odometry set \mathcal{E}_{od} , loop closure set \mathcal{E}_{lc} , threshold c_1 (default: `chi2inv(0.99, 1)`), threshold c_2 (default: `chi2inv(0.99, 2)`), and GNC continuation factor f (default: 1.4);

Output: inlier set \mathcal{I} , poses estimate $(\hat{\mathbf{R}}, \hat{\mathbf{t}})$

begin

```

// Compute regularization variables
{k_ij} ← Regularize((R_tilde_ij, t_tilde_ij), A);
// Solve TLS-ARA using GNC
{theta_hat_i} ←
  GNCforARA({theta_tilde_ij}, {kappa_ij}, E_od, E_lc, c_1, f, A);
// Convert angle to rotation matrix
{R_hat_i} ← RecoverR({theta_hat_i});
// Solve TLS-TA using GNC
I ←
  GNCforTA({R_hat_i}, {R_hat_i}, {tau_ij}, E_od, E_lc, c_2, f, A);
{R_tilde_ij} ← RecoverR({theta_hat_i});
// Final estimate
(R_hat, t_hat) ←
  SE-Sync({(R_tilde_ij, t_tilde_ij)}, {kappa_ij}, {tau_ij}, A);
return I, (R_hat, t_hat);

```

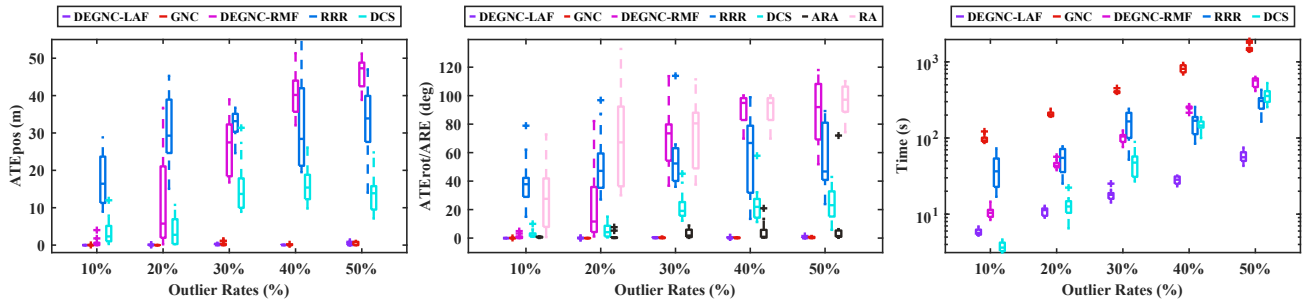


Fig. 1. **city10000** (5000 poses). Average Trajectory Error (ATE), Average Rotation Error (ARE) and the running time of the proposed approach compared to state-of-the-art techniques on the city10000 dataset. Statistics are computed over 10 Monte Carlo runs.

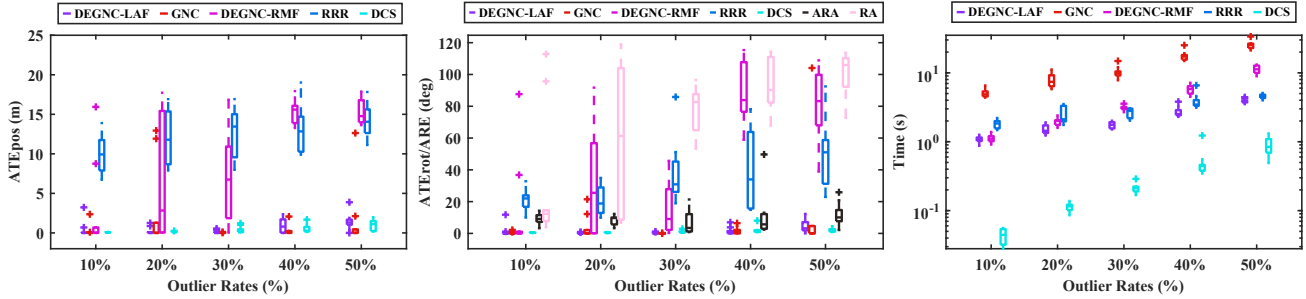


Fig. 2. **intel**. Average Trajectory Error (ATE), Average Rotation Error (ARE) and the running time of the proposed approach compared to state-of-the-art techniques on the intel dataset. Statistics are computed over 10 Monte Carlo runs.

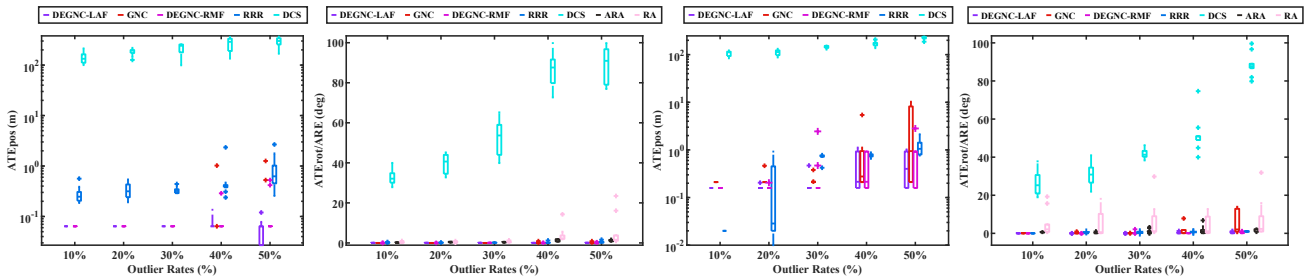


Fig. 3. **kitti.00 and kitti.05**. Average Trajectory Error (ATE) and Average Rotation Error (ARE) of the proposed approach compared to state-of-the-art techniques on the kitti.00 dataset (first to second from left) and the kitti.05 dataset (third to fourth from the left). Statistics are computed over 10 Monte Carlo runs.

TABLE II
INFORMATION OF THE DATASETS TESTED ON

Dataset	Poses	Loop Closures
city10000	5000	3385
intel	1728	786
kitti.00	4541	137
kitti.05	2761	66
manhattan	3500	1953

VII. EXPERIMENTS

A. Experimental Setup

We test the performance of DEGNC-LAF on five standard PGO benchmarking datasets: city10000, intel, kitti.00, kitti.05 and manhattan, described in [1]. Some information of these datasets are listed in Table 2. We benchmark our approach against (i) GNC [8], (ii) *DEcoupled Graduated Non-Convexity with Rotation Matrix Formulation* (DEGNC-RMF), which is the rotation matrix version of DEGNC-LAF (i.e., the first subproblem in the proposed framework is replaced by TLS-ARA with TLS-RA), (iii) *Realizing, Reversing, and Recovering* (RRR) [5] and (iv) *Dynamic Covariance*

Scaling (DCS) [4]. Since GNC takes too much time on the original city10000 dataset, we select only the first 5,000 poses with corresponding measurements of the dataset. While the datasets we test on contain no outlier initially, we select random pairs of poses and add a loop closure between each pair of poses to simulate outliers.

We performed all experiments in C++ running on a Linux machine with the Intel i7-12700KF (3.60 GHz). We use the latest accelerated version [32] of SE-Sync and SO-Sync (i.e., the rotation averaging version of SE-Sync) to respectively configure GNC and DEGNC-RMF and use 4 threads to drive them, while DEGNC-LAF is implemented with linear solver used in GTSAM [33] using only one thread. The relevant parameters of GNC follow the configuration in [8]. We selected an open source² version of DCS implemented with ceres-solver [15]. We use kernel size $\Phi = 1$ for DCS and 4 threads to drive the ceres-solver, and set the maximum number of iterations to 100, keeping default settings in the source code for all other parameters. We use the RRR algorithm implemented by the authors and set the clustering threshold $\gamma = 10$.

²<https://github.com/gisbi-kim/toy-robust-backend-slam>

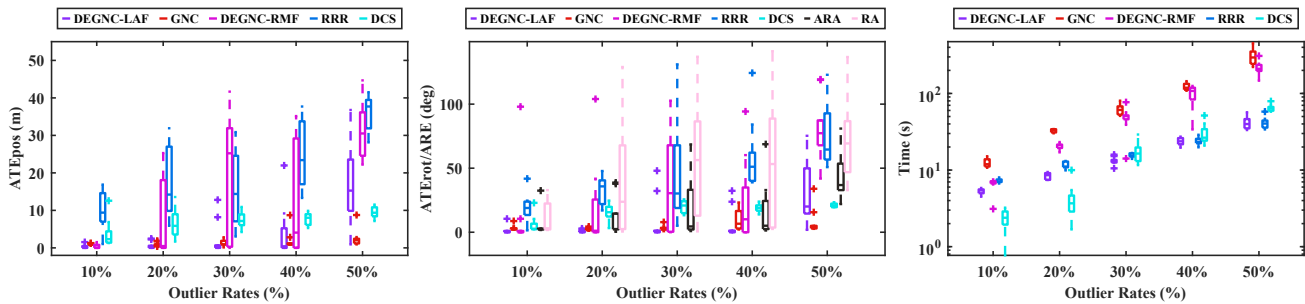


Fig. 4. **manhattan.** Average Trajectory Error (ATE), Average Rotation Error (ARE) and the running time of the proposed approach compared to state-of-the-art techniques on the manhattan dataset. Statistics are computed over 10 Monte Carlo runs.

B. Results

We evaluate the algorithms by Average Trajectory Error (i.e., the ATE_{pos} and ATE_{rot} in [34]) and running time. When evaluating ATE_{rot} we also show Average Rotation Error (ARE) of the solutions to TLS-ARA and TLS-RA with respect to optimal rotation estimates obtained by $SO-Sync$ in the absence of outliers, which are labeled as "ARA" and "RA" respectively in the figures.

city10000. Fig. 1 shows the performance of each algorithm on the city10000 dataset. DEGNC-LAF and GNC dominate the other algorithms in terms of accuracy and robustness against outliers. At the same time, DEGNC-LAF is the fastest approach in most cases, except for being slightly slower than DCS at the outlier rate of 10%. Although GNC can achieve similar accuracy to DEGNC-LAF, it is sometimes over 30 times slower than the latter, since it consumes too much time to compute optimality certificates.

intel. Fig. 2 shows the performance of each algorithm on the intel dataset. On this dataset, both DEGNC-LAF and GNC can achieve moderate robustness and accuracy. Despite this, DEGNC-LAF runs 4-7 times faster than GNC. This dataset allows DCS to maintain more stable performance compared to DEGNC-LAF and GNC while running at great speed. This is probably because the intel dataset provides a good initial guess for DCS so that it does not take a few iterations to reach the global minima.

kitti_00 and kitti_05. These two datasets do not provide initial guesses, so we use the ground-truth as the initial guesses for RRR and DCS. Loop closures in these two datasets are very sparse, so each algorithm can perform good efficiency and thus we do not show the running time in Fig. 3. DEGNC-LAF, GNC and DEGNC-RMF perform good robustness and high accuracy on the kitti_00 dataset. In addition, the solution to TLS-ARA always shows small errors on this dataset. DEGNC-LAF outperforms other algorithms on the kitti_05 dataset, being robust to 50% of outliers, while GNC begins to break at 40% of outliers.

manhattan. manhattan is a particularly challenging dataset for robust planar PGO. The odometry measurements in manhattan have large rotational errors, which causes DEGNC-LAF and DEGNC-RMF to break when solving TLS-ARA and TLS-RA in cases of high outlier rates (i.e., 50%), as shown in Fig. 4. Although GNC shows the best robustness on this dataset, it sometimes fails and takes hundreds of seconds at high outlier rates. At outlier rates

below 30%, DEGNC-LAF can achieve competitive accuracy with GNC and outperforms other algorithms, while runs 2-4 times faster than GNC.

C. Discussion

1) *Imperfect accuracy of TLS-ARA:* From the results we can see that the accuracy of solutions to the TLS-ARA problem generally exceeds those of the TLS-RA problem, which can be explained by *Remark 1*. This exhibits the superiority of angle setting to rotation matrices and ultimately results in the better robustness of DEGNC-LAF compared to DEGNC-RMF. Despite the superior accuracy of TLS-ARA to TLS-RA, the solutions to TLS-ARA can not always reach perfect accuracy (e.g., as shown in Fig. 2), which explains why we do not directly adopt the solutions to TLS-ARA as the final estimates. Fortunately, the TLS-TA problem is not too sensitive to the rotation accuracy deficiencies caused by TLS-ARA and thus can usually provide a correct set of outliers under mild rotational errors.

2) *Robustness of DEGNC-LAF vs. GNC:* We note that comparing DEGNC-LAF with GNC in terms of robustness presents opposite results on the kitti_05 and manhattan datasets. This can be explained by the fact that the odometry measurements in kitti_05 have low rotational noise but relatively high translational noise, while in manhattan the opposite is true. In scenarios like kitti_05, DEGNC-LAF can obtain a good estimate of rotation by solving the first sub-problem, allowing the estimate to effectively resist outliers even with high translational noise, and thus performs better robustness than GNC.

VIII. CONCLUSION

We proposed a specialized framework for global outlier rejection over planar pose graph. Decoupling the robust PGO problem and introducing a linear representation for planar rotation are the keys to the proposed framework. This framework requires only linear solvers instead of global non-linear solvers, for which preconditions are necessary to reach global optima and are often computationally intensive. We believe the proposed method can be an effective alternative to the standard and general-purpose GNC algorithm for robust planar PGO. This is especially true when the scenario allows for relatively accurate rotation estimation on odometry or when it is necessary to maintain a dense distribution of loop closures.

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