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Generalization of Impact Response Factors for Proprioceptive Collaborative Robots

Carlos Relaño[1] Daniel Sanz-Merodio[2], Miguel López[2] and Concepción A. Monje[1]

***Abstract*— Physical Human-Robot Interaction(pHRI) requires taking safety into account from the design board to the collaborative operation of any robot. For collaborative robotic environments, where human and machine are sharing space and interacting physically, the analysis and quantification of impacts becomes very relevant and necessary. Furthermore, analyses of this kind are a valuable source of information for the design of safer, more efficient pHRI. In the definition of the first parameter for dynamic impact analysis, the dynamic impact mitigation capacity was considered for certain configurations of the robot, but the design characteristics of the robot, such as the inertia of actuators, were not included. This paradigm changed when MIT presented the "impact mitigation factor" (IMF) with which, in addition to considering the ability of a certain robot to mitigate impacts for every configuration, it was possible to quantify backdriveability by taking the inertia of actuators into account for the calculation of the factor. However, IMF was proposed as a method to analyse floating robots like. This paper presents the Generalised Impact Absorption Factor (GIAF), suitable for both floating and fixed-base robots. GIAF is a valuable design parameter, as it provides information about the backdriveability of each joint, while allowing the comparison of impact response between floating and fixed-base robotic platforms. In this work, the mathematical definition of GIAF is developed and examples of possible uses of GIAF are presented.**

I. Introduction

The physical interaction of a robot with the environment remains a major challenge in robotics, as it plays a key role in interaction in unstructured environments or in human-robot interaction. Today's robotics enables repetitive and high-precision activities to be performed in known environments with ease. This has been of great help in industry by enabling the automation of processes in highly controlled environments. However, as technology and needs advance, some of these tasks involve occasional or repetitive impacts, especially in not fully controlled environments such as human-robot interactions and collaborative workspaces. Analysing how these impacts are mitigated is important because, as in the example above, the impacts are generated and absorbed by the workers or the environment.

There are many different parameters influencing the impact mitigation capacity of a robot, both at link and actuator level as mass-specific/torque ratio and energy efficiency [1]. Therefore, in impact mitigation, gear efficiency and friction are important factors to take into account. Passive dynamic mechanisms can also be found in the literature to minimise impact, such as series elastic actuation (SEA) [2] or variable stiffness actuators (VSAs) [3], and through impedance control [4]. However, these approaches have not demonstrated a sufficiently large bandwidth for highly dynamic movements.

This work focuses on impacts occurring at the end-effector, where the forces generated by the collision propagate through all joints of the robot. Depending on the configuration of the robot, an uneven distribution of these forces could damage the robot and could, for example, affect the production line of a factory. This type of impact has been studied in other works, and methodologies for planning the configuration of the robot joints are presented. In [5], the impact force generated from the interaction is minimised by designing the pre-impact configuration.

However, these parameters do not allow direct quantification of the effect of the inertia of the motors on the inertial backdrivability of the mechanism. For this purpose, the Impact Mitigation Factor (IMF) was developed in [1]. In this work, the authors argue that, in complex mechanisms, the actuator placement and the structure of the robot joint determine the reflected actuator inertias, which ultimately affect the backdrivability. While backdrivability includes both velocity-dependent and inertia-dependent effects [6], the inertia-dependent effects are much more difficult to regulate through closed-loop impedance control [4], [7]. Therefore, these passive inertial effects depend primarily on the reflected inertias of the actuator, determining the degree to which the body inertia contributes to impacts. Therefore, the reflected inertia of the actuator is an important design feature for the higher loads felt in links, gearboxes and other transmission components [1]. However, the IMF methodology presented is focused on floating robots, such as the Cheetah [8], and is not directly applicable to a fixed-based robot. Therefore, it is necessary to find a new factor applicable to both cases, which provides information on the backdrivability and normalised to compare between different robotic platforms.

The structure of the work is as follows. Section II presents a review of existing impact parameters, their mathematical implementation and their applicability, from dynamic shock absorption depending on the robot configuration to the use of these parameters with a robot design approach. A new parameter to analyse impact absorption, the generalized impact absorption factor (GIAF), also solves the problem with fixed base robots. In Section III, examples of the GIAF parameter for a fixed-base commercial robot are analysed. The effect of the inertia and other actuator design parameters on the presented parameter are also investigated. Finally, Section

[1] Systems Engineering and Automation Department, *University Carlos III of Madrid*, Madrid, Spain. {crelano, cmonje}@ing.uc3m.es

[2] ARC Robotics, *Arquimea Research Center*, Santa Cruz de Tenerife, Spain. {dsanz, mlopez}@arquimea.com

IV summarizes the main points studied in the article. From our point of view, this parameter extends the possibilities for the design and use of floating and fixed-base robots and their application in impact planning for physical human-robot interaction.

II. Parameter Definition

A. Preliminary work and Background

This section introduces the theoretical approach to the evolution of impact factors. The first approach of manipulability presented in [9] is based on and extends the concept of the Jacobian matrix. In a robot with n degrees of freedom, the velocity of the end-effector $v \in \mathbb{R}^m$ and the joint velocities $\dot{\theta} \in \mathbb{R}^n$ are kinematically related with the Jacobian matrix J, as follows:

$$v = J\dot{\theta} \tag{1}$$

The *manipulability measure* is a scalar value defined as

$$\omega_m = \sqrt{det([J][J^T])} = \sigma_1\sigma_2...\sigma_m \tag{2}$$

where $[J]$ is the singular value decomposition (SVD) of J and $(\sigma_1\sigma_2...\sigma_m)$ are the singular values of $[J]$.

From this equation, the concept of *manipulability ellipsoid* is defined in $\mathbb{R}^m$ with the principal axes in the direction of the eigenvectors of $[J]$ and the magnitudes of the singular values.

This ellipsoid defines the directions on the end-effector where velocities can be easily generated.

The dual approach based on force/torque ($\tau = J^T F$) provides the *manipulability force measure*, a scalar that defines the directions in which greater static force can be generated.

$$\omega_f = 1/\omega_m = 1/(\sigma_1\sigma_2...\sigma_m) \tag{3}$$

To extend this concept to an impulse contact, in [10] a full dynamic model of a robot with the impulse forces is considered. Assuming a completely inelastic collision with no deformation, the generalized force with this constraint is:

$$\tau_{Const} = J^T\lambda \tag{4}$$

where λ is the vector of the Lagrange multipliers.

Assuming that the bodies in contact are rigid, the collision is modeled with a constant "coefficient of restitution" e, that characterizes the collision type. Thus, by the law of conservation of energy:

$$[(v_1 + \Delta v_1) - (v_2 + \Delta v_2)]^T n = -e(v_1 - v_2)^T n \tag{5}$$

where v_1 and v_2 are the velocity of the body 1 and 2 while Δv_1 and Δv_2 are the changes of v_1 and v_2 after the collision and n is the normal vector to the contact plane.

This coefficient of restitution e will be obtained from the previous and post impact velocities $e = -{v_z}^+/{v_z}^-$ [11]. A value of $e = 0$ means a completely inelastic collision.

Assuming also that constraint force is an impulse collision:

$$\Gamma = J^T\lambda \tag{6}$$

Thus, from the dynamics equation of the robot:

$$H(\theta)\ddot{\theta} + h(\theta,\dot{\theta}) - \tau = \Gamma \tag{7}$$

where the term $h(\theta,\dot{\theta})$ includes gravity and Coriolis effects.

Integrating both sides of this equation into an infinitesimally short time interval:

$$\lim_{\Delta t\to 0}\int_{t_0}^{t_0+\Delta t} H(\theta)\ddot{\theta}dt + \lim_{\Delta t\to 0}\int_{t_0}^{t_0+\Delta t}(h(\theta,\dot{\theta}) - \tau)dt = \lim_{\Delta t\to 0}\int_{t_0}^{t_0+\Delta t}\Gamma dt \tag{8}$$

Under the assumption of an infinitesimally short time interval of collision, the joint position remains unchanged and joint angular speeds are finite so the integral in the second term of the previous equation is canceled and can be written as

$$H(\theta)[\dot{\theta}(t_0+\Delta t) - \dot{\theta}(t_0)] = \int_{t_0}^{t_0+\Delta t}\Gamma dt \tag{9}$$

When $\Delta t \to 0$, the right side of the equation converges to a finite value:

$$\Gamma_\delta = \int_{t_0}^{t_0+\Delta t}\Gamma dt = J^T\int_{t_0}^{t_0+\Delta t}\lambda_\delta dt \tag{10}$$

The speed at the collision point of the robot is

$$\dot{X}_{Coll}(t_0) = J\dot{\theta}(t_0) \tag{11}$$

The velocity variation in the robot's end-effector at impact is $\Delta\dot{X}_{Coll}$. From the previous equations, and taking into account that $JH^{-1}(\theta)J^T$ is invertible, the variation of torque of the robot joints $\Delta\Gamma_\delta$ can be expressed in terms of θ and $\Delta\dot{X}_{Coll}$ as

$$\Delta\Gamma_\delta = J^T(JH^{-1}(\theta)J^T)^{-1}\Delta\dot{X}_{Coll} \tag{12}$$

introducing the *Operational Space Inertia Matrix* (OSIM) as

$$\Lambda = (JH^{-1}(\theta)J^T)^{-1} \tag{13}$$

Therefore, taking into account the coefficient of restitution of the impact (5), the increment of torque felt by the robot joints in the system can be expressed in relation to the OSIM matrix as

$$\Delta\Gamma_\delta = J^T\left(\frac{-(1+e)(\Delta\dot{X}_{Coll})^T n}{n^T\Lambda n}\right) \tag{14}$$

This equation defines the per-joint torque reaction to an impact at the end-effector of the robot. The *dynamic impact measure* is introduced in [10] based on the concepts introduced in (14) as the scalar

$$\omega_{di} = \sqrt{det([J^{+T}][H]^2[J^+])} \tag{15}$$

where J^+ is a pseudoinverse of J.

This parameter ω_{di} models the capacity of the robot to withstand impacts in different directions at the end-effector. And the related *dynamic impact ellipsoid*, defined by the eigenvector and eigenvalues of $([J^{+T}][H]^2[J^+])$, extends the concept of manipulability to dynamic environments.

A similar approach based on the generalised inertia tensor $G = JH^{-1}J^T = \Lambda^{-1}$ can be found in [12], where a *generalized inertia ellipsoid* was presented. An extension of this concept to free-floating links was presented in [9].

B. Impact mitigation factor

In [1], Wensing et al. introduced a new impact metric that not only quantifies the impact absorption capacity of the robot in some position, but also quantifies the backdrivability upon an impact, allowing for design comparison across different robots. This parameter is the *impact mitigation factor* (IMF).

The IMF parameter calculation has been developed based on a floating body system, so the base coordinate system $\theta_b \in \mathbb{R}^6$ and the joint coordinate system $\theta_j \in \mathbb{R}^n$ are defined. Therefore, the dynamics of the system can be completely described, as developed in (7), through

$$\begin{bmatrix} H_{bb}(\theta) & H_{bj}(\theta) \\ H_{jb}(\theta) & H_{jj}(\theta) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_b \\ \ddot{\theta}_j \end{bmatrix} + h(\theta, \dot{\theta}) - \tau = \Gamma \quad (16)$$

From equations 12 and 13, the contact impulse $\rho \in \mathbb{R}^3$ with an impact with velocity $\Delta\dot{X}_{Coll}$ can be written as

$$\rho = -\Lambda \Delta\dot{X}_{Coll} \quad (17)$$

The IMF parameter is then presented as a relationship between the system with all joints locked from the base coordinate system

$$\Lambda_L = (J_b H_{bb}^{-1}(\theta) J_b^T)^{-1} \quad (18)$$

and the OSIM of the system Λ, where $J_b \in \mathbb{R}^{3x6}$, $J = [J_b \; J_j]$.

The contact impulse in a locked system is defined as

$$\rho_L = -(J_b H_{bb}^{-1}(\theta) J_b^T)^{-1} \Delta\dot{X}_{Coll} \quad (19)$$

To compare how free dynamics of the system mitigate the impulse, $\Lambda\Lambda_L$ is defined as a term that determines the inertia that the system feels upon in the impact in comparison with the case of the locked system.

The difference of the impulse in each case is then

$$\rho_L - \rho = (I - \Lambda\Lambda_L^{-1})\rho_L = \Xi\rho_L \quad (20)$$

where $\Xi = I - \Lambda\Lambda_L^{-1}$ is defined as the the impact mitigation matrix and the *impact mitigation factor* is defined as

$$\xi = det(\Xi) \quad (21)$$

The range of this parameter is $0 \le \xi \le 1$. If the IMF $\xi = 1$, the system under analyses has perfect inertial backdrivability *in comparison with the blocked system*. Therefore, the IMF is introduced as a metric to quantify the effectiveness of the actuator and robotics arm design in mitigating impact forces.

In the development of this work, the IMF has been calculated with a Franka Emika robot, which is fixed-base robot. In the calculation of the IMF, the fixed base has been simulated as a very large mass. The calculation result shows that the IMF gives a value of $\xi = 1$ in every position of the robot. This is because, in all cases, the backdrivability of the robot is perfect with respect to a very big mass acting as a floating base for the system at study. The conclusion is that this IMF parameter cannot be directly used for fixed-base robots.

Accordingly, in order to have a dynamic impact parameter that also takes into account the backdrivability of the system and can be applied both to floating and fixed-base robots, a new parameter is proposed in this paper, the *generalized impact absorption factor*.

C. Generalized impact absorption factor

Following the previous IMF factor development, the mass matrix is partitioned into the matrix that accounts for the kinetic energy of the rigid body links H_{rb} and a matrix H_{mot} related with the kinetic energy of the actuator mounted between the links:

$$H = H_{rb} + H_{mot} \quad (22)$$

With this approach, the OSIM matrix becomes

$$\Lambda = (J(H_{rb} + H_{mot})^{-1}(\theta)J^T)^{-1} \quad (23)$$

Developing the free dynamics terms in 23 gives

$$\Lambda_F = (J(H_{rb})^{-1}(\theta)J^T)^{-1} \quad (24)$$

Assuming all joints of the robot to be free, the contact impulse can be written as

$$\rho_F = -\Lambda_F \Delta\dot{X}_{Coll} \quad (25)$$

From (17) and (25) the impact of the robot without rotor inertia in the joints can be rewritten as

$$\rho_F = \Lambda_F \Lambda^{-1} \rho \quad (26)$$

and therefore:

$$\rho - \rho_F = (I - \Lambda_F \Lambda^{-1})\rho = Z\rho \quad (27)$$

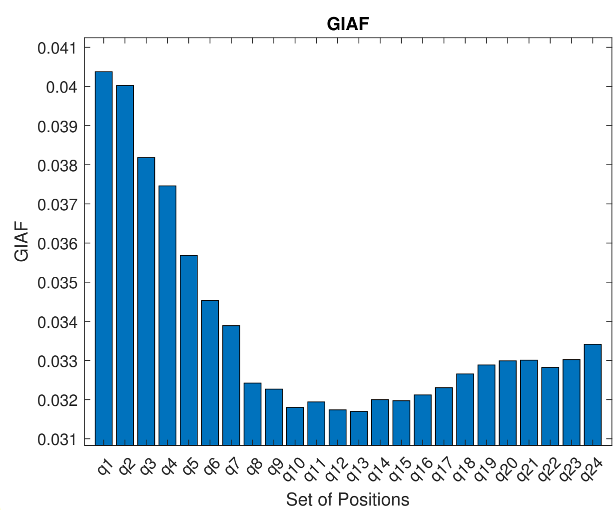

Fig. 1: GIAF results for 24 different redundant joint configurations on the Franka Emika Panda robot.

where $det(Z) = det(I - \Lambda_F \Lambda^{-1})$ is the *generalized impact increment factor* (GIIF) and defines how much the contact inertia of the real robot has increased with respect to the robot without rotor inertia.

The scalar *generalized impact absorption factor* (GIAF) ζ is then defined as

$$\zeta := det(\Lambda_F \Lambda^{-1}) \tag{28}$$

The range of the GIAF is $0 \leq \zeta \leq 1$. A value of GIAF $\zeta = 1$ defines a system whose joints can move with no inertia, while $\zeta = 0$ defines a system with the joints completely locked by the inertia of the actuators. Therefore, the GIAF is defined taken into account the rotor without inertia, that means the free dynamics of the robot instead of in relationship with the locked system, as in the IMF.

In summary, the GIAF is a valuable metric for evaluating the effectiveness of impact mitigation designs in every class of dynamic robot. The closer value of GIAF to one, the more significant the reduction in impact forces and the lower the risk of injury or damage.

An extension of this parameter, of particular interest for the design and control of robots, is the *directional GIAF (dGIAF)*, which determines the system's ability to absorb impacts in a given direction. The *directional GIAF*, given a direction $\mathbf{x}$, is defined as

$$\zeta_x = \frac{\mathbf{x}^T \Lambda_F \mathbf{x}}{\mathbf{x}^T \Lambda \mathbf{x}} \tag{29}$$

The range of this parameter is also $0 \leq \zeta_x \leq 1$. Thus, ζ_x evaluates how close the movement in a certain direction is to the free dynamics.

III. Parameter Analysis and Results

Throughout this section, the application of the GIAF parameter presented above will be analysed. The study of this parameter will be carried out using a Franka Emika collaborative robot, a 7 DoF robot commonly used in research. This analysis is focused on the results provided by the GIAF in different configurations. These 24 different joint configurations on the Franka Emika Panda are chosen using the redundancy of the 7 DoF manipulator to explore configurations while preserving the same Cartesian position for the end-effector. The position selected in the experiments can be found in Appendix A.

This analysis is divided into two sections. First, the study shows how the GIAF identifies which configuration responds better to an impact. This calculation is done within the space of redundant joints with the end-effector in a fixed position. Second, an extrapolation of the characteristics of the Franka robot is performed. The behavior of the GIAF is analysed after a theoretical modification of the reflected inertia of the joints. The response upon impact after a theoretical change in the gear ratio of the joints is also analysed.

A. GIAF analysis

Figure 1 shows the variation of the GIAF for the positions shown in Appendix A, using the actuators reflected inertia provided by Franka. The GIAF quantifies the impact absorption capacity for redundant positions, thus allowing to choose safer configurations. The reflected inertia can be modified by adapting the position of the center of mass and the inertia matrix of the links. From this calculation, extrapolating the values of reflected inertia, the upper graph in Figure 2 shows how the GIAF parameter provides, in addition to the ability to absorb impacts from redundant configurations, the capability to quantify the backdrivability upon impact, since it yields higher values if the system has less inertia and lower values if a greater reflected inertia is assumed.

Following the same methodology, the lower graph in Figure 2 analyzes how the GIAF, and therefore the backdrivability of the system, would be affected by a supposed increase or decrease in the gear ratio of the actuators on the joints. The graphs show that increasing the gear ratio decreases the GIAF, while decreasing the gear ratio increases the GIAF values. Therefore, there is a great potential of the usability of the GIAF as an information-based design tool.

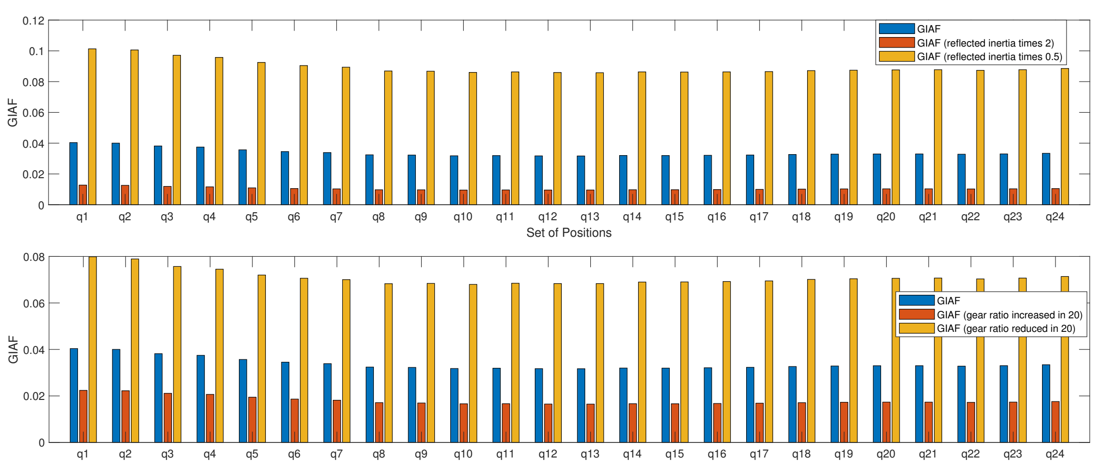

Fig. 2: GIAF response to reflected inertia and the ratio of gear-box of the joint modification.

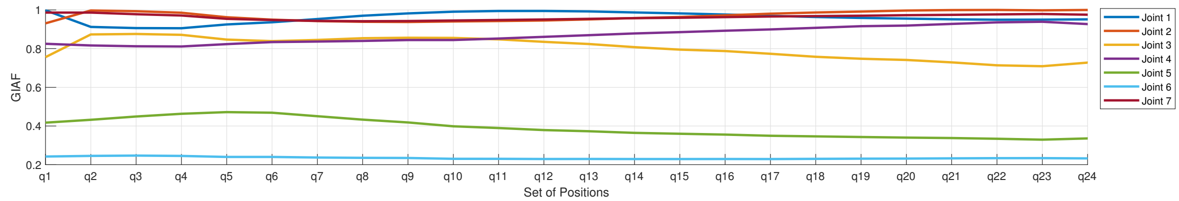

Fig. 3: GIAF value characterising the effect of each joint in the redundant configurations described in Appendix A.

B. GIAF for joints analysis

The GIAF provides valuable information to investigate the individual effect of each joint in the capacity to absorb impact energy for redundant configurations. The methodology proposed characterises the effect of per-joint reflected inertia on the robot's capability to absorb impacts. The aim is to investigate which joints have more influence and need to absorb more load upon impact for a certain configuration of the robot. With this purpose, the reflected inertia of all other joints is set to zero and the GIAF parameter is calculated.

This procedure is performed for every joint in the system. The closer the GIAF gets to one, the greater the energy absorption for that specific joint upon impact. Figure 3 shows the result of applying this method to the redundant configurations presented in Appendix A. It can be observed that joints five and six are the ones that have the lowest GIAF values, concluding that the reflected inertias of these actuators are the most relevant in impact mitigation for these joint configurations. These joints would be the ones to receive the highest load at the moment of impact. Therefore, these are the first joints whose inertia should be analysed in order to reduce the effects of impact. A redesign of these joints could, for example, improve safety in human-robot interactions. This strategy is directly applicable to joints of variable stiffness when absorbing impacts, providing additional value to what is presented in [3].

C. dGIAF analysis

Finally, as proposed in the previous section, it is possible to perform an analysis through GIAF of the effect of impacts in a given direction x. It can be of interest in cases where repetitive impacts in a particular direction occur. Figure 4 shows a polar representation of the dGIAF in two different planes (XY plane and XZ plane) for configuration $q1$. It can be seen in both planes how the dGIAF value varies according to the direction of impact in each plane. In the XY plane, for instance, it can be seen that if the impact occurs in the 150° direction, the dGIAF is higher than in the case of the 90° direction. Therefore, the dGIAF is demonstrated to be a relevant indicator for the analyses of the impact direction effect.

IV. Conclusion

This work presents a new generalised factor for impact analysis in collaborative robots, GIAF. It provides a new perspective on the paradigm of analysing the effects of the actuator inertia on impact, both for floating and fixed-base robotic platforms. The reflected inertia of actuators is a very relevant design parameter when it comes to the analysis of the effect caused by impacts on robots, and the GIAF provides valuable information to this respect. It allows the designer to quantify the effect of the impact on the end-effector and to compare the performance of different joint configurations, providing information about the backdrivability of the system both in floating and fixed-base configurations.

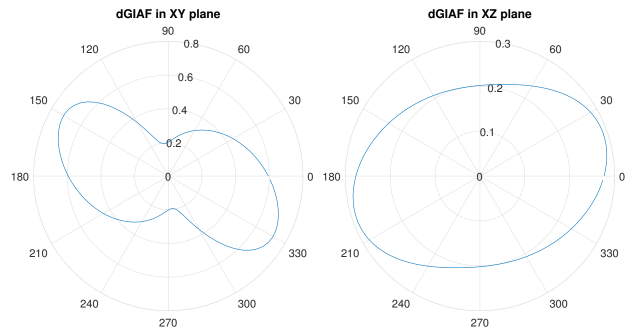

Fig. 4: Representation of the dGIAF in the xy and xz planes, for configuration $q1$.

Theoretical examples of the dynamics of a real robot, Franka Emika Panda, are presented to analyse the effects of varying the reflected inertia of the actuators. Furthermore, it is possible to analyse these effects for a given impact direction and a methodology is proposed to analyse the individual effect on each joint.

These results are very promising and future research steps will focus on the experimental validation of the approach using real robots.

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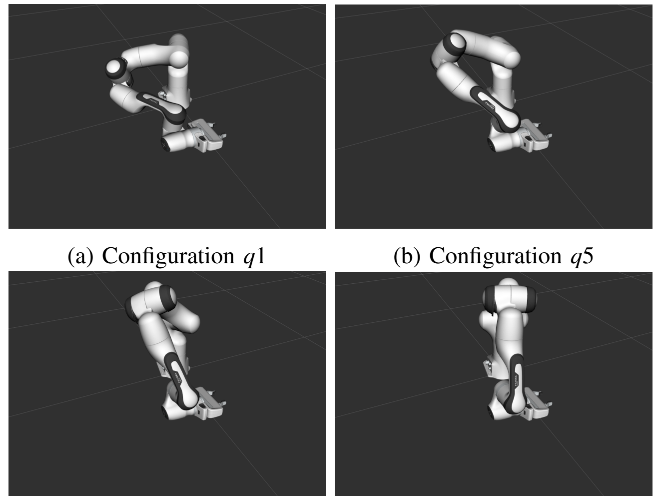
Appendix

In this appendix, the 24 different joint configurations used for the GIAF calculation are presented. Table I lists the corresponding joint angles in radians considering the comercial Franka Emika robot. Figure 5 shows some of the joint configurations in a simulated 3D environment.

TABLE I: Franka robot's joint angle values for redundant positions in impact factor calculation

Config.	Joint1	Joint2	Joint3	Joint4	Joint5	Joint6	Joint7
q1	-2.01	1.76	0.96	-2.44	-0.51	2.15	-1.06
q2	-1.92	1.66	0.98	-2.46	-0.35	2.2	-1.17
q3	-1.75	1.38	0.98	-2.46	0.03	2.23	-1.48
q4	-1.66	1.27	0.96	-2.46	0.21	2.23	-1.61
q5	-1.57	1.15	0.94	-2.46	0.38	2.2	-1.73
q6	-1.48	1.05	0.89	-2.44	0.52	2.15	-1.85
q7	-1.4	0.98	0.84	-2.43	0.63	2.09	-1.92
q8	-1.31	0.89	0.79	-2.43	0.72	2.02	-1.99
q9	-1.22	0.84	0.72	-2.41	0.79	1.97	-2.04
q10	-1.13	0.79	0.65	-2.41	0.86	1.92	-2.09
q11	-1.05	0.75	0.58	-2.39	0.91	1.87	-2.13
q12	-0.96	0.72	0.49	-2.39	0.96	1.82	-2.15
q13	-0.87	0.7	0.42	-2.39	0.99	1.78	-2.18
q14	-0.79	0.66	0.35	-2.37	1.03	1.73	-2.2
q15	-0.7	0.65	0.26	-2.37	1.06	1.68	-2.23
q16	-0.61	0.65	0.17	-2.37	1.1	1.64	-2.25
q17	-0.52	0.63	0.1	-2.37	1.13	1.61	-2.25
q18	-0.44	0.63	0.02	-2.36	1.15	1.55	-2.27
q20	-0.26	0.65	-0.16	-2.36	1.22	1.47	-2.3
q21	-0.16	0.65	-0.24	-2.36	1.24	1.41	-2.32
q22	0.00	0.63	-0.1	-2.36	1.19	1.5	-2.29
q23	0.17	0.65	-0.26	-2.36	1.24	1.41	-2.32
q24	0.35	0.68	-0.42	-2.36	1.31	1.33	-2.34



(a) Configuration $q1$ (b) Configuration $q5$

(c) Configuration $q11$ (d) Configuration $q19$

Fig. 5: Franka robot's selected redundant positions for impact factor calculation

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