2023 IEEE International Conference on Robotics and Automation (ICRA 2023)
May 29 - June 2, 2023. London, UK

Loosely-coupled localization fusion system based on track-to-track fusion with bias alignment

Soyeong Kim[1] Jaeyoung Jo[1] Paulo Resende[2] Benazouz Bradai[2] and Kichun Jo[1]

Abstract— **The localization system is an essential element in robotics, which can provide accurate position information. Multiple localization systems can be integrated for reliable localization operations because there are various methods for measuring the position or processing algorithms. Significantly, the track-to-track (T2T) fusion method can fuse multiple localization systems using each system's estimate without accessing the sensor's low data. However, most T2T fusion-based localization systems ignore slowly varying biases, such as drift errors, odometry errors, and offsets among multiple maps. This can degrade the localization performance because a slowly varying bias is directly reflected in the localization estimate. Therefore, a slowly varying bias must be considered in the fusion process to derive reliable estimates. This study proposes a T2T fusion-based localization system that considers a slowly varying bias. First, the slow-varying bias difference between the systems was estimated. Because each localization system can have a different bias, the estimated bias difference was used to align it with the reference system. Second, a fused estimate can be obtained by T2T fusion using bias-aligned estimates. The proposed fusion system can also be used without limiting the number of inputs to the localization system. The proposed system was compared with various T2T-based localization fusion algorithms for verification in a simulation environment, and it exhibited the best performance in RMSE error comparison.**

I. INTRODUCTION

In intelligent transportation systems, various localization systems exist for accurate positioning. Many methods to measure the position of a vehicle are found, such as global positioning system (GPS), inertial navigation system (INS), light detection and ranging (LiDAR) sensor-based map-matching localization, and high-definition (HD) map-matching localization. Each system has advantages and disadvantages, depending on the characteristics of the sensor or algorithm. For example, because HD map matching localization predominantly utilizes lane information, it is vital in lateral position estimation, but its performance for longitudinal estimation is weak. For LiDAR map matching localization, it can perform robust localization for an area with many surrounding topographical features. However, its performance deteriorates in plain areas without any features. The weaknesses of each system can be addressed by integrating multiple localization systems to derive reliable estimates.

This research was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2019R1G1A1099806, and No.2020R1C1C1007739).

[1]Soyeong Kim, Jaeyoung Jo and Kichun Jo are with the Department of Smart Vehicle Engineering, Konkuk University, 120, Neungdong-ro, Gwangjin-gu, Seoul, Republic of Korea. kichun@gmail.com
[2] Paulo Resende and Benazouz Bradai are with the Driving Assistance Research Center, Valeo, CEDEX 93012 Bobigny, France. paulo.resende@valeo.com

Loosely coupled fusion can obtain the best estimate by fusing the estimated states of multiple systems and covariance data. The track-to-track (T2T) fusion method is applied for loosely fusing multiple localization systems, mainly used in the object tracking field. T2T fusion joins the locally estimated track in individual systems to derive more accurate results with reduced uncertainty. Multiple localization systems can be integrated without any access to the small amount of data in the system using T2T fusion. It has been applied in various fields, such as cooperative perception [1], mapping [2], SLAM [3], and localization fusion [4], [5], [6], [7]. Specifically, localization fusion fields integrate various localization systems considering the system uncertainty modeled as a white-noise Gaussian model. However, these studies did not consider the slowly varying bias reflected in the localization estimates at low frequencies. A GPS-based localization system has various bias error components, such as drift and multipath errors. For a map-matching localization system, maps are created under different conditions, and an offset can exist between multiple maps, such as calibration, drift, or odometry errors. This offset between the multiple maps can cause localization bias errors. Owing to this slowly varying bias, problems arise when T2T fusion is directly applied to a localization fusion system. First, the assumption of T2T fusion cannot be satisfied because bias is not considered for the exact location. Second, a slowly varying bias is directly reflected in the results of the localization system, which degrades localization performance.

This paper proposes a T2T fusion-based localization fusion system that considers the slowly varying bias reflected in localization. There are two steps for fusing the localization system for accurate and reliable positioning considering slow-varying bias: 1) slow-varying bias alignment with the Kalman filter (KF), and 2) localization fusion with various T2T fusion methods. The first step estimates the bias difference between the reference and target based on the KF. We aligned the target bias with the reference bias using the estimated bias difference. Second, T2T fusion was conducted to fuse the localization system with a bias-aligned estimate. In addition, various T2T fusion techniques have been applied to derive the optimal fusion methods. Finally, we obtain more accurate and reliable localization estimates. We conducted simulation-based verification to verify the proposed system. The validity of this study was verified through a comparative analysis with a localization fusion system that did not consider slow-varying biases. In addition, various T2T techniques were analyzed for optimal localization fusion.

The contributions of this study are as follows.

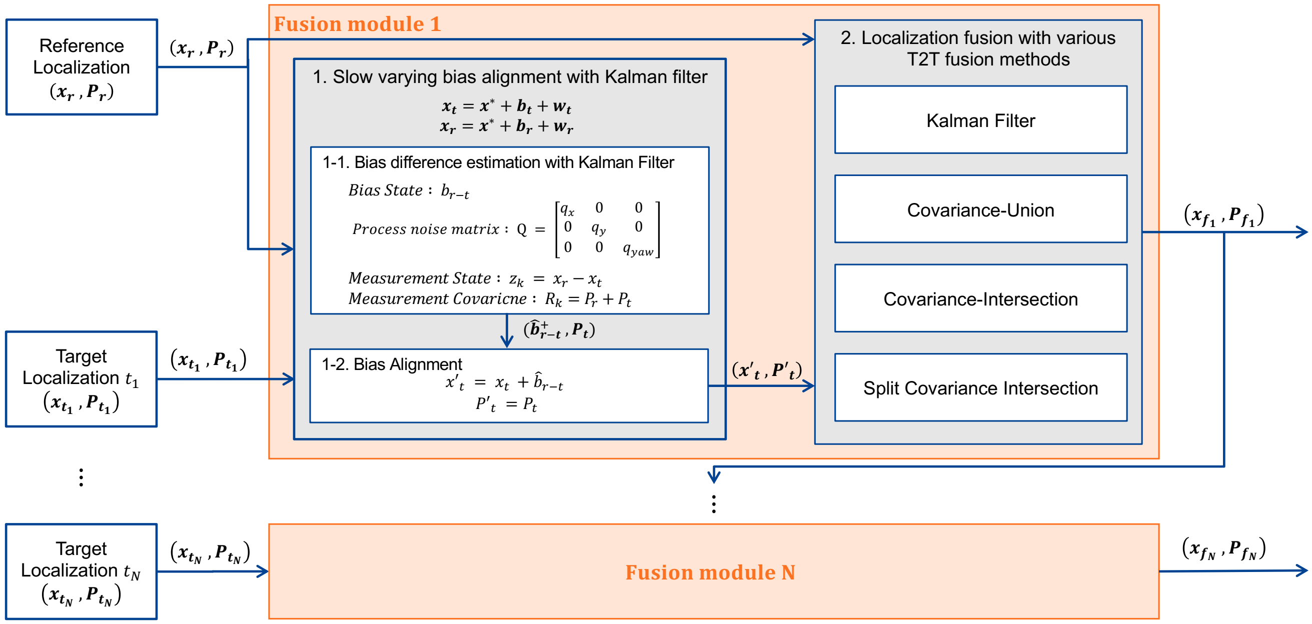

Fig. 1. The proposed system architecture. The main processes are divided into two parts: 1) slow varying bias alignment with Kalman filter (KF) and 2) localization fusion with various T2T fusion methods. In the first part, slow varying bias is estimated and aligned based on the KF, resulting in aligned target localization data. In the second part, localization fusion is conducted with various T2T fusion methods using aligned target and reference localization data. Additionally, the fusion of two or more localization systems is possible through the above structure because it has scalability.

- **Low-frequency bias alignment:** The localization bias error can be reduced by estimating and correcting the slow varying bias, which can be reflected in the localization estimates.
- **High-frequency noise correction:** By adopting the optimal T2T fusion technique, an accurate and reliable localization estimate can be obtained.
- **Flexible localization fusion system:** The number of localization systems applied in a fusion system is not restricted. Only a selection of target and reference localization systems is required.

The remainder of this paper is organized as follows. In Section II, the problem is formulated, and notations are introduced. Section III presents the system architecture. In Sections IV and V, we describe the proposed algorithm in detail. Section VI describes the verification results, and we conclude the paper in the final section VII.

II. Problem Description

This study aimed to fuse localization estimates from various systems. There are two types of localization systems: target and reference. Each target localization system can be aligned and fused to a reference localization system. Both localization systems have white Gaussian noise and a slowly varying bias error. Each system's estimated state and covariance can be obtained as x_t and P_t for the target system and x_r and P_r for the reference system. Ultimately, we wish to obtain the fused localization state x_f and covariance P_f.

The input states x_r and x_t can be modeled as follows:

$$\begin{aligned} x_r &= x^* + b_r + w_r, w_r \sim N(0, P_r) \\ x_t &= x^* + b_t + w_t, w_t \sim N(0, P_t). \end{aligned} \tag{1}$$

where x_r and x_t are state vectors that can be constrained by $SE(2)$ and $SE(3)$. For $SE(2)$, the state vectors are x, y, yaw in 2D space and x, y, z, yaw in 3D space for $SE(3)$. w_r and w_t are the system noise with covariances P_r and P_t respectively. x^* represents the true state of the localization. b_r and b_t denote the slowly varying biases in the reference and target localization systems, respectively.

Because each system has a different bias, it must be aligned to obtain a fused estimate for the same state. By estimating the difference b_{r-t} in the bias and compensating for the target state, the target state can obtain the same bias as the reference state. The bias difference b_{r-t} can be estimated using the bias estimation function

$$b_{r-t} = BiasEstimation(x_r, P_r, x_t, P_t) \tag{2}$$

When we obtain the bias difference b_{r-t}, the target bias can be aligned to the reference bias with the summation of the estimated bias difference b_{r-t}. The bias-aligned target state x'_t can be obtained as follows:

$$x'_t = x^* + b_t + b_{r-t} + w_t \tag{3}$$

Because the $b_t + b_{r-t}$ term is aligned with b_r, the bias-aligned target state is arranged as follows:

$$x'_t = x^* + b_r + w_t. \tag{4}$$

Finally, T2T fusion is applied to fuse the reference and bias-aligned target estimates. Our ultimate objective is to estimate the true state accurately and reference bias $x^* + b_r$. In a T2T fusion scheme, the fused state and covariance are determined as follows:

$$\begin{aligned} x_f &= T2T_x(x_r, P_r, x'_t, P'_t) \\ P_f &= T2T_P(x_r, P_r, x'_t, P'_t). \end{aligned} \tag{5}$$

where $T2T_x$ and $T2T_P$ are the T2T fusion functions, which were set as the optimum method in Section V.

III. System overview

The overall system comprises two parts, as shown in Fig. 1. The first step was a slow-varying bias alignment with the KF. This step is subdivided into the bias difference estimation using the KF and the bias alignment step. In the bias difference estimation step, the difference between the target and reference biases can be estimated using the estimate's state and covariance. The KF estimates the bias difference considering the system noise. Subsequently, the target bias is aligned using the estimated bias difference from the reference bias in the bias-alignment step. Consequently, we obtained a bias-aligned target state and covariance.

The second step involves localization fusion using various T2T fusion methods. This step employs bias-aligned localization and reference localization data as inputs. Four T2T fusion methods were applied to integrate bias-aligned target and reference localization systems, and the optimal T2T method was derived after a comparative analysis. Finally, the fused localization state and covariance are obtained as the output.

IV. Slow varying bias alignment with the Kalman filter

We estimated the slowly varying bias differences between the target and reference estimates. A filter that considers the uncertainty of each estimate is needed to estimate the slowly varying bias differences. The KF can estimate system noise characteristics using the Q matrix, which represents the covariance of the system noise. Therefore, we adopted the KF to estimate the bias difference between the target and reference biases. Subsequently, by aligning the target bias with the reference bias using the estimated bias difference, it was possible to estimate the same true state, including the same bias.

A. Bias difference estimation with the Kalman Filter

Bias estimation with the KF was conducted using the state and covariance results of the target and reference localization systems. The KF for estimating the slowly varying bias is modeled as follows:

$$\begin{aligned} \hat{b}_{r-t,k} &= F_k \hat{b}_{r-t,k-1} + w_{r-k,k} \\ P_{r-t,k} &= F_k P_{r-t,k-1} F_k^T + Q_{r-t,k-1} \end{aligned} \tag{6}$$

where F_k is the identity matrix because the bias is a slowly varying variable, $\hat{b}_{r-t,k}$ is the predicted state vector at time step k, and $w_{r-k,k}$ denotes the process noise. $Q_{r-t,k-1}$ is the covariance of process noise, which must be tuned to an adequate value by considering the system noise characteristics.

The measurement can be modeled as:

$$z_{r-t,k} = H_k b_{r-t,k} + v_{r-t,k} \tag{7}$$

where $z_{r-t,k}$ is the measurement state and $v_{r-t,k}$ is the observation noise. H_k is set as an identity matrix because the measurement and state are equivalent. The difference between the states of the two localization systems must be modeled as a measurement model to estimate the slow-varying bias as follows:

$$\begin{aligned} z_k &= x_{r,k} - x_{t,k} \\ R_k &= P_{r,k} + P_{t,k}. \end{aligned} \tag{8}$$

The measurement state z_k and covariance R_k are represented as the summation of the reference and target estimates. Because two or more random variables are utilized as inputs, they can be treated as multivariate normal-distribution problems. If the two variables are independent, then the following formula is established [8]:

$$\begin{aligned} X \sim N(\mu_x, \sigma_x^2), Y \sim N(\mu_y, \sigma_y^2) \\ X + Y \sim N(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2) \end{aligned} \tag{9}$$

By applying the above system and measurement models, we obtain the estimated bias state $\hat{b}_{r-t}$ and covariance P_{r-t}.

B. Bias Alignment

The slowly varying bias in the target localization estimate can be aligned to the reference bias using the estimated bias information obtained in the previous step. Given the estimated bias state $\hat{b}_{r-t}$ and covariance P_{r-t}, the bias-aligned localization state x_t' and covariance P_t' are calculated as follows:

$$\begin{aligned} x_t' &= x_t + \hat{b}_{r-t} \\ P_t' &= P_t + \hat{P}_{r-t} \end{aligned} \tag{10}$$

Because both the estimated bias state $\hat{b}_{r-t}$ and the target state x_t are random Gaussian distributions, the target bias can be aligned to the reference bias with a simple summation. The covariance of the estimated bias can be set as the summation of the target and bias covariances because they are independent random variables. Consequently, the bias-aligned target localization estimate can be aligned with the same formation as the reference estimate in (1).

V. Localization fusion with T2T fusion methods

Different localization systems are fused in this step using T2T fusion methods with the aligned target and reference localization estimates as inputs. Furthermore, various T2T techniques are applied and analyzed to select the optimal fusion algorithm. Four T2T fusion techniques have been introduced and proven in many research areas: KF, covariance intersection filter, covariance union filter, and split covariance intersection filter.

A. Kalman Filter

The KF has been applied in various research fields, such as dynamic filtering and sensor fusion. Here we treat the KF as the fusion method. It does not consider the correlation between estimates; therefore, it only derives the optimal fusion results when the estimates are independent. However, if there is a correlation between the estimates, fusion consistency is not guaranteed, and optimistic fusion results are produced.

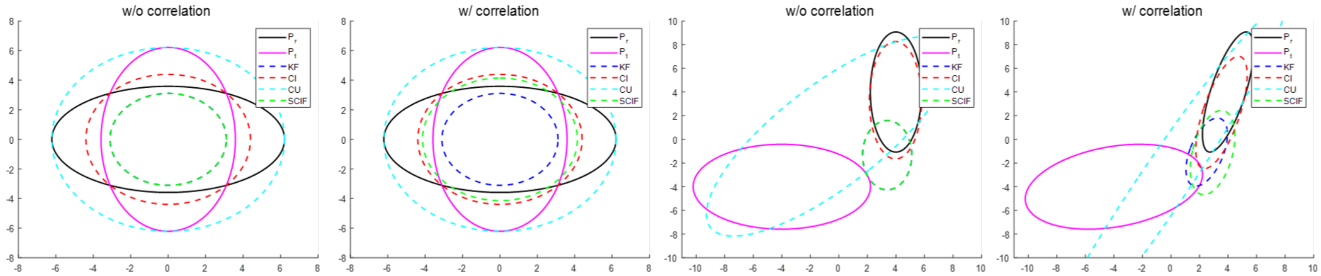

Fig. 2. Covariance ellipses of P_r (black line) and P_t (magenta line). The dotted lines represent fusion results: blue is KF, red is CIF, cyan is CUF, and green is SCIF. The first and third plots represent the fusion cases with no correlation. The second and fourth plots show a fusion case with correlation.

The fused estimate f can be determined using the following formula for the bias-aligned and reference localizations as inputs:

$$\begin{aligned} P_f^{-1} &= P_r^{-1} + P_t'^{-1} \\ x_f &= P_f(P_r^{-1}x_r + P_t'^{-1}x_t'). \end{aligned} \tag{11}$$

B. Covariance Intersection Filter

The covariance intersection filter (CIF) combines tracks with unknown correlations, assuming a correlation exists between the estimates. The covariance intersection filter neglects the independent information between estimates because it treats them as fully correlated. This leads to pessimistic and conservative results that generate a larger covariance than the true data. In addition, if the difference in the estimates exceeded the covariance boundary, biased results were obtained for small covariances.

The fused estimate f for the covariance intersection filter is determined as follows:

$$\begin{aligned} P_f^{-1} &= \omega P_r^{-1} + (1-\omega)P_t'^{-1} \\ x_f &= P_f(\omega P_r^{-1}x_r + (1-\omega)P_t'^{-1}x_t') \end{aligned} \tag{12}$$

where

$$\omega = argmin(det(P_f)). \tag{13}$$

ω can be computed by using an optimization function to minimize $det(P_f)$, as proposed in [10].

C. Covariance Union Filter

The covariance union filter (CUF) was first introduced in [11], which deals with all source data uncertainties. It can fuse two inconsistent estimates by covering the region present in both ellipses of the two estimates. However, overly conservative estimates can be obtained because they provide consistent estimates for all possible distributions.

The unified fused results were obtained using the following formula:

$$\begin{aligned} U_r &= P_r + (x_f - x_r)(x_f - x_r)^T \\ U_t &= P_t + (x_f - x_t')(x_f - x_t')^T \end{aligned} \tag{14}$$

$$P_f = max(U_r, U_t) \tag{15}$$

$$x_f = argmin(det(P_f)). \tag{16}$$

The fused state x_f is determined by optimizing (16) to minimize the determinant of P_f.

D. Split Covariance Intersection Filter

The covariance intersection and the KF guarantee fusion consistency when applied under appropriate circumstances. The KF makes preserving the correlation information challenging because it treats only independent information. The CIF provides a pessimistic estimate because it neglects possible independent information. Both method do not guarantee fusion consistency when the correlation is unknown. The split covariance intersection filter (SCIF) was introduced in [13] to overcome the drawbacks of the Kalman and CIF. The SCIF is under the assumption of the split form $\{X, P_d + P_i\}$, where covariance component P_d means the maximum degree of correlation with others, and P_i implies the degree of its absolute independence.

Given two estimates, $\{x_r, P_{rd} + P_{ri}\}$ and $\{x_t, P_{td} + P_{ti}\}$, the fused results can be calculated as follows:

$$\begin{aligned} P_r &= P_r d/\omega + P_r i \\ P_t &= P_t d/(1-\omega) + P_t i \end{aligned} \tag{17}$$

$$P_f^{-1} = P_r^{-1} + P_t^{-1} \tag{18}$$

$$x_f = P_f(P_1^{-1}x_r + P_2^{-1}x_t') \tag{19}$$

where ω is determined by optimizing the objective function that minimizes the determinants of P_f.

E. Comparative study of fusion methods

Comparative studies were conducted using covariance ellipse visualization to select the optimal T2T method. A comparative analysis was conducted by focusing on the location and size of the ellipse, which represents the fused state and covariance. Confidence can be estimated from the size of the ellipse; the smaller the size, the higher the confidence of the estimate.

Regarding T2T fusion, keywords were related to overconfidence and underconfidence. The fusion consistency can be violated for overconfidence because the fused result obtains more confidence than the input estimates. Conversely, overly conservative results are obtained for underconfidence. Ultimately, the goal is to obtain a fusion result for an appropriate ellipse that is not overly conservative and maintains the consistency of the two estimates. We set four estimated fusion scenarios to analyze the four fusion methods to select the optimum T2T method, as shown in Fig.2. The first and

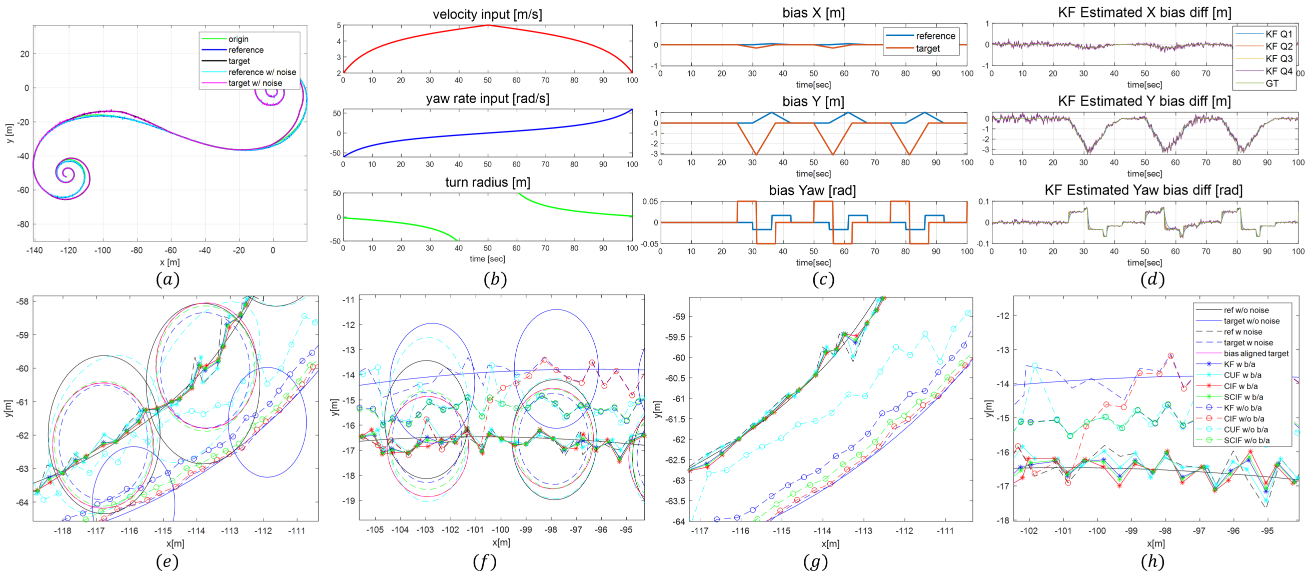

Fig. 3. (a) Simulation trajectory setup for bias case 2 in the table I and Q2 for Q matrix in table II. (blue: reference trajectory, black: target trajectory, cyan: reference trajectory with noise, magenta: target trajectory with noise) (b) Simulation vehicle input setup. (c) Offset setup. (blue: reference offset, red: target offset) (d) Bias estimation results for X, Y, and yaw. (e)-(h) T2T Fusion results. (e) and (f) also show the covariance of estimates as ellipses. (black solid line: reference trajectory (ground-truth), blue solid line: target trajectory, black dashed line: reference trajectory with noise, blue dashed line: target trajectory with noise, a blue solid line with a star: KF, yellow solid line: bias alignment result, a cyan solid line with the star: CUF, a red solid line with the star: CIF, a green solid line with the star: SCIF, every solid line with a circle: T2T fusion without bias alignment.)

second plots represent the fusion of two estimates with the same mean without and with correlation, respectively. The third and fourth plots display cases with different means, without and with correlation, respectively.

The KF and SCIF have the same result if there is no correlation. If there is a correlation and the means are equal, as shown in the second plot, the KF ellipse forms a smaller ellipse than P_r and P_t since correlation is ignored. In contrast, CIF and SCIF create consistent ellipses considering the correlation. However, if the two estimates are far from the covariance radius (third and fourth plots), looking at the result of CIF, the covariance is excessively skewed toward the small estimate. In all four plots, it can be seen that SCIF exhibits good fusion results without excessive convergence to one side or violation of the fusion consistency. Therefore, SCIF was selected as the appropriate fusion method for localization fusion.

VI. Verification

The purpose of the verification was to evaluate the performance of bias alignment and T2T fusion in a simulation environment and to select the optimal algorithm through the analysis of various T2T algorithms.

A. Environmental setup

Since roads in the real world have various curvatures, such as straightforward, steep, or gentle curves, we generated trajectories with different turning radii and velocities to evaluate the proposed algorithm.

Both the reference and target trajectories exhibit high-frequency noise and low-frequency bias. The verification in simulation is performed with a time interval of 0.1 seconds and time stamps of 1000 times. The high-frequency noise in each localization estimate followed a normally distributed random variable proportional to the covariance matrix of the estimate.

TABLE I

Various cases for bias period and max bias size

		Reference		Target	
	Period	x	y	x	y
Case 1	100 ms	1.250 m	0.062 m	0.416 m	0.003 m
Case 2	250 ms	3.125 m	0.156 m	1.041 m	0.052 m
Case 3	500 ms	6.250 m	0.312 m	2.083 m	0.104 m

TABLE II

Q matrix configuration

	$q_x(m)$	$q_y(m)$	$q_{yaw}(degree)$
Q1	0.15	0.15	1
Q2	0.25	0.25	2
Q3	0.35	0.35	3
Q4	0.45	0.45	5

Table I shows various simulation cases for the period and magnitude of the slow varying bias added to the target trajectory. As shown in (6), the Q matrix in the KF for bias alignment must be tuned to an appropriate value. In this simulation, the Q matrix was set as a diagonal matrix, and each element q_x, q_y, and q_{yaw} was set to various values, as listed in Table II.

B. Verification results

The bias alignment and T2T fusion results were compared with a ground truth trajectory (reference trajectory without noise) to evaluate the performance of the proposed algorithm and calculate the RMSE (root mean square error).

Slow varying bias estimation Fig. 3(d) shows the estimated results of slow varying bias using the KF for various Q matrix values. The larger the value of the Q matrix, the closer

TABLE III
COMPARISON OF FUSION RESULTS FOR EACH T2T ALGORITHM FOR VARIOUS SIMULATION CASES IN TABLE I
(NONE: RESULTS OF BIAS ALIGNMENT, KF: KALMAN FILTER, CIF: COVARIANCE INTERSECTION FILTER, CUF: COVARIANCE UNION FILTER, SCIF: SPLIT COVARIANCE INTERSECTION FILTER)

		RMSE(m)				
Bias period	T2T method	Q1	Q2	Q3	Q4	Only T2T
Case 1	None	0.2577	0.2358	0.2241	0.2172	-
	KF	0.1846	0.1806	0.1871	0.1951	**0.3064**
	CIF	0.2190	0.1977	0.1869	0.1969	0.4199
	CUF	0.2533	0.2464	0.2417	0.2406	0.3455
	SCIF	0.1990	**0.1799**	0.1838	0.1936	0.3109
Case 2	None	0.2670	0.2367	0.2248	0.2193	-
	KF	0.1993	0.1880	0.1952	0.2046	1.020
	CIF	0.2339	0.2037	0.1896	0.2026	1.2147
	CUF	0.2534	0.2494	0.2468	0.2440	**0.8371**
	SCIF	0.2160	**0.1865**	0.1913	0.2031	1.0934
Case 3	None	0.2724	0.2314	0.2142	0.2051	-
	KF	0.1912	0.1755	0.1787	0.17870	0.7630
	CIF	0.2300	0.1960	0.1839	0.1915	0.7765
	CUF	0.2537	0.2406	0.2293	0.2251	1.7812
	SCIF	0.2096	0.1762	**0.1760**	0.1842	**0.6805**

the estimated offset to the ground-truth offset because of the greater trust in the measurement value. Therefore, when a slowly varying bias exists, the first row in each case in Table III shows that the higher the Q matrix value, the lower the RMSE value because the bias-aligned target trajectory approximates the reference trajectory.

T2T fusion with bias alignment Fig. 3 (e)-(f) shows the trajectory that fuses the bias-aligned target trajectory with the reference trajectory and the trajectory that fuses the reference trajectory with the target trajectory. In the KF, the state is not excessively biased to one side, even if the sizes of the two covariances differ. However, KF forms a smaller ellipse than the target and reference ellipses, resulting in inconsistent fused results. The CIF result is skewed to a smaller covariance between the reference and bias-alignment trajectories because it does not consider independence. In contrast, the SCIF shows the optimum result with a value between KF and CIF without skewing to a specific source.

Table III shows that the result after each T2T fusion method has a small RMSE when an appropriate Q matrix value is selected rather than a high Q matrix value. This is because the bias alignment result, biased towards the reference trajectory, loses the noise characteristic of the target trajectory. Therefore, the optimal T2T method was selected as SCIF, and the Q matrix was tuned to an acceptable value through this simulation.

T2T fusion without bias alignment If T2T fusion is performed without bias alignment, it has a relatively high RMSE value because the reference and target trajectories are fused, containing the remaining bias. As shown in Fig. 3(e), when the covariance size of the target trajectory is smaller than the covariance size of the reference trajectory, the CUF is not biased toward the target trajectory. This is because it tries to include the covariance of both trajectories. However, different T2T fusion algorithms are biased toward the target trajectory with a smaller covariance size, especially in CIF.

VII. CONCLUSION

This paper presents a framework for the loosely coupled fusion of multiple localization systems based on the T2T fusion method with bias alignment. The proposed system consists of two parts:1) slow varying bias estimation and alignment with the KF, and 2) localization fusion with T2T fusion methods. Considering the slowly varying bias in the localization system, we developed a robust and reliable localization fusion system. Furthermore, various T2T fusion methods were applied to fuse multiple localization systems, and the proposed split-covariance intersection filter-based system exhibited the best performance with the smallest RMSE error. The proposed method also has scalability because all localization systems can be applied by selecting whether it will be a reference or target localization. In the future, we plan to apply various fusion methods to evaluate the proposed methods in the real world. In addition, the estimated bias information was utilized to merge multiple maps used in multiple localization systems. Thus, we aimed to develop a more accurate and robust localization fusion system.

REFERENCES

[1] A. Lima, P. Bonnifait, V. Cherfaoui and J. A. Hage, "Data Fusion with Split Covariance Intersection for Cooperative Perception," 2021 IEEE International Intelligent Transportation Systems Conference (ITSC), 2021, pp. 1112-1118.

[2] H. Li and M. Yang, "Multi-Vehicle Cooperative Local Mapping Using Split Covariance Intersection Filter," 2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2019, pp. 2153-2158.

[3] S. Fang, H. Li and M. Yang, "Multi-Vehicle Cooperative SLAM Using Iterated Split Covariance Intersection Filter," 2021 IEEE Intelligent Vehicles Symposium (IV), 2021, pp. 947-952.

[4] L. Li and M. Yang, "Joint Localization Based on Split Covariance Intersection on the Lie Group," in IEEE Transactions on Robotics, vol. 37, no. 5, pp. 1508-1524, Oct. 2021.

[5] C. Pierre, R. Chapuis, R. Aufrère, J. Laneurit and C. Debain, "Range-Only Based Cooperative Localization for Mobile Robots," 2018 21st International Conference on Information Fusion (FUSION), 2018, pp. 1933-1939.

[6] S. Fang, H. Li and M. Yang, "Adaptive Cubature Split Covariance Intersection Filter for Multi-Vehicle Cooperative Localization," in IEEE Robotics and Automation Letters, vol. 7, no. 2, pp. 1158-1165, April 2022.

[7] Song, Haryong, Wonsub Choi, and Haedong Kim. "Robust vision-based relative-localization approach using an RGB-depth camera and LiDAR sensor fusion." IEEE Transactions on Industrial Electronics 63.6 (2016):3725-3736.

[8] D. S. Lemons and P. Langevin, An Introduction to Stochastic Processes in Physics. Baltimore, MD, USA: JHU Press, 2002.

[9] Jo, Kichun, Keonyup Chu, and Myoungho Sunwoo. "GPS-bias correction for precise localization of autonomous vehicles." 2013 IEEE Intelligent Vehicles Symposium (IV). IEEE, 2013.

[10] Julier, Simon, and Jeffrey K. Uhlmann. "General decentralized data fusion with covariance intersection." Handbook of multisensor data fusion. CRC Press, 2017. 339-364.

[11] Uhlmann, Jeffrey K. "Covariance consistency methods for fault-tolerant distributed data fusion." Information Fusion 4.3 (2003):201-215.

[12] Li, Hao, Fawzi Nashashibi, and Ming Yang. "Split covariance intersection filter: Theory and its application to vehicle localization." IEEE Transactions on Intelligent Transportation Systems 14.4 (2013):1860-1871.

[13] Julier, Simon, and Jeffrey K. Uhlmann. "General decentralized data fusion with covariance intersection." Handbook of multisensor data fusion. CRC Press, 2017. 339-364.

[14] Li, Hao, et al. "Track-to-track fusion using split covariance intersection filter-information matrix filter (scif-imf) for vehicle surrounding environment perception." 16th International IEEE Conference on Intelligent Transportation Systems (ITSC 2013). IEEE, 2013.

[15] Carrillo-Arce, Luis C., et al. "Decentralized multi-robot cooperative localization using covariance intersection." 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2013.