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Effective Combination of Vertical,Longitudinal and Lateral Data for Vehicle Mass Estimation

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Abstract— **Real-time knowledge of the vehicle mass is valuable for several applications, mainly: active safety systems design and energy consumption optimization. This work describes a novel strategy for mass estimation in static and dynamic conditions. First, when the vehicle is powered-up, an initial estimation is given by observing the variations of one suspension deflection sensor mounted on the rear. Then, the estimation is refined based on conditioned and filtered longitudinal and lateral motions. In this study, we suggest using these extracted events on two different algorithms, namely: the recursive least squares and the prior-recursive Bayesian inference. That is to express the results in a deterministic and statistical sense. Both simulations and experimental tests show that our approach encompasses the benefits of various works in the literature, preeminently, robustness to resistive loads, fast convergence, and minimal instrumentation.**

I. INTRODUCTION

When paving the way to autonomous driving, virtual sensing offers a plethora of solutions to enhance vehicle behavior in terms of safety, handling, and comfort [1]. Given this perspective, the vehicle's inertial parameters virtual sensor is of critical importance, especially that of the mass. In fact, the performance of safety systems such as Anti-lock Brake System (ABS), Electronic Stability Program (ESP), Roll Stability Control (RSC), and active suspension systems, depend on the accuracy of the vehicle model used designing them[2]. Since the load can vary by 50 % for passenger vehicles and by 400 % for heavy-duty vehicles (HDV)[3], real-time tracking of the mass information is essential to have a suitable model, thus making the controllers mentioned above adaptive and optimal. Moreover, the energy demand for different power-train architectures is significantly impacted by the vehicle mass and the terrain. As stated in [4], a 10 % change in the mass leads to a 2.4 to 4.1 % change in energy consumption. Thus, accurate information on this parameter adapts the eco-navigation to the vehicle environment when using efficient strategies such as speed profile optimization and look-ahead control [5]-[6]. In this context, it has been demonstrated that knowing the actual weight of the vehicle allows a more accurate estimation of the driving range of the Battery electric vehicles (BEV), it also allows more to implement more efficient strategies to get the best performance of the combustion engine and the battery capacity for Hybrid Electric Vehicles (HEV)[7]. Considering the application areas described above, an algorithm of vehicle mass estimation before being embarked especially for in economy-priced vehicles should respect some requirements. The algorithm should require little computation resources to run on low-power platform till providing real-time execution. It needs to be versatile ; it benefits from different dynamics of the vehicle when moving,it does not rely only on a specific motion, and it is independent from any other contextual information. It must be also robust to road load resistances such as slope and banking angles, rolling resistance, to aerodynamic drags as well as to uncertainties and noises comping from the measurements. Furthermore, it must be able to converge very fast within 1 to 5 % of the real mass and should be able to return an estimation of the error on the estimation of the mass. Finally, it should consider being relatively inexpensive as it does not require to implement costly sensors. Many research studies are present in the literature, and the approaches therein, tend to respect most of the requirements cited above or find a trade-off between them. [8] provides a classification of existing works based on two criteria. The first is whether the algorithm is continuously updating its estimate or probing for specific excitations of the vehicle motions; the former is referred to as "averaging," while the latter is called "event-seeking." The second criterion is the dynamics used to estimate the mass, as they can be longitudinal, lateral, vertical, or a combination of these. Furthermore, the observer can be model-based or data-driven, and can pursue estimation in a deterministic or statistical manner. [9] estimates the mass and road grade simultaneously using recursive least squares (RLS) with multi-rate forgetting factors. The method makes use of a full longitudinal dynamic model with predefined coefficients for air drag and rolling resistance. In most of the tests, the inferred grade and mass estimates are accurate; the estimator, however shows overshoots when gear shifts occur and with no sustained excitations. [8]-[10] suggest high-frequency data extraction for vehicle mass estimation. Their approach effectively filters out the effect of resistive forces and seeks for sharp longitudinal events to update the estimation using RLS. The independence from road load coefficients (slope, air drag, and rolling resistance) is the key advantage of this idea. Aside from longitudinal dynamics-based observers, [11] focuses on the lateral motion for both vehicle parameters (including mass) and states estimation. An Extended Kalman Filter (EKF) is used for this purpose. On the other hand, [2] capitalizes on a complete vertical dynamics model to output the mass and the road roughness

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using Dual Kalman Filter (DKF). The observer requires four suspension deflection sensors and a vertical accelerometer. A setback of this approach is the impact of suspension sensors' miscalibration and drift on the estimation: for example, the magnetic field of Linear Variable Differential Transformers (LVDTs) can be affected by vibrations, road excitations, and temperature variations. Furthermore, the residual voltage must be compensated in order to define the sensor's true null position. [12] increases the observer's availability by considering both longitudinal and lateral dynamics. For each motion, an RLS algorithm is used for estimation, which is fed beforehand by conditioned inputs from a supervisor block.
All of the preceding studies estimate the optimal mass in the least squares sense. However, in the presence of uncertain measurements or non-Gaussian sensor noise, providing a distribution of possible values rather than a single value is more advantageous. [13] uses Bayesian inversion on the longitudinal dynamic equation to generate a posterior probability distribution of mass and road grade. When compared to Least Squares, the results are more statistically reliable and have a lower mean error. Data-driven approaches are another alternative for model-based observers. For example, [14] uses a feed-forward neural network (FFNN) for parallel estimation of mass and road grade. The reported results outperform the traditional filtering techniques. However, building reliable training datasets and explaining the neural network results remain the drawback of data-driven observers. Table I summarizes all of the works described in accordance with the classification criteria.

The proposed algorithm in this study is designed to meet the majority of the requirements by capitalizing on concepts from previous works while avoiding their limitations. When starting the vehicle, the algorithm estimates the mass from the variations of the rear suspension deflection sensor. When the vehicle moves, this previous rough estimation gets refined by analyzing longitudinal and lateral excitations (event-seeking). In this way, the effect of resistive force is filtered using a method similar to [8]. Retrieved data from the sensors get conditioned to identify the specific

TABLE I
CLASSIFICATION OF PREVIOUS WORKS IN THE LITERATURE

Reference	Averaging/Event-seeking	Estimation Approach	Used Dynamics
[8]	Averaging	Model-based: RLS	Longitudinal
[9]-[10]	Event-Seeking	Model-based: RLS	Longitudinal
[11]	Averaging	Model-based: EKF	Lateral
[2]	Averaging	Model-based: DKF	Vertical
[12]	Event-Seeking	Model-based: RLS	Longitudinal And Lateral
[13]	Averaging	Model-Based: Bayesian Inversion	Longitudinal
[14]	Averaging	Data-driven: FFNN	Longitudinal

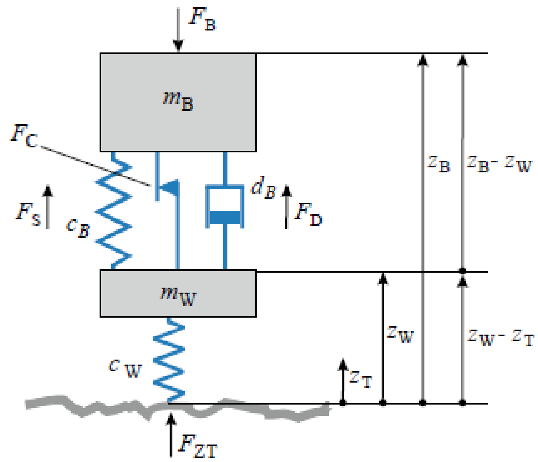

Fig. 1. Quarter-car suspension model

events suitable for estimation. Finally, the mass is estimated using either the recursive least squares (RLS) method or recursive Bayesian inference. The remainder of this paper is organized in the following way. Section 2 presents the estimation algorithm. It first introduces for each step the vehicle dynamics equations used for estimation, a general overview of RLS and prior-recursive Bayesian inference used to incrementally refine the mass estimation and terminates by presenting the complete architecture of the algorithm. Section 3 presents and discusses the results obtained by simulation as well as during real experiments done on cars. Section 4 presents some future works and concludes.

II. VEHICLE MASS ESTIMATION

A. *Initialization with suspension deflection sensor*

Assuming the steady state case and an even ground, the vertical force F_{ZT} of the quarter car suspension model depicted in Figure 1 is as follow :

$$F_{ZT}= c_B\,(z_B - z_W) = c_B z_{BW} \tag{1}$$

Where: c_B is the suspension stiffness and z_{BW} is the relative position measured by the suspension deflection sensor.
In this initial approximation, we consider that the load is equally distributed on the rear. Hence, the vertical forces can be expressed as:

$$F_{Zrr}=F_{Zrl}= \frac{Ml_f}{2l} g= c_{Brr}\ z_{BWrr} \tag{2}$$

Where subscripts rr and rl refer to rear right and rear left respectively, and:
M : Mass of the vehicle (kg)
l_f : front track (m)
l : wheelbase (m)
g : standard acceleration due to gravity (m/s²)
The initial mass $M^{initial}$ is the sum of the known empty vehicle mass M^{idle} and the change noticed from the suspension deflection sensor Δz_{WBrr}.

$$M^{initial}=M^{idle}+\frac{2\ l\ c_{Brr}\ \Delta Z_{WBrr}}{l_f\ g} \quad (3)$$

B. Vehicle Longitudinal Model

With respect to longitudinal motion, the vehicle is subjected to traction $F_{traction}$, brake F_{brake}, air drag $F_{air\ drag}$, rolling resistance $F_{rolling\ resistance}$ and road grade $F_{road\ grade}$ forces. The force balance equation of the longitudinal dynamics can be written:

$$Ma_x = F_{traction} - F_{brake} - F_{air\ drag} - F_{rolling\ resistance} - F_{road\ grade} \quad (4)$$

Where: a_x = Longitudinal acceleration (m/s²)
As demonstrated in [8], the contribution of resistive forces in high frequencies is negligible. Thus by subtracting equation (4) over a small time step, the effect of these forces is eliminated and we obtain:

$$M\Delta a_x = \Delta(F_{traction} - F_{brake}) \quad (5)$$

The traction force can be computed from the wheel torque C_{tq_wheel} estimated by the vehicle's electronic control units (ECUs) ,and the tire rolling radius r_{wheel} :

$$F_{traction} = \frac{C_{tq\_wheel}}{r_{wheel}} \quad (6)$$

The braking force is linearly related to the braking pressure P_{brake} if the wheels are not locked:

$$F_{brake} = P_{brake}K_{brake} \quad (7)$$

Where K_{brake} is a known constant.To account for sufficient excitations and assure pure longitudinal motion, the gradient of longitudinal acceleration Δa_x and that of the inertial forces $\Delta(F_{traction} - F_{brake})$ are conditioned. Table II outlines the conditions for the longitudinal model.
Based on the idea that traction and braking forces are predominant in longitudinal motion, the observer is only activated when the conditions presented in table II are verified. This allows decoupling mass estimation from unknown road load coefficients.

C. Vehicle Lateral Model

To gain more availability to update the estimation, the algorithm also analyzes the lateral motion. The lateral dynamics equation using the linear single-track model [15] and in presence of banking angle can be written as:

$$Ma_y = F_{yf}\cos(\delta_f) + F_{yr} - F_{lat\ air\ drag} - F_{banking} \quad (8)$$

Where:
a_y : Lateral acceleration (m/s²)
F_{yf} : Front axle lateral force (N)
F_{yr} : Rear axle lateral force (N)
$F_{lat\ air\ drag}$: Force due to lateral air drag (N)
$F_{banking}$: Force due to road baking angle (N)
δ_f : Front steering wheel angle (rad)
Similar to the subsection above, the effect of lateral air resistance and road inclination angle can be eliminated by differentiating Equation (8) over a small time step:

$$M\Delta a_y = \Delta(F_{yf}\cos(\delta_f) + F_{yr}) \quad (9)$$

In the linear region of the tires, the front and rear axles lateral forces can be determined by :

$$\begin{cases} F_{yf} = C_{\alpha f}\alpha_f \\ F_{yr} = C_{\alpha r}\alpha_r \end{cases} \quad (10)$$

Here $C_{\alpha f}$, $C_{\alpha r}$ are the front and rear cornering stiffnesses and α_f, α_r their respective side slip angles.
A kinematic model independent from the mass is used to estimate the lateral forces. For this purpose, a Linear Kalman Filter (LKF) was applied to (11) to estimate the lateral velocity V_y.

$$\begin{cases} \begin{bmatrix} \dot{v_x} \\ \dot{v_y} \end{bmatrix} = \begin{bmatrix} 0 & \dot{\psi} \\ -\dot{\psi} & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix} \\ v_x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} \end{cases} \quad (11)$$

Then, α_f and α_r are computed in function of the estimated lateral velocity $\widehat{v}_y$,the yaw rate $\dot{\psi}$, and the measured longitudinal velocity v_x as:

$$\begin{cases} \alpha_f = \mathrm{atan}\left(\frac{\widehat{v_y}}{v_x}\right) + \frac{l_f\dot{\psi}}{v_x} - \delta_f \\ \alpha_r = \mathrm{atan}\left(\frac{\widehat{v_y}}{v_x}\right) - \frac{l_r\dot{\psi}}{v_x} \end{cases} \quad (12)$$

It is worth mentioning that accelerations (a_x and a_y), and yaw rate ($\dot{\psi}$) are defined in the car inertial frame and can be acquired via the Inertial Measurement Unit (IMU) embarked in the vehicle. Figure 2 depicts all the variables described in this section.
In the same fashion, activation conditions of the pure lateral motion are highlighted in Table 3.

D. Unifying data

The vehicle mass estimation is now formulated as a linear regression problem. Depending on the current motion

TABLE II
CONDITIONS FOR ACTIVATING THE LONGITUDINAL MOTION OBSERVER

The measured lateral acceleration should be less than 0.2 m/s²
The absolute gradient longitudinal acceleration $\lvert\Delta a_x\rvert$ should be greater than 0.01 m/s²
The absolute gradient of the net longitudinal force $\lvert\Delta(F_{traction} - F_{brake})\rvert$ exceeds 10 N

TABLE III
CONDITIONS FOR THE LATERAL MOTION

The measured lateral acceleration $\lvert a_y\rvert$should be less than 4 m/s²
The absolute gradient lateral acceleration $\lvert\Delta a_y\rvert$ should be greater than 0.01 m/s²
The longitudinal velocity v_x should be constant
The absolute steering wheel angle $\lvert\delta_f\rvert$ should be greater than 15°

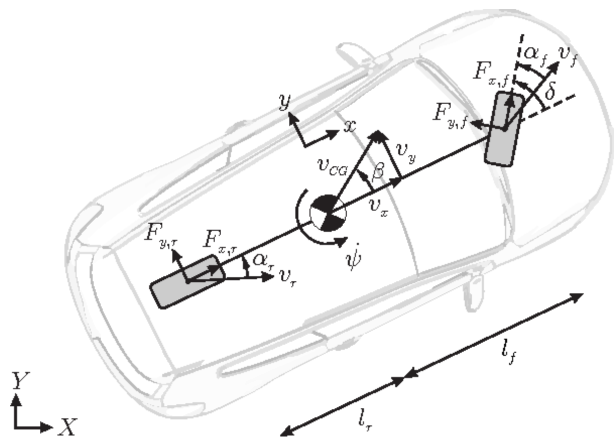

Fig. 2. Double-track model (The longitudinal forces $F_{x,f}$ and $F_{x,r}$ are neglected in the pure lateral model.

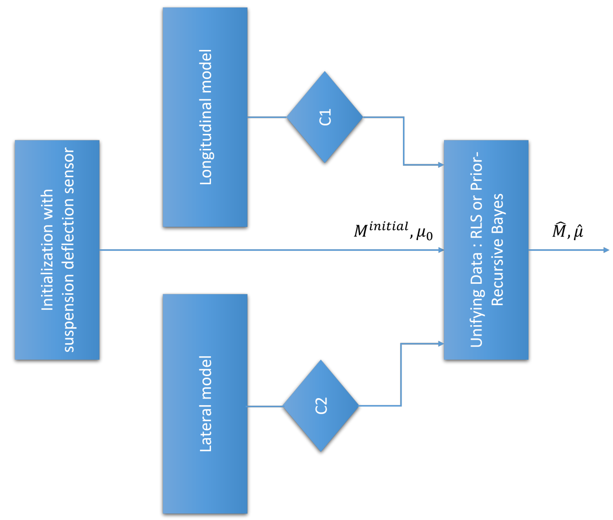

Fig. 3. Block diagram showing the 4 blocks and the conditions that constitute the vehicle mass estimation

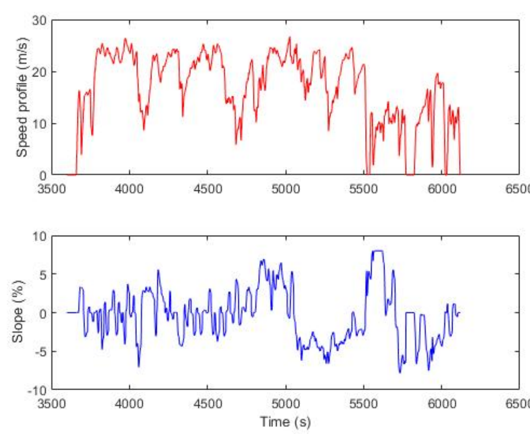

Fig. 4. Speed profile and Road slope of Experiment 1

(longitudinal or lateral), the regressors and the measurements will change as:

$$\begin{cases} \Delta(F_{traction}-F_{brake})=M\Delta a_x+\mu \text{ if C1 is True} \\ \Delta(F_{yf}\cos(\delta_f)+F_{yr}) = M\Delta a_y+\mu \text{ if C2 is True} \end{cases} \tag{13}$$

C1 and C2 refer to the activation condition of the longitudinal and lateral blocks respectively and μ to the model imprecisions.

Hereafter, two different algorithms may be used to estimate mass and model imprecisions. The first one is RLS, it recursively outputs the optimal parameters in the least squares sense. The second one is prior-recursive Bayes, which fits the model sequentially on partitions of data and updates the probability distribution using the posterior from the previous stage as the prior in the new stage based on the most recent measurements. Figure 3 shows the global architecture of the proposed algorithms.

1) 1^{st} Method : Recursive Least Squares : The system (13) can be written as follows: $y = \varphi\theta$ where the measurement y and the regressor φ change in function of C1 or C2. $\theta = [M, \mu]$ are the parameters to be estimated.

After initializing the first state $\theta_0 = [M^{initial}\ \mu_0]$, the update law of RLS is :

$$\widehat{\theta}_k = \widehat{\theta}_{k-1} + K_k\left(y_k - \varphi_k\widehat{\theta}_{k-1}\right) \tag{14}$$

$$K_k = P_{k-1}{\varphi_k}^T(\varphi_k P_{k-1}{\varphi_k}^T)^{-1} \tag{15}$$

$$P_k = (I - K_k\varphi_k)P_{k-1}(I - K_k\varphi_k)^T \tag{16}$$

K_k and P_k are the gain and the estimation error covariance matrices respectively.

2) 2^{nd} Method : Prior-Recursive Bayes : Bayesian inference uses Bayes theorem[16] to update the probability distribution of the parameters based on new measurements.

Step 1: The initial belief is the prior $p(\theta)$ based on $M^{initial}$ observed from the first block.

Step 2: Starting from this initial belief, for each partition that contains n pairs of (y_i, φ_i), we update the posterior as:

$$p(\theta \mid Y, \varphi) = \frac{p(Y \mid \varphi, \theta)\, p(\theta)}{\int p(\varphi, Y, \theta)\, d\theta} \tag{17}$$

$p(Y \mid \varphi, \theta)$ is called the likelihood and it describes the probability of the observations giving the regressors and the parameters.

Iteration Steps: each time new data are available, the posterior is updated according to the equation used on Step 2; the current posterior is used as the prior when performing the update.

III. Simulation and Experimental Results

A. Simulation Results

The validity of the proposed estimator has been first verified by simulation using a high-fidelity vehicle model in AMESIM and implementing the estimator logic in MATLAB/Simulink. Two experiments were constructed: The first one is a segment of a European driving cycle where the speed

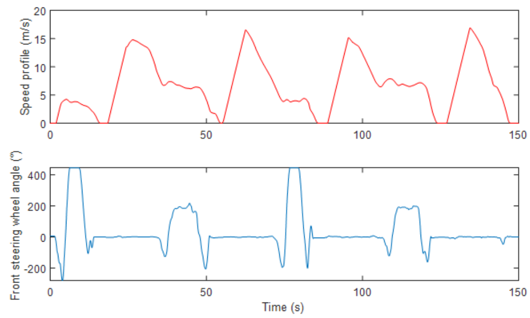

Fig. 5. Speed profile and Steering angle of Experiment 2

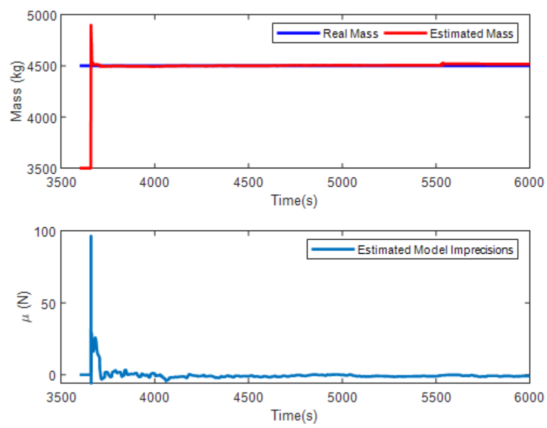

Fig. 6. Estimated Mass and μ from Experiment 1

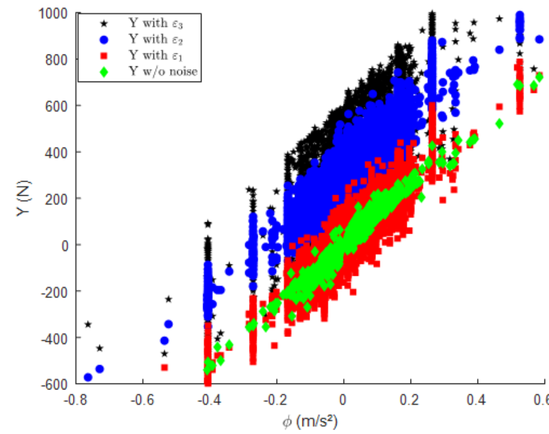

Fig. 7. Extracted measurements and regressors with three sets of noise

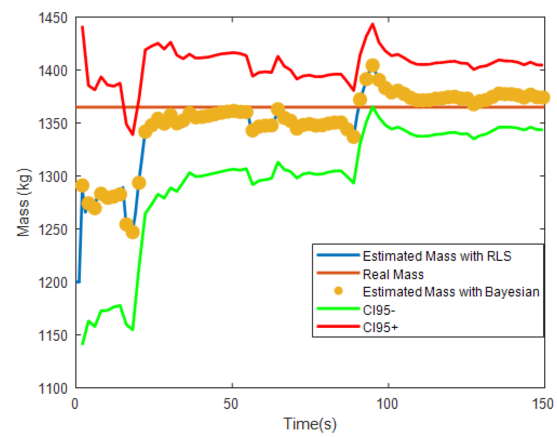

Fig. 8. Estimation Results for Experiment 2 with ε_3 : credible interval of Bayesian is marked with red & green solid lines

profile and the slope are depicted in Figure 4 , the second combines accelerating, braking and cornering. For the latter, speed profile and steering wheel angle are plotted in Figure 5. Note that the initialization with suspension deflection sensor is not considered in simulation part, but will be included in the real data test. Lastly, the time step for differentiating (4) and (8) is $dt = 0.02s$.

The aim of Experiment 1 is to confirm the estimator robustness to resistive loads (slope in this case). For simplicity, only the estimator with RLS is used and since it is a straight line driving, the lateral event-seeking block is not functional. As shown in Figure 6, the estimator converges to the real mass rapidly (within 20 seconds) and the steady error is 0.38 %. In addition to that, the peak of the estimated μ corresponds to that of the estimated mass which gives the intuition to discard this value.

Experiment 2 sights to advocate the enhanced availability of the algorithm when using longitudinal and lateral motions. Moreover, it compares RLS and Prior-Recursive Bayesian with three sets of added noise ε,as shown in Figure 7: $\varepsilon_1 \sim \mathrm{N}\left(0, 60^2\right), \varepsilon_2 \sim \mathrm{N}\left(300, 60^2\right), \varepsilon_3 \sim \mathrm{U}\left(0, 600\right)$.To keep the model analytically tractable and without using Monte Carlo sampling techniques for the Bayesian observer, we choose the normal-inverse-gamma conjugate model (details are handed in [17]). This is justified when assuming a normal distribution with unknown variance for the prior.

Both observers consistently produce an estimation error of less than 1 % for all simulation sets. One plus of the Bayesian observer is the credible interval (also called confidence interval) which shows that there is 95 % probability that the true estimate lies within it. In our case, this interval starts wide and tends to narrow for each partition of measurements with a batch size $n = 100$. For example, the estimation result for the case with noise ε_3 is depicted in Figure 8. In terms of availability, the observer was activated 32 % of the time with a contribution of 10 % for the longitudinal block and 22 % for the lateral one. As can be seen, in a maneuver where longitudinal excitations are insufficient, benefiting from the lateral motion is advantageous to often update the estimation.

B. Experimental Results

To further verify the proposed approach, a real drive test has been conducted on the "RENAULT Lardy" track. The test vehicle is a Plug-in Hybrid Electric Vehicle (PHEV) SUV with a suspension deflection sensor mounted on the rear right. Off-line simulation was achieved using the recorded CAN Bus data that were synchronized at a sampling frequency of 100 Hz. The test was divided into two sessions:

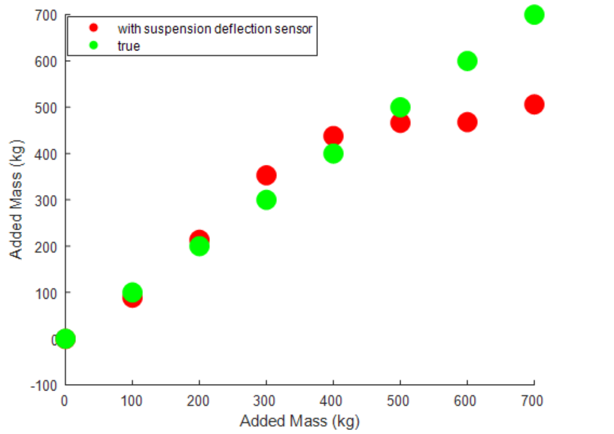

Fig. 9. Results of the static test with the rear suspension stiffness $c_{Brr} = 30kN/m$

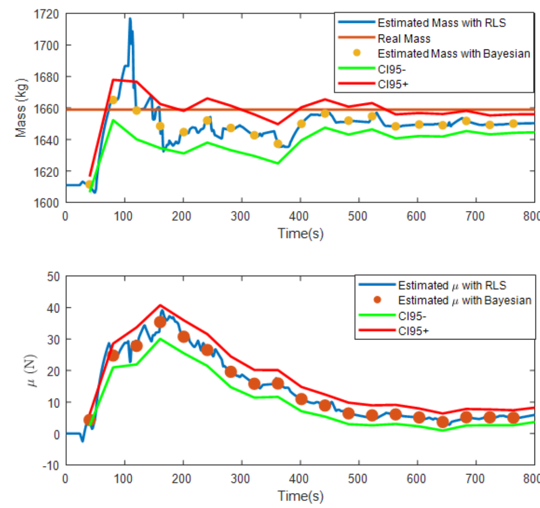

Fig. 10. Estimation Results on "RENAULT Lardy" track; Mass estimation result (Upper figure) and Model imprecisions result (Bottom Figure)

First, a static test where 8 ballast samples with a weight difference of 100 kg were added sequentially to feature the initialization phase of our algorithm. Then, a dynamic test with an actual value of 1659 kg.

The results of the first test shows that up to a linear range, the estimated mass with the suspension sensor is very precise. However, the error increases nearing the saturation range (see Figure 9). In addition to that, the performance could worsen also in some contexts such as the presence of road grade or miscalibrations of the sensor. Thus, despite being a better estimation than the empty vehicle load, it is mandatory to correct it via dynamics conditions.

For the second test, the change noticed from the suspension deflection sensor was $\Delta z_{WBrr} = 2.64\ mm$, which yields an initial mass $M^{initial} = 1611\ kg$. Over the course of driving, the estimation is enhanced and the proposed algorithms achieve a steady error of 0.54 %. Figure 10 summarizes the results with a differentiating time $dt = 0.2s$ and a batch size for the Bayesian observer $n = 200$. Finally, the longitudinal block was available 15 % of the time, while the lateral was around 22 %.

IV. Conclusion and Future Work

This paper addresses the problem of vehicle mass estimation and proposes a novel approach that is active once the vehicle is powered-up. At the static state, we initialize our algorithm by observing the compression/bounce of one rear suspension deflection sensor. Under dynamic driving conditions, we employ a new formulation of the longitudinal and lateral equations based on the idea that resistive forces can be ignored at high frequencies. Therefore, we extract data suitable for estimation. These data are fed into two observers to output the mass: RLS, which produces the optimal value in the least square sense, and Prior-Recursive Bayes, which produces a probability distribution of the estimated values and thus can quantify its confidence level.

The proposed approach proved its effectiveness via simulation and experimental results in terms of the following features: 1) The initial step enables to have a first value of mass better than the idle load, which fastens the convergence of the observer. 2) The problem is formulated as a linear regression model that is independent and robust to unknown resistive load coefficients. 3) The steady error achieved for both observers is less than 1 %. 4) Relying on longitudinal and lateral motions increases the availability of the estimation, thus excitations from one direction can compensate the insufficient ones from the other.

Future work will focus on two aspects: 1) Incorporate Markov Chain Monte Carlo (MCMC) sampling techniques to enhance the performance of Bayesian observer in the case of unknown statistical characteristics of the measured data.2) Deploy the proposed algorithm as an upstream observer to infer other vehicle parameters such as air drag and rolling resistance coefficients. These parameters are mostly identified by the car manufacturers but the process is time-consuming.

References

[1] M. A. Reche, "Vehicle Dynamics Virtual Sensing and Advanced Motion Control for Highly Skilled Autonomous Vehicles," 2019.

[2] B. L. Boada, M. J. L. Boada, and H. Zhang, "Sensor Fusion Based on a Dual Kalman Filter for Estimation of Road Irregularities and Vehicle Mass under Static and Dynamic Conditions," IEEE/ASME Trans. Mechatronics, vol. 24, no. 3, pp. 1075–1086, 2019.

[3] M. Druzhinina, A. Stefanopoulou, and L. Moklegaard, "Adaptive con- tinuously variable compression braking control for heavy-duty vehicles," J. Dyn. Syst., Meas., Control, vol. 124, no. 3, pp. 406–414, Sep. 2002.

[4] R. B. Carlson, H. Lohse-Busch, J. Diez, and J. Gibbs, "The measured impact of vehicle mass on road load forces and energy consumption for a BEV, HEV, and ICE vehicle," SAE Int. J. Altern. Powertrains, vol. 2, no. 1, pp. 105–114, 2013.

[5] E. Hellström, M. Ivarsson, J. Åslund, and L. Nielsen, "Look-ahead control for heavy trucks to minimize trip time and fuel consumption," Control Eng. Pract., vol. 17, no. 2, pp. 245–254, 2009.

[6] N. Lin, C. Zong, and S. Shi, "Method for switching between traction and brake control for speed profile optimization in mountainous situations," Energies, vol. 11, no. 11, 2018.

[7] A. Ritter, F. Widmer, B. Vetterli, and C. H. Onder, "Optimization-based online estimation of vehicle mass and road grade: Theoretical analysis and experimental validation," Mechatronics, vol. 80, no. October, p. 102663, 2021.

[8] H. K. Fathy, D. Kang, and J. L. Stein, "Online vehicle mass estimation using recursive least squares and supervisory data extraction," Proc. Am. Control Conf., pp. 1842–1848, 2008.

[9] A. Vahidi, A. Stefanopoulou, and H. Peng, "Recursive least squares with forgetting for online estimation of vehicle mass and road grade: Theory and experiments," Veh. Syst. Dyn., vol. 43, no. 1, pp. 31–55, 2005.

[10] J. Ghosh, S. Foulard, and R. Fietzek, "Vehicle Mass Estimation from CAN Data and Drivetrain Torque Observer," SAE Tech. Pap., vol. 2017-March, no. March, 2017.

[11] T. A. Wenzel, K. J. Burnham, M. V. Blundell, and R. A. Williams, "Dual extended Kalman filter for vehicle state and parameter estimation," Veh. Syst. Dyn., vol. 44, no. 2, pp. 153–171, 2006.

[12] K. Han, I. Kim, S. Kim, and K. Huh, "Real-time vehicle mass estimator for active rollover prevention systems," Trans. Korean Soc. Mech. Eng. A, vol. 36, no. 6, pp. 673–679, 2012.

[13] X. Shen and Y. Zhang, "Estimating vehicle mass and road grade through Bayesian inversion," IFAC-PapersOnLine, vol. 54, no. 10, pp. 235–240, 2021.

[14] S. Torabi, M. Wahde, and P. Hartono, "Road grade and vehicle mass estimation for heavy-duty vehicles using feedforward neural networks," 4th Int. Conf. Intell. Transp. Eng. ICITE 2019, pp. 316–321, 2019.

[15] M. Kanitz, Automotive control, vol. 59, no. 9. 2006.

[16] M. B. Hooten, D. S. Johnson, and B. M. Brost, "Making Recursive Bayesian Inference Accessible," Am. Stat., vol. 75, no. 2, pp. 185–194, 2021.

[17] 2022. [Online]. Available: https://fr.mathworks.com/help/econ/what-is-bayesian-linear-regression.html#bvinfmm-1. [Accessed: 14- Sep-2022].