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Multi-Robot 3D Gas Distribution Mapping: Coordination, Information Sharing and Environmental Knowledge

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Abstract— **Environmental monitoring and mapping operations are an essential tool to combat climate change. An important branch of this domain concerns the construction of reliable gas maps. Adaptive navigation strategies coupled with multi-robot systems improve the outcome of an environmental mapping mission by focusing more efficiently on informative areas. This direction is yet to be explored in the context of gas mapping, which presents peculiar challenges due to the hard-to-sense and expensive-to-model nature of the underlying phenomenon. In this paper, we introduce the application of a multi-robot system to a gas mission with severe time constraints. We study the impact of information-based navigation strategies, coupled with increasing levels of coordination among the robots, on information gathering and consequent map reconstruction performance. We also focus on proposing solutions that inject additional knowledge into the system to enhance the final mapping outcome. We tested the strategies through extensive high-fidelity simulation experiments, and we compared the proposed approaches to three relevant baseline methods.**

I. Introduction

Environmental monitoring and mapping missions are an essential tool to combat climate change. In recent years, robots have been employed for these tasks, increasing their efficiency and reducing the threat to human and animal lives in hazardous regions. Environmental mapping tasks span from methods relying on images, such as mapping of weeds for agriculture [1], to approaches that rely on scarcer data, such as mapping chemical components in water [2] or in the air [3]. Navigation methods for environmental mapping aim at improving the quality of the final map by focusing on areas of interest, while taking into account the time and energy constraints of the robotic platform. To this end, adaptive sampling techniques are currently one of the main topics tackled by the research community in these fields. Another interesting direction for environmental mapping algorithms is the employment of multi-robot systems, whose architecture can be exploited to improve the information gathering procedure. In this paper, we explore multi-robot adaptive sampling techniques for gas mapping, an application characterized by an underlying phenomenon that is hard to sense and model, and for which only punctual data points can be retrieved.

Gas Distribution Mapping (GDM) is the field concerned with producing a reliable map of a gas distribution in a given environment. Ideal platforms to perform this and other gas detection tasks in indoor or GNSS-denied spaces are Nano Aerial Vehicles (NAVs). Their movement range can capture the tridimensionality of the gas dispersion, and their propellers cause a smaller perturbation compared to bigger drones, preserving their sensing capabilities [4]–[6]. Gas data is collected by mounting a gas sensor on the NAV, sampling usually at a frequency in the range of 1-10 Hz, and obtaining a punctual measurement of the distribution. A prominent estimation approach for the underlying gas distribution is the Kernel DM+V method, presented in [7] and extended to 3D in [8]. This method provides the mean and variance of the gas distribution by interpolating the sensed gas values using a Gaussian kernel. The kernel interpolation allows to infer the gas distribution of an area of a few tens of centimeters around where the measurement was taken. The method was successfully used for gas mapping missions with one flying vehicle in 2D [9], [10] and in 3D [4].

In the field of GDM, navigation is largely achieved using the lawnmower movement, a non adaptive strategy consisting of a preplanned trajectory with lengthy stops at measurement locations [5], [9]. The resolution of the lawnmower scan is highly impacted by the time budget allocated for the exploration of the affected volume, leading to coarser scans when time is limited. Informative Path Planning (IPP) uses informative quantities, such as entropy, to guide navigation towards areas of high information content. This technique is used often for navigation during environmental monitoring operations [11], such as temperature field estimation [12] or weed monitoring [1]. The stochastic and time variant nature of the gas dispersion phenomenon makes the employment of IPP strategies for robotic gas detection challenging. In particular, sensing capabilities are degraded by the wake of the propellers when employing a UAV, providing less precise information to the IPP algorithm. Limitations in flight time also impact the amount of information that the robot can gather during a mission. Nonetheless, there are a few examples of IPP strategies being applied to GDM successfully, showing improved performance with respect to preplanned trajectories [4], [10], [13]. Most of the work on GDM only exploits locally interpolated information.

Environmental mapping missions can benefit from the employment of a Multi-Robot System (MRS) architecture coupled with IPP. There are several perks to this: in a given time-frame, more information about the phenomenon of interest is gathered, the system benefits from increased robustness, both for hardware failure and redundancy in sensing, and the mapping mission can be conducted in less time or achieve larger coverage. A majority of approaches

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in this field is concerned with obtaining exhaustive coverage of the concerned volume [14]. In the scope of this work, we consider that our mission is time-sensitive. Therefore, while we want to cover as much volume as possible, we want the robots to focus on informative areas.

Multi-robot system mapping approaches often couple adaptive sampling strategies with additional measures to improve the mapping outcome. Several approaches allocate optimally a set of predetermined locations to the robot's team, usually based on information content and location [15]. Other approaches exploit prior knowledge of the phenomenon to make informative decisions [16], or rely on a large amount of data, often coming from cameras [17], [18]. Moreover, global knowledge can be injected in the planning in the form of attractor landmarks [17] or by exploiting an underlying model of the phenomenon [19]. These techniques are coupled with different levels of coordination and data sharing between the agents. Reliance on prior data, on a model or on predefined targets indicates that these methods not only exploit the sensed input data, but need additional elements to effectively guide the exploration. In the scope of gas mapping, data is particularly scarce and punctual compared to other fields, this makes the employment of non-myopic approaches difficult, even in the context of a MRS. Additionally, prior knowledge of the dispersion phenomenon is rarely available. Finally, the stochastic and time-variant nature of gas dispersion makes it hard and environment-dependent to model, making the integration of additional information into the navigation algorithm difficult.

Multi-robot systems have rarely been employed in the GDM domain. A mapping method based on Gaussian kernels that uses a MRS composed of UAVs is presented in [20]. However, the navigation strategy used is the preplanned lawnmower movement in 2D, with 1/3 of the area allocated to each of the three drones. An informative path planning based approach for GDM is proposed in [21]. The authors show that IPP strategies outperform random walk navigation, but the method is constrained to 2D maps with robots moving very slowly, at 2 m/min. Multi-robot architectures have been more frequently explored for Gas Source Localization (GSL) applications, which differ from GDM in terms of their objective but not for the amount of available data. In [22], the source localization is improved by exploiting coordination strategies among robots, while in [19], IPP planning coupled with a model of the gas distribution is used for source localization by a team of robots.

In this paper, we investigate the effects of information sharing, coordination and environmental knowledge for a MRS performing a gas mapping mission in 3D. The contributions of the paper are:

- Inspired by [22], we propose coordination strategies for a MRS performing a GDM mission.
- We assess the impact of the addition of a spatial clustering strategy.
- We assess the impact of a model-based navigation strategy, taken from [23], that injects global knowledge in the system.
- We compare our approaches to three baselines: multi-robot preplanned trajectory, single robot model-free navigation with clusters and single robot model-based navigation.

For all approaches, the 3D Kernel DM+V/W was chosen to estimate the gas distribution because of its light computational requirements. The gas mapping mission is conducted by NAVs moving continuously, to maximize the amount of information gathered within the time constraints. The methods presented in this work take into account the limitations of the target hardware platform and are tested in a high-fidelity simulator.

II. METHODOLOGY

This section outlines the methodology adopted for this paper. It starts with a brief overview of the gas distribution estimation algorithm, followed by a discussion about the information-based navigation strategy employed during this work. Then, the coordination strategies employed for the MRS are introduced. Finally, an overview of the model-based strategy, as well as the underlying collision avoidance strategy, are presented. With model-based, we refer to strategies that employ an underlying model of the gas distribution for navigation, in contrast with model-free, where such knowledge is not available. For the purpose of tractability, the experimental volume is divided in N cells, of size $10*10*10cm^3$. The methods are presented in a generalized way for a system of r robots.

A. Gas Map Estimation

We use the 3D Kernel DM+V/W algorithm, presented in [8] to estimate the gas distribution in the explored volume. This algorithm builds a 3D map of the gas dispersion by weighting the collected gas samples with a multivariate Gaussian function. The wind information, retrieved, for example, with an on board anemometer, can also be included in the estimation. However, in the scope of this work we assume that the wind intensity and direction are constant and known throughout the experiment. For each new sample, the corresponding weight is found by evaluating a Gaussian kernel at the distance between the location of the measurement and the center of cell. The shape of the kernel is controlled by the covariance matrix and depends on the kernel width, which encodes the amount of extrapolation on individual readings. We selected the parameters of this algorithm in accordance with previous work [4], [23].

The gas estimation inside each cell is associated to a confidence value $\alpha^{(k)}$:

$$\alpha^{(k)}(\sigma_0) = 1 - e^{-\frac{\Omega^{(k)}(\sigma_0)}{\sigma_\Omega^2}} \tag{1}$$

where $\Omega^{(k)}(\sigma_0)$ is the integrated weight map and σ_Ω is a scaling parameter. The confidence map attributes higher confidence to cells for which more gas values were gathered. In previous work, we kept the confidence always equal to 1, in order to compensate for the continuous movement of the robot. However, we decided to reintroduce this value in this work because of the degradation in performance we observed for the lawnmower movement from simulation to reality in [4], which was due also to the fact that the lawnmower was not gathering a lot of values for each cell. We therefore believe that, by using the confidence map in simulation, we can obtain results closer to the real world.

B. Navigation

The adaptive IPP strategy employed to select the next goal positions for model-free strategies uses the Kullback-Leibler Divergence (KLD) [24] to maximize the information gathered during navigation. This quantity has been successfully used in our previous work on GDM and GDM+GSL [4], [23]. The KLD quantifies the difference between two probability distributions and is computed as:

$$D_{KL}(P||Q) = \sum_{i} P(i) log_2 \left(\frac{P(i)}{Q(i)} \right) \tag{2}$$

where P is the current probability distribution of the gas samples in each cell and Q is the estimated next step probability distribution, obtained by adding one virtual sample, corresponding to the expected value of the distribution, to P. A higher value of KLD indicates discrepancies between P and Q and suggests that more exploration is needed to increase the confidence of the gas estimation in a cell.

C. Coordination Strategies

A MRS can present varying levels of coordination between its agents, which impact, for example, the amount of resources shared or the navigation decisions. Here, we discuss the coordination strategies employed in this paper.

1) Individualist Strategy: The robots independently build their own gas distribution map based on the samples that they acquire within the time limits. The robots can only use their own map to select the next goal positions, making the selection entirely reliant on local estimation. When the time budget elapses, a final gas map, built by combining the r maps produced by the agents, is delivered and evaluated. This strategy is the easiest to deploy on real hardware, since it does not require communication among the agents.

Since an estimation of the gas value can be provided by more than one robot for each cell, we propose to use three quantities to select a single gas value among the estimation guesses provided by different robots. For each cell, we select the gas value with the lowest associated variance, the highest number of samples gathered by the robots in the cell, or the highest confidence value $\alpha^{(k)}$, computed as part of the 3D Kernel DM+V/W algorithm.

2) Cooperative Strategy: In this strategy, the robots contribute to a unique gas map that is shared with all the members of the system. Concretely, this translates to updating the gas distribution map estimation using the 3D Kernel DM+V/W algorithm for each new gas sample, regardless of which robot it comes from. This strategy still relies exclusively on local estimation, but the robots have access to the information gathered by the entire fleet. As a first step in the implementation with real robots, we will carry out the mapping computation on a central base station, but future steps could exploit effectively the communication among agents to deploy this method in a distributed fashion.

During the navigation phase, each robot uses the shared map to select the next goal position. The advantage of this strategy compared to the individualist one lies in the higher amount of information that can be used by all robots to take more informative next steps.

3) Collaborative Strategy: Collaboration adds movement coordination to the cooperative strategy. Movement coordination allows the agents to improve the allocation of their goal positions. We propose two movement allocation strategies that we named *Swap* and *Replan*.

In the *Swap* strategy, when a robot reaches its goal position, it will identify a new goal position using the IPP-based navigation criterion and make sure that it falls beyond a safe distance d_s from the other robots' goals. The new goal position and the current goal positions of the other robots are optimally allocated by minimizing the total distance travelled, using the Hungarian algorithm [25]. This strategy pushes the robots to explore away from each other and, by keeping them apart, decreases the interference of the propellers among the drones during physical experiments. The effect of the d_s parameter is also studied.

In the *Replan* strategy, when a robot reaches a goal position, it will find r possible next goals. The goals are selected with a greedy method that maximizes the navigation criterion, explained in Section II-B, and are at a safe distance d_s from each other. The goals are then allocated to the robots based on the shortest distance covered, using the Hungarian algorithm. All the robots immediately switch to their new goal position. The rationale behind this strategy is that a robot replans for all members of the system based on the newly acquired information since the last replanning, which could offer more insights on where to go next.

In our previous work [4], we showed the benefits of coupling IPP strategies with a spatial clustering method. Consequently, we decided to apply the same method to the *Swap* and *Replan* strategies. The method consists in dividing the volume in K clusters using the K-means algorithm, and explore all the clusters sequentially. The number of clusters chosen for this work was set to four. We chose to have clusters that are big enough to avoid excessive interference among the drones, and to avoid biasing the mission to achieve high volume coverage, instead of trying to focus primarily on the plume. Although the addition of clusters does not change the reliance on local data only, the system now can count on a more spatially constrained navigation, which pushes for exploration. The cluster method corresponds to a less spatially constrained version of visiting predefined waypoints.

D. Model-Based Navigation

GaSLAM [23] is an algorithm for simultaneous gas mapping and source localization. It organically combines two state of the art methods, 3D Kernel DM+V/W for mapping and Source Term Estimation (STE) [26] for localization, to produce a reliable gas map and localize the gas source within it. The sensed gas data is interpolated using the 3D Kernel DM+V/W algorithm, and the resulting map is used by the STE to create a belief of the source location. Since STE is a model-based algorithm, we effectively have two maps of the environment at all times: one is the map generated by the Kernel algorithm (Kernel Map), which only contains locally sensed data, and one is the map given by the STE model (STE Map), which uses the belief on the STE parameters to guess the gas distribution in the whole environment.

Navigation is vector based. In the initial exploratory phase, the navigation vector is the sum of a vector pointing to the highest KLD and a vector pointing to the belief of the source. The following exploitative phase, which starts when the belief of the source location position is deemed good enough, focuses on exploring areas where the difference between the Kernel Map and the STE Map is high. In this work, if the source location prediction converges to a good belief, the final map is considered to be the STE Map, otherwise the Kernel Map is used. For more details, see [23].

In this paper, we apply the best performing coordination strategy to a MRS running GaSLAM to observe the effects of the injection of global knowledge. Global knowledge comes from the gas dispersion model used by the STE, which, in this case, is the Pseudo-Gaussian model [27].

E. Baselines

Three baseline methods are used in the evaluation of the multi-robot GDM algorithms proposed. The first one is the lawnmower method, which is a non-adaptive path planning strategy consisting of a zig-zag motion. The total volume is divided in r zones and each robot is allocated to one zone, in which it will perform the scan, similarly to [20]. The second baseline is a single-robot method, where navigation uses IPP and clusters, similarly to [4]. The number of clusters is set to four, to keep consistency with the division in clusters of the multi-robot methods. The final baseline consists of a single robot running the model-based GaSLAM algorithm [23].

F. Collision Avoidance

The collision avoidance method is straightforward and computationally light. It relies on the premise of a centralized architecture due to the lack of on-board obstacle detection capabilities on the target platform. At each step of the algorithm, if the distance between each pair of robots falls below a threshold, the robots are given new goal positions located in the opposite direction of their current distance vector. The new waypoints $\mathbf{w}$ are calculated as follows:

$$\mathbf{w_n} = \mathbf{x_n} - \mathbf{d_{12}} * \mathbf{x_{th}} \qquad \mathbf{n} \in \{\mathbf{1}, \mathbf{2}\} \tag{3}$$

where $\mathbf{x_1}$ and $\mathbf{x_2}$ are the current 3D positions of the robots, d_{12} is the normalized distance between the robots and $\mathbf{x_{th}}$ is the threshold. Once the new waypoint is reached, the robots will carry on the exploration based on IPP navigation.

III. Performance Evaluation

In this section, the simulation setup and the evaluation metrics used in this work are presented.

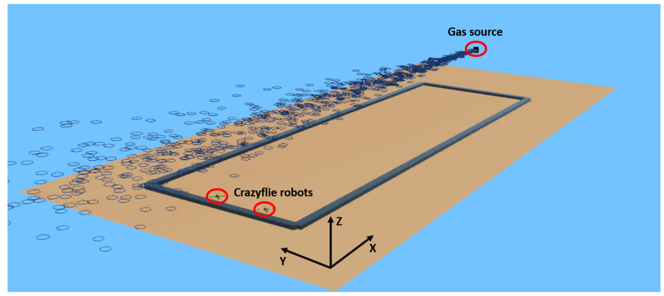

Fig. 1: View of the simulation experiments carried out in Webots. The gas source and Crazyflie robots positions are highlighted with red circles. The coordinate system used throughout the paper is also pictured.

A. Simulation Setup

The simulation experiments were carried out using Webots [28], a high-fidelity open-source simulator. A realistic model of the gas dispersion is simulated using a gas dispersion plugin [29]. The NAV fleet is composed of two simulated Crazyflie robots equipped with gas sensors and moving at a constant speed of 0.15 m/s. The fleet size is fixed to two robots because, before discussing scalability, we would like to carefully validate our methods in the real-world. In fact, we expect the scalability of these methods to be tightly tied with the environment and gas plume configuration. In past physical experiments we showed that the Crazyflie is a suitable platform for gas detection in indoor spaces [30]. The experimental volume is 7x2x0.5 m^2. Since the objective of this paper is to evaluate the performance of coordination strategies in missions with severe flight time constraints and where full coverage of the volume cannot be achieved using a lawnmower scan, we decided to halve the flight time budget of the NAV to 2 minutes and 15 seconds. We opted to halve the time instead of doubling the volume in order to be able to compare our simulation results to physical results in the future. To be consistent with reality, where all robots have the same time budget, single robot experiments are carried out with the same time limitations as multi-robot experiments. The experiments were repeated ten times for each strategy, and the robots starting positions were randomized on one edge of the volume (corresponding to the Y-axis). A picture portraying the simulation setup and clarifying the coordinate system can be seen in Figure 1.

The biggest drawback of our simulation setup is that it does not take into account the effect that NAVs have on the gas plume. In particular, in past experiments, we observed a significant degradation in performance for the lawnmower movement in the real world, not present for informative path planning techniques [4]. We tried to mitigate this effect by reintroducing the confidence map, as explained in Section II-A. In the scope of a multi-robot setup, the presence of several vehicles could worsen the sensing conditions. To this end, we try and draw conclusions about how far apart we can keep the robots in our algorithm while still obtaining satisfactory performance.

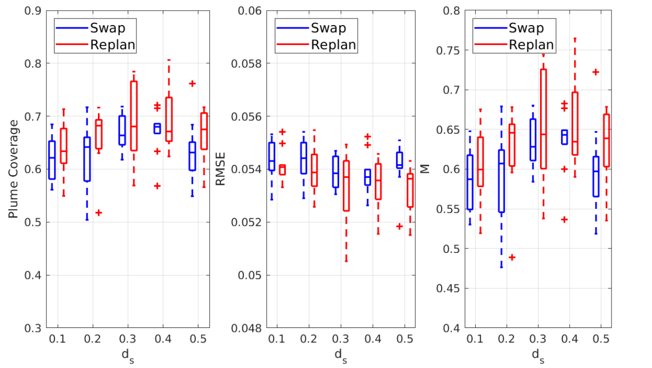

Fig. 2: Impact of the safe distance d_s on the performance of the *Swap* and *Replan* collaborative strategies.

B. Evaluation Metrics

We evaluate the navigation strategies with plume coverage and Root Mean Square Error (RMSE). In this work, we do not use volume coverage or shape coverage as evaluation metrics because the objective of the mapping mission is to capture details of the gas dispersion.

Plume coverage indicates the percentage of the gas plume that is detected in the final gas map. It is computed as:

$$PC = p(d_i \geq Th | g_i \geq Th) \; \forall i \in N \tag{4}$$

where d_i are cells belonging to the final gas map, g_i are ground truth cells and Th is the threshold that determines the presence of gas. In simulation this threshold is determined a priori and it is static. RMSE computes the difference between the final gas map and the ground truth map for the cells that have been updated.

In order to facilitate comparisons among the strategies, the metrics are combined in an overall metric M:

$$M = (PC * |1 - RMSE|) \tag{5}$$

The ground truth distribution is easily acquired in simulation.

IV. Results

This section presents the results of the experimental evaluation. The comparison between map merging strategies for the individualist method, the effect of the d_s parameter on collaborative strategies and a comparison between all approaches are presented.

A 1-way ANOVA test showed no significant difference between maps constructed with different merging criteria (p=0.9149). We chose to keep the variance merging method for further evaluations.

We carefully study the impact of the d_s parameter, which identifies the safe distance between goal positions in collaborative strategies. We want to keep goals far from each other to increase exploration and decrease interference in real experiments. However, goals that are too far away could reduce the exploitation of information regarding the presence of the plume, leading to a less informative final map. A plot showing the impact of d_s is presented in Figure 2. The graph shows that performance for the *Swap* strategy increases when the d_s increases, until $d_s = 0.4m$, where we can observe the highest performance. For $d_s = 0.5m$ we notice a sharp drop in performance, which is due to the lack of exploitation of information because of the high distance kept between drones. Interestingly, the *Replan* strategy seems to be less affected by the choice of d_s, with similar average M value for all $d_s > 0.1$. This can be explained by the fact that all the goals are replanned considering the full amount of information available, allowing for the selection of informative steps even at a distance from each other. Based on this performance evaluation, we decided to pick a value of $d_s = 0.4m$ for the *Swap* strategy and of $d_s = 0.2m$ for the *Replan* strategy.

The comparison between all strategies is presented in Figure 3. All multi-robot strategies outperform the single robot model-free strategy. The addition of cooperation and information sharing between the robots improves slightly on the individualist strategy, but still performs worse than lawnmower. The collaborative strategies, with and without spatial clustering, outperform the lawnmower in the overall metric, with the addition of clusters achieving a boost in performance. Moreover, only the addition of the clustering method allows to obtain comparable RMSE values to the lawnmower. Additionally, the variance of the collaborative methods is relatively low and comparable to the lawnmower's one, while the individualist and cooperative strategies present high variance. These results show the benefits of the coupling of information sharing and movement coordination for a multi-robot gas mapping mission under severe time constraints. Interestingly, the *Swap* strategy outperforms the *Replan* strategy significantly when clustering is applied. This seems to indicate that planning one goal at a time, coupled with task allocation based on distance, is preferred in the reduced volume of the cluster.

We applied the *Swap* collaborative strategy to a team of two robots performing GaSLAM. Figure 3 shows that the model-based strategy is able to capture the plume very well. The outliers in the Plume Coverage plot correspond to two experiments where the robots could not converge to a good belief of the source on time, decreasing the overall performance. The single-robot GaSLAM strategy presents a huge variance in results. This is due to the underlying model converging less frequently to a good belief of the source location with only one robot. The RMSE of the model-based approaches, both with single and multi-robot architectures, presents a very high variance, highlighting the drawback of relying on modelled data instead of directly sensed ones.

A slice of the resulting gas maps for the best runs of several strategies, together with the ground truth, are presented in Figure 4. The plot highlights the limitations of the non-adaptive strategy of the lawnmower, which delivers a patchy map of the environment. The map of the individualist strategy shows that this is the one with the lowest exploration component, and significant improvement on this front is visible in the *Swap* map. The injection of additional elements, in the form of spatial division through clustering and with the

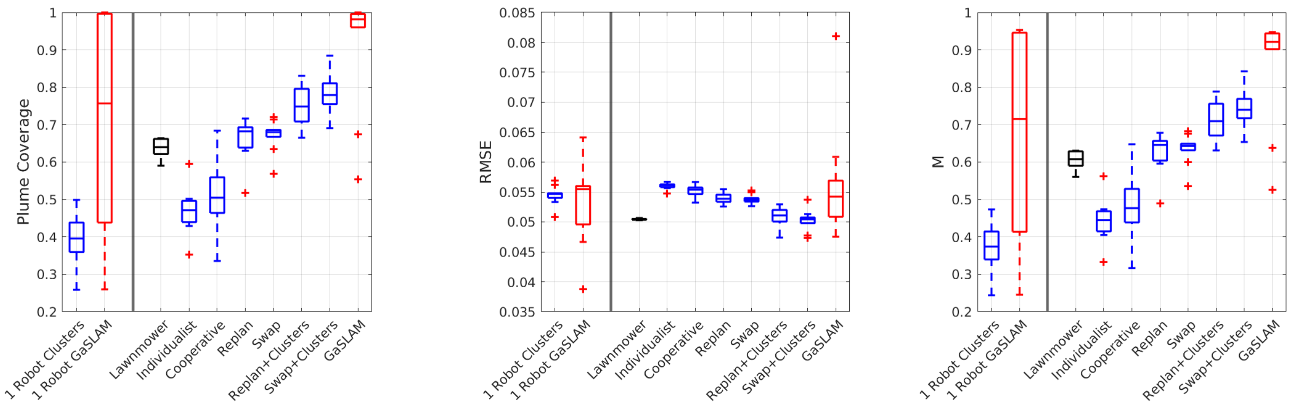

Fig. 3: Performance comparison between all strategies. Single-robot strategies are on the left side of each plot. Blue plots correspond to model-free navigation strategies, red plots to model-based navigation strategies and black plots to the results coming from the lawnmower. Higher values are better for plume coverage and metric, lower values are better for RMSE.

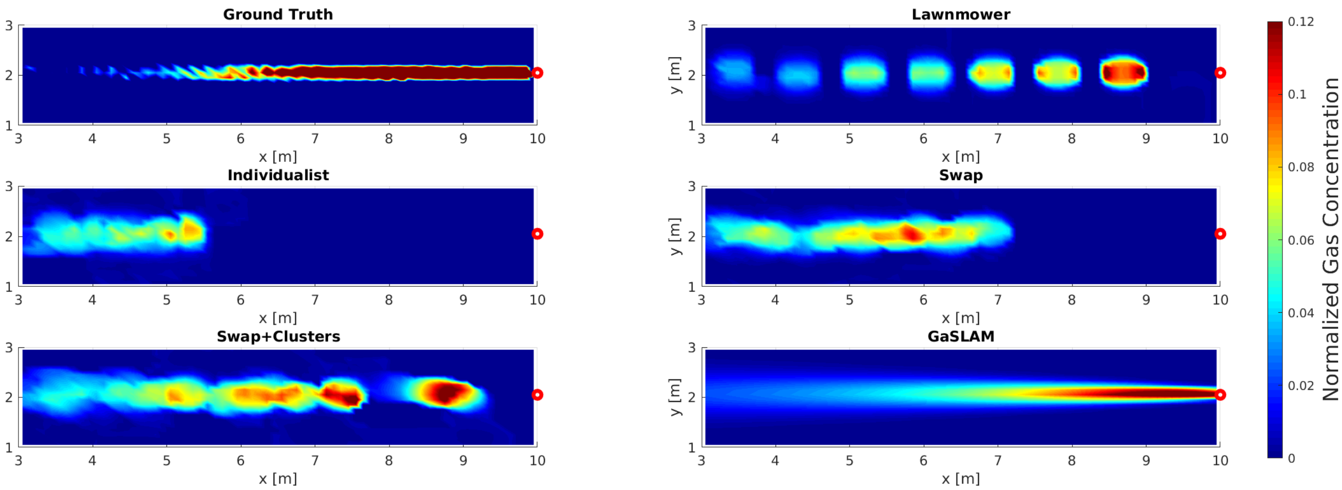

Fig. 4: Resulting gas distribution maps for some of the proposed strategies. The red circle represents the gas source position. Note that the Z-axis is omitted to simplify the visualization. The coordinate origin is in a corner of the simulated wind tunnel, which is reflected by the fact that X and Y-axis do not start from 0 in the figure.

model-based approach, significantly increase the quality of the final maps, which are able to capture almost the entirety of the plume. It must be noted that, while the GaSLAM map closely resembles the ground truth, reliance of a model makes it hard to use this methods in all scenarios where the model does not hold, for example in the presence of obstacles or of multiple gas sources.

V. Conclusion

In this paper, we study the effects of information sharing, coordination, and environmental knowledge on a multi-robot system performing a gas mapping mission. We show the impact of a multi-robot architecture, which significantly improves the final gas map. We also demonstrate the boost in performance of increasing coordination for model-free exploration under severe time constraints with respect to the lawnmower and single robot baselines. Moreover, we boost the performance of our collaborative strategies by injecting guided spatial exploration through clustering and model-based navigation. One important conclusion that can be drawn is that a solely cooperative strategy, relying only on information sharing, cannot outperform the non-adaptive path planning strategy. Collaboration, global knowledge or additional spatial guidance are needed to outperform it. This conclusion highlights the competitiveness of the lawnmower strategy for scenarios where sensing capabilities are scarce and limited to a small area.

The results obtained in this paper will serve as a solid base to conduct the physical experiments, as they explore and compare several different strategies, highlighting their advantages and drawbacks. Further steps will involve the deployment of the algorithms in a distributed fashion by employing robots with higher on-board capabilities than Crazyflies, which will allow, for example, to improve the collision avoidance strategy and move towards an autonomous MRS. Finally, the model-based strategy could be coupled with non-myopic navigation strategies, to further boost its performance.

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