

Planning Assembly Sequence with Graph Transformer

Lin Ma¹, Jiangtao Gong^{1✉}, Hao Xu², Hao Chen², Hao Zhao¹, Wenbing Huang¹ and Guyue Zhou¹

Abstract—Assembly Sequence Planning (ASP) is the essential process for modern manufacturing, proven to be NP-complete thus its effective and efficient solution has been a challenge for researchers in the field. In this paper, we present a graph-transformer based framework for the ASP problem which is trained and demonstrated on a self-collected ASP database. The ASP database contains a self-collected set of LEGO models. The LEGO model is abstracted to a heterogeneous graph structure after a thorough analysis of the original structure and feature extraction. The ground truth assembly sequence is first generated by brute-force search and then adjusted manually to be in line with human rational habits. Based on this self-collected ASP dataset, we propose a heterogeneous graph-transformer framework to learn the latent rules for assembly planning. We evaluated the proposed framework in a series of experiments. The results show that the similarity of the predicted and ground truth sequences can reach 0.44, a medium correlation measured by Kendall’s τ . Meanwhile, we compared the different effects of node features and edge features and generated a feasible and reasonable assembly sequence as a benchmark for further research. Our dataset and code are available on: https://github.com/AIR-DISCOVER/ICRA_ASP.

I. INTRODUCTION

To facilitate automatic assembly in a wide range of fields, such as the furniture industry, auto manufacturing industry, and arts and crafts industry, assembly sequence planning is a key procedure after product design.

The dominant methods for ASP problems are assembly-by-disassembly planning and the combinatorial optimization algorithm. To be detailed, the main task of Assembly-by-disassembly planning is finding a feasible removing path by recursively removing parts from the assembly product [1] [2]. Moreover, the combinatorial optimization algorithm aims at determining the precedence relationship among all parts under complex constraints [3] [4] using dynamic programming.

ASP is proven to be NP-complete, so its effective and efficient solution has been a challenge for researchers in the field. Consequently, there is much room for progress in this research. Deep learning methods have been widely used in various areas and proven to be effective, so we expect to use deep learning methods to learn the latent rules of an assembly problem from previous experience. However, the two dominant methods mentioned above can only find a solution for a specific assembly model. As a result, there is no dataset specialized for ASP problems to train a deep learning model.

The datasets most related to our task are illustrated as follows. A main application of ASP is generating step-by-step

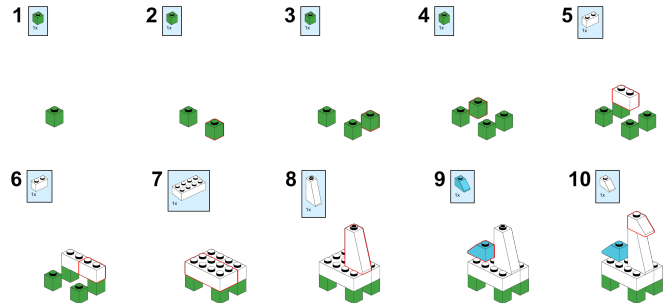


Fig. 1. An example of Assembly sequence planning

instructions for furniture and household articles, which are also universally used in many research [5]. 80 IKEA furniture models are present and applied to assembly environment design in [6]. These furniture models are made up of several parts without complex block relationships, so the main task is to ensure stability or visibility in the assembly process. Moreover, a large-scale part-level 3D dataset containing various objects in indoor scenes was released [7]. This dataset enables many 3D generative tasks at the part level, such as estimating the location and pose of parts [8] and learning a consistent part order for a given object category [9]. Except for furniture and household items, 3D shapes simulated by the LEGO model are generated via sequence assembly [10] [11]. However, the aforementioned assembly datasets are mostly designed or collected for generative tasks rather than sequence assembly planning, so a dataset specifically for the ASP problem is demanded. A suitable assembly dataset composed of a moderate number of instances and containing common block relationships would provide us with more opportunities to establish universal solutions to the ASP problem.

However, the collection of assembly instances is no easy task for the following reasons. Firstly, the instances are supposed to be sufficient for training and homogeneous with each other. Nevertheless, the majority of industries only release a few templates. Secondly, even if we have the templates, the assembly sequences, namely, the ground truth, are always not available.

In this paper, we present a self-collected LEGO dataset for the ASP task. Structures built from LEGO are complex enough to approach real-world design and have a large number of models designed and uploaded by users. Unfortunately, not all models are practical, so we made some adjustments, which will be illustrated in section III. Moreover, we propose a graph-transformer framework for ASP. Our goal is to train a network to learn the latent rules of the assembly sequence and automatically generate a feasible

¹Institute for AI Industry Research (AIR), Tsinghua University, 10080, Haidian District, Beijing, P.R.China. lastnamefirstname@air.tsinghua.edu.cn

²Qianzhi Technology, China hao.xu.chn@gmail.com

assembly sequence, such as Fig.1.

The main contribution of this work can be summarized as follows.

- 1) A self-collected ASP dataset using user-defined models from LEGO Studio and feasible assembly sequences by brute force method and manual adjustment as ground truth;
- 2) A graph-transformer framework for ASP problem, with a heterogeneous graph attention network to encode the LEGO models, which are decoded with the attention mechanism to generate assembly sequences;
- 3) A series of experiments to evaluate the effectiveness of the proposed framework and offer the benchmark for this ASP problem.

II. RELATED WORK

Assembly sequence planning ASP problems are usually solved by assembly-by-disassembly planning and the combinatorial optimization algorithm. The assembly-by-disassembly method finds a feasible disassembly order via motion planning [12]. Being NP-complete, it requires numerous search operations even to find a feasible disassembly path. Thus researchers compress the search space using a tree structure, such as ‘‘Rapidly-exploring random tree’’ [13] and ‘‘Expansive Voronoi tree’’ [14]. Nevertheless, the combinatorial optimization algorithm always implements the heuristic methods, such as Ant Colony Optimization algorithm [15], Gray Wolf optimization algorithm [16], and Artificial Neural Networks [17].

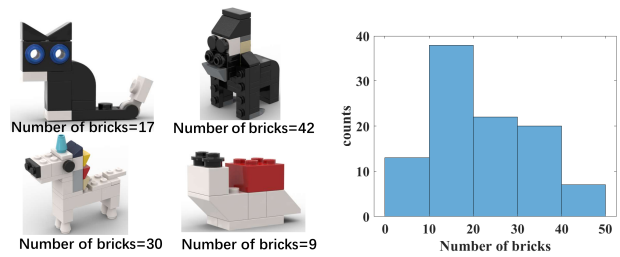
ASP problems have not been extensively studied by deep learning methods. To the best of our knowledge, the only method related to deep learning is proposed in [18]. We believe there is still much room for progress in this research.

Graph-transformer The transformer has become the standard architecture for a wild variety of fields. In particular, its variants have been modified to explore the latent feature of graph-structured data and fulfill different tasks for natural language processing [19], biological molecular structure [20] [21], social networks [22], and academic network [23].

In a heterogeneous graph, different types of objects are mapped to different types of nodes, and the same is true for edges [24] [25]. This abstraction is more appropriate for containing more complete information. Accordingly, LEGO models are similarly devised to heterogeneous graphs in our dataset. Different node types represent different LEGO bricks and different edge types for different relations between LEGO bricks.

Almost all of the tasks of the graph-transformer framework are node classification, graph classification, link prediction, and text generation, but barely see its application in sequence planning. To the best of our knowledge, this is the first attempt to utilize graph-transformer in an assembly sequence planning task.

LEGO model As an abstraction of real-world objects, the LEGO model is a powerful tool for promoting and testing human cognitive abilities[26]. Moreover, cognitive loads are usually studied during LEGO assembly tasks [27].



(a) (b)
Fig. 2. Overview of the LEGO models.

Since LEGO models are complex enough to approach real-world design, it is common for researchers and designers to regard the LEGO model as an abstraction of a physical object. By studying the LEGO model, we can increase our cognition of the real world and be inspired to solve practical problems. For example, LEGO models are used to approximate actual 3D shapes while ensuring that the final products have some good properties such as connectivity and stability [28] [29]. Furthermore, the LEGO models abstracted from physical objects are applied to the deep generative models for 3D shape generative tasks [10] [11].

In the above scenario, the LEGO model is abstracted into LEGO Model Representation graph [30] [10], with a single vertex in the graph representing a LEGO brick (regular brick, plate, or tile) and an edge representing the linkage between two LEGO bricks. This is consistent with real-world assembly problems. For example, furniture and cars are abstracted to graph structures [7] [1].

III. LEGO-ASP DATASET

A. Dataset summary

We collect 100 LEGO animal models created and uploaded by individual users in LEGO Studio, among which the simplest one is composed of 3 bricks and the most complex one is composed of 44 bricks. The median of brick numbers in a LEGO model is 19. As shown in Fig.2.(a), we present four models with different numbers of bricks as examples. And the statistics of the number of bricks in an individual LEGO model are demonstrated in Fig.2.(b).

The total number of bricks in all LEGO models is 2127, of which 23 types of bricks are frequently used. The rest are used only a few times or customized, but they take up a non-negligible percentage as a whole. According to the role of bricks, the frequently-used bricks can be roughly categorized into the following three types: regular brick, plate, and tile. In addition, they can also be categorized into four types according to their shape: cubic, curve, slope, and round. At the same time, the bricks are of different sizes. With different combinations of these features, a large variety of bricks exist. We present different types of LEGO bricks in Fig.3.(a). For example, the first brick is a regular brick as well as a round type. The second is a customized brick, the seventh is a plate of size 1x2. Moreover, the ratio of different types is presented in Fig.3.(b).

However, these LEGO models uploaded by individual users may not be carefully checked, so separations and

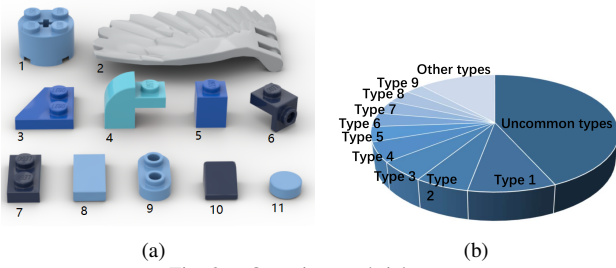


Fig. 3. Overview on brick types.

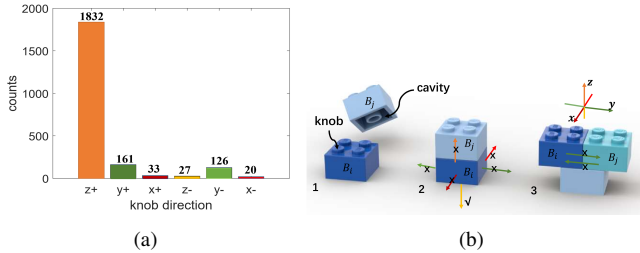


Fig. 4. Statistics on knob directions.

collisions may occur. In this case, we checked all models and fix the separations and collisions, while preserving the original shapes and structures as much as possible.

What should be clarified carefully is the mode of connection. As is shown in Fig.4.(b), two blocks are snapped together by a knob from one brick and a cavity from another, which ensures the stability of the LEGO models.

B. Heterogeneous graph

It is intuitive to regard the bricks as nodes and their relationships as edges. In this way, we convert LEGO models into graph structures. Nevertheless, there is additional information about the nodes and edges. In the next part, we will illustrate the node features and edge features extracted from the original LEGO models in detail.

1) Node features

In the LEGO model, every brick has a unique feature, so regarding bricks as identical nodes will cause information loss. Therefore, we need to categorize nodes into different types and capture the unique feature of nodes. As mentioned above, we labeled 23 types of frequently used bricks and the rest of the bricks are equally labeled as uncommon-type. In this method, the node feature corresponding to types is represented as a one-hot vector of 24 dimensions.

In addition, we also consider the positions as important features. Nevertheless, the pattern of positions is not consistent in different LEGO models for the following reasons. First, the start positions and viewing angle are randomly set when customers create their models, so the entire models are casually scattered. Second, users often rotate the models for convenience or a better perspective. Given the inconsistency of positions, we deliberately construct a coordinate system to make the different LEGO models as consistent as possible. In detail, there are three steps to determine the positions of bricks.

- The first step is to determine the direction of the coordinate system. By observing many LEGO assembly

videos and parsing LEGO product instructions, we discovered a *principle assembly direction*. This direction is consistent with the direction in which the majority of bricks' knobs orient, so we define it as the positive direction of the z -axis (vertical). Analogously, the *second assembly direction* is the direction in which bricks' knobs orient only ranks second to the *principal assembly direction*. Then the *second assembly direction* is defined as the positive direction of the y -axis (transversal). As the positive directions of the y -axis and z -axis are fixed, we can easily infer the positive direction of the x -axis (longitude) in a left-hand coordinate system. That is because the left-hand coordinate system is common in computer graphics. Moreover, we also verify the rationality of this setting by exploring the structures of the collected LEGO models. As shown in Fig.4., nearly all the knobs are oriented in the *principle assembly direction*, which is set to be the positive direction of the z -axis ($z+$ for short, and the same manner with $y+, y-, x+, x-$), and very few knobs are oriented in $z-$. Moreover, the number of knobs orient $y+$ is close to the number of knobs orient $y-$, which may correspond symmetric structures.

- The second step is to determine the origin of the coordinate system \mathcal{O} . We find the bounding box of the whole LEGO model, denoted as \mathcal{B}_m . The origin is set to be the vertex on \mathcal{B}_m , whose x, y, z coordinate is the smallest.
- Considering the coordinate system is constructed, the third step is to calculate the final positions of the nodes. For i -th node v_i corresponding to brick B_i . We denote its bounding box as $\mathcal{B}_i, i = 1, \dots, n$, where n represents the number of bricks in the LEGO model. For v_i , we set its position as the center of the \mathcal{B}_i . After we got all the positions of the nodes, we normalize them to the unit cube as the final node features of positions.

2) Edge features

In graph structure, edges between nodes represent the relationships between bricks. In the LEGO model, the most obvious relationship between bricks is the link. However, the block relationship between nodes is equally important, which determines whether the assembly process is feasible. The block relationship is defined for adjacent nodes, which shows the spatial continuity in ASP requirements. Next, we will illustrate the two types of relationships and their corresponding edges in detail.

- Link edge: if bricks B_i and B_j are connected with a knob-cavity structure. Then there is a link edge between the corresponding nodes v_i and v_j . This edge is called a link edge, denoted as relation triplet $\langle v_i, links, v_j \rangle$, and $\langle v_j, links, v_i \rangle$. As shown in Fig.4.(b) middle.
- Block edge: Compared with the link relationship, the block relationship is more intricate. If brick B_i collides with brick B_j when it moves a short distance along the direction of the axes (both positive and negative), namely, brick B_i cannot be assembled or removed

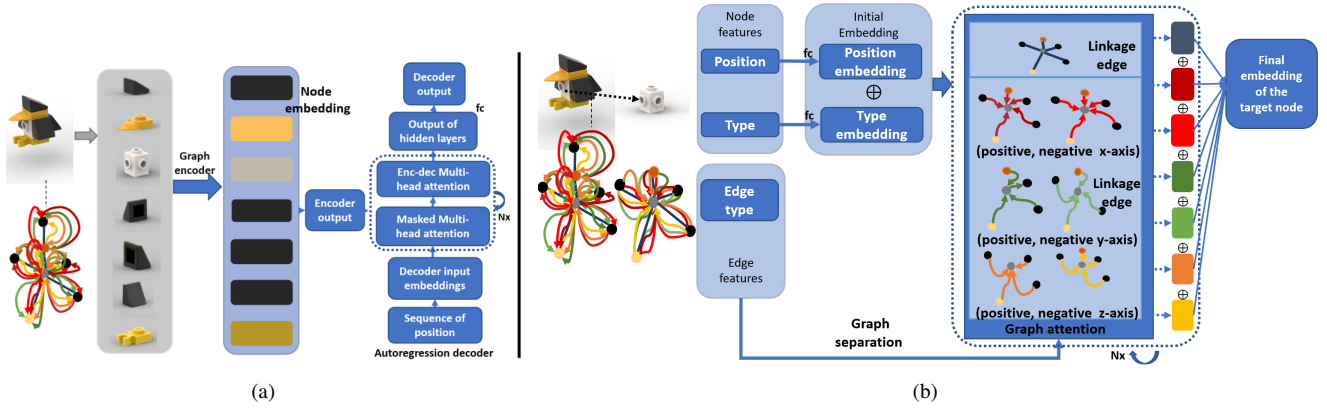


Fig. 5. Workflow: (a) framework; (b) graph encoder for one node.

under the condition that brick B_j is already assembled. Then there is a directed edge from the corresponding node v_j to v_i . In this way, the block relationship has directions. We treat different block directions as different edges. It is also denoted as a relation triplet, for example, $\langle v_i, \text{block in directions of the positive } x\text{-axis}, v_j \rangle$. As shown in Fig.4.(b) middle, brick B_i can only be snapped together with B_j along with the direction of the positive z -axis. It is blocked by B_j in the direction of the x -axis (positive, negative), y -axis (positive, negative), and z -axis (negative). What should be clarified is that the block relationship exists not only between linked bricks but also between adjacent bricks. As shown in Fig.4.(b) right, B_i and B_j are placed side-by-side and blocked with each other in the positive and negative directions of the y -axis.

So far, we have constructed edge-heterogeneous graph structures. As mentioned in part A, we have fixed all the separations, that is, all the bricks are connected with the knob-cavity structure, which guarantees the connectivity of the generated graph structure.

C. Ground Truth

In the LEGO Studio platform, users can freely create their characters. They can causally add or remove the bricks even if other bricks are blocking them. However, it is infeasible in physical assembly. Therefore, we cannot use this creation sequence as the ground truth for assembly, even though the LEGO Studio has preserved it. In order to efficiently obtain a feasible and reasonable assembly sequence as ground truth, we adopt a two-step process.

- First, we generate a feasible assembly sequence in the assembly-by-disassembly approach proposed in [31]. However, the pattern of our self-collected LEGO models is different from the humanoid LEGO models, so the generated assembly sequence is not as good as it is in [31]. Therefore, there is an additional modification step.
- Secondly, considering the reasonability of the assembly sequence, we manually adapt the assembly sequence according to the following principles. (1) The order of

substructures with similar semantic meaning is sequential. (2) For spatial continuity, we modified the situation if two bricks were located far away but assembled one after another.

IV. GRAPH-TRANSFORMER FOR ASP PROBLEM

Considering the particularity of the ASP problem in the LEGO model, we propose a new graph-transformer framework to predict an assembly sequence. The graph-transformer framework is an end-to-end framework for which the input is a heterogeneous graph and the output is the predicted assembly positions.

In Section III, we abstract the LEGO model to the graph structure, so the following statement will be based on the terminology of the graph structure. Our framework consists of two parts, graph encoder and transformer decoder. The workflow of our framework is shown in Fig.5.(a). For conciseness, the normalization layer is left out and will also not be mentioned in the following model description.

A. Heterogeneous Graph encoder

1) *Graph attention*: The majority of transformer-type models adopt the tuple (Q, K, V) to perform the attention mechanism, represented in the following expression:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V,$$

where d_k is the dimension of v . We degenerate this expression to node level, for the i -th node:

$$\text{Attention}(Q_i, K, V) = \sum_{j=1}^n \text{softmax}\left(\frac{Q_i K_j}{\sqrt{d_k}}\right)V_j.$$

In the above expression, n is the number of tokens. Transferring to our setup, n is the number of nodes in the graph structure. This means all of the nodes in the graph structure have an impact on the i -th node and they are treated equally. To this end, the graph structure has not been well explored and utilized, for which we adopt the treatment in [32]. We perform masked attention—only consider the neighbor nodes j for i -th node, $j \in \mathcal{N}_i$ where \mathcal{N}_i is neighbor set of i

(connect i -th node with edges). In this way, the attention of i -th node becomes:

$$\text{Attention}(Q_i, K, V) = \sum_{j \in \mathcal{N}_i} \text{softmax}\left(\frac{Q_i K_j}{\sqrt{d_k}}\right) V_j.$$

2) *Heterogeneous edges for attention mechanism*: In the above part, we restricted the attention mechanism to the neighbors, which preserves the graph structure to a certain extent. However, it does not take into account the types of edges. As mentioned in section III, there are link edges and block edges. Meanwhile, the block edges have different directions. In the assembly process, they are of great difference, so we discriminate them in the graph-transformer framework. We no longer use shared linear transformations for Q, K, V but represent each type of edge with an individual linear transformation:

$$\text{Attention}(Q_i, K, V) = \sum_{t=1}^7 \sum_{j \in \mathcal{N}_i^t} \text{softmax}\left(\frac{Q_i^t K_j^t}{\sqrt{d_k}}\right) V_j^t.$$

where the $t = 1, 2, \dots, 7$ represent different edge types, that is, link edge and six types of block edge; \mathcal{N}_i^t represents the set of neighbors connected with t -th edge type and Q^t, K^t, V^t are obtained under the linear transformation corresponding to t -th edge type, seeing Fig.5.(b).

As it is shown in Fig.5., this heterogeneous graph attention is equivalent to dismembering the heterogeneous graph into homogeneous “subgraphs” according to the types of edges. After applying graph attention independently, we aggregate the attention of all “subgraphs”. Iterating this process in a stack of N identical layers, we got the final encoding output.

B. Transformer decoder

As shown in Fig.5.(a), the decoder part of our model is identical to the autoregression decoder of vanilla transformer, so we will not illustrate it in detail. Nevertheless, we would like to clarify that the input and output of our decoder are sequences of position, that is, our target is to predict the next predicted position. In this way, we set the next node (or brick to be assembled) to be the one whose distance to the predicted location is the shortest. To this end, we set the loss function to be the Mean Squared Error (MSE) between the real sequence of positions and the predicted sequence of positions.

V. EXPERIMENT

This section consists of four parts. First, we state the metrics used to evaluate the similarity of the predicted and the ground truth assembly sequence. Second, we demonstrate the impact of anomalous data items. Third, we test the effect of node types and edge types in ablation studies. Finally, we provide qualitative and quantitative results of our model on an in-process task.

A. Metrics

Since there is no mature algorithm to check whether a sequence is feasible, we propose using the following two metrics to evaluate the quality of the predicted assembly sequence: Kendall’s τ and the Regularized Location Swap

Deviation (RLSD). The following Kendall’s τ is used to measure the similarity of node priorities.

$$\tau = \frac{N_{\text{concordant}} - N_{\text{discordant}}}{n(n-2)/2}.$$

Where n represents the length of the sequence and the $N_{\text{concordant}}$ and $N_{\text{discordant}}$ respectively represent the number of concordant pairs and discordant pairs. We would like to demonstrate the concordant pairs in our setup with the following example: if brick B_i is assembled earlier (or later) than brick B_j in both ground truth and predicted sequence, then we call it a concordant pair and call it discordant pair for the opposing situation.

The second metric is Regularized Location Square Deviation. The conventional LSD can be viewed as the mean squared error (MSE) for components’ locations in the sequence.

$$\text{LSD}(S_1, S_2) = \frac{1}{n} \sum_{i=1}^{n-1} (i[S_1] - i[S_2])^2.$$

Where S_1, S_2 is two sequences and $i[S]$ is the location of i on the sequence S . However, the value of LSD greatly depends on the length of a sequence, thus it is divided by a coefficient of regularization $(n+1)(n-1)/2$. In this way, the value of RLSD is constrained in $[0, 1]$ whether the sequence is long or short. The less the value, the better the predicted sequence.

B. Impact of large graphs

According to Fig.2. in Section II, there is a large difference in the number of bricks in the individual LEGO model, which means the number of nodes in a graph varies. The more nodes a graph has, the more complex its structure is. And it will tend to have a hierarchical structure, so its pattern will no longer be consistent with short ones. Thus it will be an outlier with a high probability. To this end, we eliminate the top 20% largest graphs and 40% largest graphs from the dataset, The result is shown in TABLE I.

TABLE I
IMPACT OF LARGE GRAPHS

	whole data	smallest 80%	smallest 60%
Kendall’s τ \uparrow	0.1700	0.2267	0.4482
RLSD \downarrow	0.3948	0.3711	0.2440

The smallest 80% means we use only 80% of data items whose number of nodes is smallest, in which 80% * 75% is the training set and the rest 80% * 25% is the test set. The same is true for the smallest 60%. The result in the next parts is also obtained under the setting: *size of training set* : *size of test set* = 3 : 1

The results show that when the number of nodes is small, the pattern of the graphs is more consistent. Under this condition, the graph-transformer framework is able to capture general rules. Kendall’s τ is around 0.44, which is not a strong correlation but a moderate one. However, we still regard it as meaningful for the following reasons. First,

Kendall’s τ is larger than 0 significantly, which means our predicted sequence has a positive correlation with the ground truth, so it captures the trend of assembly to a certain degree. Second, in this method, we can obtain a sequence whose similarity to a feasible solution is 44 percent. Based on this sequence, we drastically reduce the search space of an NP-complete problem. Moreover, the RLSD is around 0.24, meaning that transferring this sequence to the ground truth requires much less swapping of elements in the sequence. Finally, we are aiming at generating a benchmark, which could be improved for future research.

C. Ablation study

In this part, a series of ablation studies are conducted to show the effectiveness of different graph features. The results are shown in TABLE II. First, we explore the node features. Since the output of our graph-transformer framework is a sequence of positions, thus, as features of nodes, the position is uneliminable for the model. To this end, we only explore the effect of the node type. Unexpectedly, rather than improving the performance, node type has a negative impact on the performance. After careful consideration, we find a possible reason to account for this situation. It is the random combination of different types of bricks that ensures that a variety of 3D shapes can be generated. Consequently, the type of nodes does not have an effect in predicting the order of the assembly and acts as piecemeal information during training. Moreover, we explore the effect of edge type. The results show that there has been a huge improvement when we add both the link and block edge to the graph-transformer framework. This result implies that both link and block edges are critical factors for the ASP problem, and they play different roles.

TABLE II
ABLATION STUDY

	position		position & type	
	$\tau \uparrow$	RLSD \downarrow	$\tau \uparrow$	RLSD \downarrow
Link edge	0.3717	0.2892	0.2764	0.3240
Block edge	0.3630	0.2944	0.3338	0.2954
Link & Block	0.4482	0.2440	0.4325	0.2422

D. In-process task

In this part, we design an in-process assembly sequence prediction task. Assuming that a LEGO model is partially assembled, we test the performance of our graph-transformer framework by completing the following assembly procedure. In detail, we set the degrees of completion as 20%, 40%, 60% and 80%. Based on these half-finished LEGO models, we complete the assembly sequence and obtain the results shown in Fig.6.

As presented in Fig.6, with the increase of the degree of completion, Kendall’s τ increases and the RLSD drops, which means the accuracy of our prediction is increased rapidly. Based on a given half-finished model, the graph-transformer framework has a preliminary understanding of the model and captures the unique features of the individual

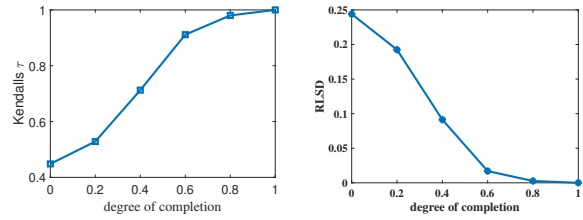


Fig. 6. Result of an in-process task.

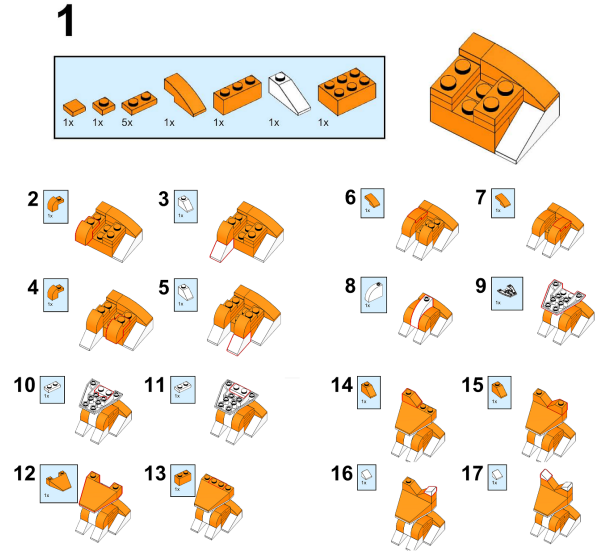


Fig. 7. Result of the in-process task.

models. Consequently, it is a great advance over the results obtained by the direct prediction in part B. Meanwhile, we present an example of this in-process task in Fig.7. We initialize the model to be 40%-completed and labeled it as 1 in the figure. The following results are feasible and reasonable. In summary, our graph-transformer framework performs well on in-process tasks.

VI. CONCLUSION AND FUTURE WORK

In this paper, we present a self-collected dataset of LEGO models for the ASP problem and formulate it into a heterogeneous graph structure, which we believe can be applied to other tasks such as LEGO generative models. In addition, we proposed a graph-transformer model for the heterogeneous graphs to predict an assembly sequence. However, our model does not perform well on large graphs. In the future, we would like to organize the nodes in a hierarchical structure. In this way, the task for large graphs can be divided into hierarchical subtasks, which can be solved efficiently and efficiently by our graph-transformer framework.

ACKNOWLEDGEMENT

We are grateful to Qianzhi Technology and Qimeng for supporting this project, especially regarding the feasible assembly sequence generation.

REFERENCES

- [1] S. Dorn, N. Wolpert, and E. Schömer, “An assembly sequence planning framework for complex data using general voronoi diagram,” in *2022 International Conference on Robotics and Automation (ICRA)*. IEEE, 2022, pp. 9896–9902.
- [2] Y.-J. Tseng, F.-Y. Yu, and F.-Y. Huang, “A green assembly sequence planning model with a closed-loop assembly and disassembly sequence planning using a particle swarm optimization method,” *The International Journal of Advanced Manufacturing Technology*, vol. 57, no. 9, pp. 1183–1197, 2011.
- [3] P. Jiménez, “Survey on assembly sequencing: a combinatorial and geometrical perspective,” *Journal of Intelligent Manufacturing*, vol. 24, no. 2, pp. 235–250, 2013.
- [4] S. Ghandi and E. Masehian, “A breakout local search (bls) method for solving the assembly sequence planning problem,” *Engineering Applications of Artificial Intelligence*, vol. 39, pp. 245–266, 2015.
- [5] M. Agrawala, D. Phan, J. Heiser, J. Haymaker, J. Klingner, P. Hanrahan, and B. Tversky, “Designing effective step-by-step assembly instructions,” *ACM Transactions on Graphics (TOG)*, vol. 22, no. 3, pp. 828–837, 2003.
- [6] Y. Lee, E. S. Hu, and J. J. Lim, “Ikea furniture assembly environment for long-horizon complex manipulation tasks,” in *2021 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2021, pp. 6343–6349.
- [7] K. Mo, S. Zhu, A. X. Chang, L. Yi, S. Tripathi, L. J. Guibas, and H. Su, “Partnet: A large-scale benchmark for fine-grained and hierarchical part-level 3d object understanding,” in *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition*, 2019, pp. 909–918.
- [8] R. Zhang, T. Kong, W. Wang, X. Han, and M. You, “3d part assembly generation with instance encoded transformer,” *IEEE Robotics and Automation Letters*, vol. 7, no. 4, pp. 9051–9058, 2022.
- [9] R. Wu, Y. Zhuang, K. Xu, H. Zhang, and B. Chen, “Pq-net: A generative part seq2seq network for 3d shapes,” in *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, 2020, pp. 829–838.
- [10] R. Thompson, E. Ghaleb, T. DeVries, and G. W. Taylor, “Building lego using deep generative models of graphs,” *arXiv preprint arXiv:2012.11543*, 2020.
- [11] J. Kim, H. Chung, J. Lee, M. Cho, and J. Park, “Combinatorial 3d shape generation via sequential assembly,” *arXiv preprint arXiv:2004.07414*, 2020.
- [12] S. Sundaram, I. Remmler, and N. M. Amato, “Disassembly sequencing using a motion planning approach,” in *Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation (Cat. No. 01CH37164)*, vol. 2. IEEE, 2001, pp. 1475–1480.
- [13] S. M. LaValle *et al.*, “Rapidly-exploring random trees: A new tool for path planning,” 1998.
- [14] S. Karaman and E. Frazzoli, “Sampling-based algorithms for optimal motion planning,” *The international journal of robotics research*, vol. 30, no. 7, pp. 846–894, 2011.
- [15] Z. Han, Y. Wang, and D. Tian, “Ant colony optimization for assembly sequence planning based on parameters optimization,” *Frontiers of Mechanical Engineering*, vol. 16, no. 2, pp. 393–409, 2021.
- [16] M. F. F. Ab Rashid, “A hybrid ant-wolf algorithm to optimize assembly sequence planning problem,” *Assembly Automation*, 2017.
- [17] C. Philip Chen, “Design of a real-time and/or assembly scheduler on an optimization neural network,” *Journal of Intelligent Manufacturing*, vol. 3, no. 4, pp. 251–261, 1992.
- [18] M. Zhao, X. Guo, X. Zhang, Y. Fang, and Y. Ou, “Aspw-drl: assembly sequence planning for workpieces via a deep reinforcement learning approach,” *Assembly Automation*, 2019.
- [19] D. Cai and W. Lam, “Graph transformer for graph-to-sequence learning,” in *Proceedings of the AAAI conference on artificial intelligence*, vol. 34, no. 05, 2020, pp. 7464–7471.
- [20] V. P. Dwivedi and X. Bresson, “A generalization of transformer networks to graphs,” *arXiv preprint arXiv:2012.09699*, 2020.
- [21] C. Ying, T. Cai, S. Luo, S. Zheng, G. Ke, D. He, Y. Shen, and T.-Y. Liu, “Do transformers really perform badly for graph representation?” *Advances in Neural Information Processing Systems*, vol. 34, pp. 28 877–28 888, 2021.
- [22] G. N. Chandrika, K. Alnowibet, K. S. Kautish, E. S. Reddy, A. F. Alrasheedi, and A. W. Mohamed, “Graph transformer for communities detection in social networks,” *CMC-COMPUTERS MATERIALS & CONTINUA*, vol. 70, no. 3, pp. 5707–5720, 2022.
- [23] Z. Hu, Y. Dong, K. Wang, and Y. Sun, “Heterogeneous graph transformer,” in *Proceedings of The Web Conference 2020*, 2020, pp. 2704–2710.
- [24] S. Yao, T. Wang, and X. Wan, “Heterogeneous graph transformer for graph-to-sequence learning,” in *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, 2020, pp. 7145–7154.
- [25] K. P. Kumar and M. L. Gavrilova, “User identification in online social networks using graph transformer networks,” in *2021 18th International Conference on Privacy, Security and Trust (PST)*. IEEE, 2021, pp. 1–10.
- [26] R. Horikoshi, “Teaching chemistry with lego® bricks,” *Chemistry Teacher International*, vol. 3, no. 3, pp. 239–255, 2021.
- [27] Z. Yang, J. Shi, W. Jiang, Y. Sui, Y. Wu, S. Ma, C. Kang, and H. Li, “Influences of augmented reality assistance on performance and cognitive loads in different stages of assembly task,” *Frontiers in psychology*, vol. 10, p. 1703, 2019.
- [28] H. Xu, K.-H. Hui, C.-W. Fu, and H. Zhang, “Computational lego technic design,” *arXiv preprint arXiv:2007.02245*, 2020.
- [29] B. Stephenson, “A multi-phase search approach to the lego construction problem,” in *Ninth Annual Symposium on Combinatorial Search*, 2016.
- [30] J. W. Kim, K. K. Kang, and J. H. Lee, “Survey on automated lego assembly construction,” *to be added*, 2014.
- [31] G. Zhou, L. Luo, H. Xu, X. Zhang, H. Guo, and H. Zhao, “Brick yourself within 3 minutes,” in *2022 International Conference on Robotics and Automation (ICRA)*. IEEE, 2022, pp. 6261–6267.
- [32] P. Veličković, G. Cucurull, A. Casanova, A. Romero, P. Lio, and Y. Bengio, “Graph attention networks,” *arXiv preprint arXiv:1710.10903*, 2017.