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Minimally Constrained Multi-Robot Coordination with Line-of-Sight Connectivity Maintenance

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Abstract— **In this paper, we consider a team of mobile robots executing simultaneously multiple behaviors by different subgroups, while maintaining global and subgroup line-of-sight (LOS) network connectivity that minimally constrains the original multi-robot behaviors. The LOS connectivity between pairwise robots is preserved when two robots stay within the limited communication range and their LOS remains occlusion-free from static obstacles while moving. By using control barrier functions (CBF) and minimum volume enclosing ellipsoids (MVEE), we first introduce the LOS connectivity barrier certificate (LOS-CBC) to characterize the state-dependent admissible control space for pairwise robots, from which their resulting motion will keep the two robots LOS connected over time. We then propose the Minimum Line-of-Sight Connectivity Constraint Spanning Tree (MLCCST) as a step-wise bilevel optimization framework to jointly optimize (a) the minimum set of LOS edges to actively maintain, and (b) the control revision with respect to a nominal multi-robot controller due to LOS connectivity maintenance. As proved in the theoretical analysis, this allows the robots to improvise the optimal composition of LOS-CBC control constraints that are least constraining around the nominal controllers, and at the same time enforce the global and subgroup LOS connectivity through the resulting preserved set of pairwise LOS edges. The framework thus leads to robots staying as close to their nominal behaviors, while exhibiting dynamically changing LOS-connected network topology that provides the greatest flexibility for the existing multi-robot tasks in real-time. We demonstrate the effectiveness of our approach through simulations with up to 64 robots.**

I. INTRODUCTION

Connectivity maintenance is critical to ensure effective information exchange among robots. It is often achieved through constraining robots' motion due to their limited communication range, so that the proximity-based communication graph remains as one connected component when robots are moving, commonly referred to as maintaining *global connectivity* [1]–[6]. Most of the existing methods use either local methods that maintain graph connectivity by preserving the initial connectivity graph topology [7]–[9], or global methods that keep the second smallest eigenvalue of the graph Laplacian positive through a secondary connectivity controller [1]–[3], [6], [10].

However, these works assume a simplistic connectivity model where pairwise robots can always communicate as long as they are within the limited communication range, and may not apply to some realistic environments. For instance, occlusions from solid obstacles in between robots may cause disruptions in information exchange, e.g. thick walls may cutoff the Bluetooth communication. Therefore, it is desired to take realistic factors such as the 'Line-of-sight' (LOS) problem into account [11], [12]. In [11], explicit visibility subgraphs and blind-spots in the workspace are computed to allow agents to navigate safely within each others' sights, but when new agents previously not in the same visibility graph emerge in LOS, an emergency brake will be performed while recomputing visibility subgraphs, leading to overly conservative maneuver. LOS-aware formation and connectivity control are presented in [12], [13] where robot behaviors are dominated by the connectivity-oriented design. As a result, robots may not progress toward achieving the primary goal when tasked to spread out and disperse over a wide area. Thus how to maintain LOS connectivity while providing the greatest flexibility for robots is an important challenge.

On the other hand, addressing complicated communication topology requirements beyond global connectivity is also a critical challenge. In many multi-robot applications, it usually requires robots to perform more than one task with different designated subgroups by simultaneously executing multiple behaviors [14]–[17] or sequences of behaviors [18]–[20] for a set of sub-tasks. For example, consider a team of autonomous ground and aerial vehicles split into multiple subgroups based on their capabilities and tasked to simultaneously explore widely separated regions with various local tasks executed by different subgroups. Achieving efficient local collaboration requires robots in the same subgroup to be connected as one component and global connectivity across different subgroups is also needed for global situation awareness. How to ensure LOS connectivity both within each subgroup and across subgroups remains also as a challenge.

In this work, we propose a novel method to consider collision avoidance and global and subgroup LOS connectivity maintenance in multi-robot coordination. The **key contribution** is three-fold. ***First***, we propose a novel notion of Line-of-sight Connectivity Barrier Certificate using Control Barrier Functions (CBF) [21] that defines admissible control space for preserving pairwise LOS connectivity with formal guarantees. ***Second***, by integrating LOS Connectivity Barrier Certificate and a graph theoretic approach, we formulate the minimally constrained LOS-aware coordination problem as a step-wise bi-level optimization problem, and propose the Minimum Line of Sight Connectivity Constraint Spanning Tree (MLCCST) method to *co-optimize* (a) the global and subgroup connectivity topology to enforce, and (b) control

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deviation from nominal controllers subject to the LOS connectivity constraints, thus providing greatest flexibility for the nominal task-related robot motion. ***Third***, we supply formal theoretical analysis on our proposed MLCCST approach.

II. PRELIMINARIES

Consider a robotic team $\mathcal{S}$ consisting of N mobile robots. Each robot $i \in [\![1, N]\!]$ is located at the position $\mathbf{x}_i \in \mathbb{R}^d$ with dynamics $\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i) + g_i(\mathbf{x}_i)\mathbf{u}_i$, where $f_i : \mathbb{R}^d \rightarrow \mathbb{R}^d$ and $g_i : \mathbb{R}^d \rightarrow \mathbb{R}^{d \times q}$ are locally Lipschitz continuous and $\mathbf{u}_i \in \mathbb{R}^q$. The workspace consists of free space and occupied space $\mathcal{C}_{\text{obs}} = \bigcup_{k=1}^{K} \mathcal{O}_k$ by K static polyhedral obstacles $\mathcal{O}_k \subset \mathbb{R}^d, \forall k$.

Communication models: Denote $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ as the ***communication graph*** of the robotic team, where each node $v \in \mathcal{V}$ represents a robot. Conventionally, if $||\mathbf{x}_i - \mathbf{x}_j|| \leq R_{\text{c}}$ with $R_{\text{c}} \in \mathbb{R}$ as the limited communication range, then it is assumed the two robots are ***connected*** and can communicate with the undirected edge $(v_i, v_j) \in \mathcal{E}$ (i.e.$(v_i, v_j) \in \mathcal{E} \iff (v_j, v_i) \in \mathcal{E}$). However, in obstacle-populated environments, the robots can only effectively communicate with others that are *not only* within the limited communication *but also* in line-of-sight (LOS) free from occlusions by all the obstacles, i.e. two robots v_i, v_j are ***LOS connected*** with the undirected LOS edge $(v_i, v_j) \in \mathcal{E}^{\text{los}}$ if $\mathbf{x}_i(1-\beta) + \mathbf{x}_j\beta \notin \mathcal{C}_{\text{obs}}$, $\forall \beta \in [0, 1], (v_i, v_j) \in \mathcal{E}$ (Occlusion-free condition). Hence, we define the ***LOS communication graph*** as $\mathcal{G}^{\text{los}} = (\mathcal{V}, \mathcal{E}^{\text{los}}) \subseteq \mathcal{G}$ with $\mathcal{E}^{\text{los}} \subseteq \mathcal{E}$ as the set of LOS communication edges. In this case, maintaining LOS connectivity between two robots in $(v_i, v_j) \in \mathcal{E}^{\text{los}}$ requires satisfying (a) connectivity constraint $||\mathbf{x}_i - \mathbf{x}_j|| \leq R_{\text{c}}$ and (b) occlusion-free condition.

A. Safety and Connectivity Constraints using CBFs

Assume the K static polyhedral obstacles can be commonly represented by F discretized obstacles located along the boundary of the static obstacles [22], [23]. Each discretized obstacle is denoted as $o \in \{1, ..., F\}$. Consider joint robot states $\mathbf{x} = \{\mathbf{x}_1, ..., \mathbf{x}_N\} \in \mathcal{X} \subset \mathbb{R}^{dN}$, the discretized joint obstacle states $\mathbf{x}^{\text{obs}} = \{\mathbf{x}_1^{\text{obs}}, \dots, \mathbf{x}_F^{\text{obs}}\} \in \mathcal{X}^{\text{obs}} \subset \mathbb{R}^{dF}$, the minimum inter-robot safe distance $R_{\text{s}} \in \mathbb{R}$, the minimum obstacle-robot safe distance $R_{\text{obs}} \in \mathbb{R}$ and the limited communication range R_{c}. The desired set on $\mathbf{x}$ for any pairwise robots i, j and obstacle o satisfying inter-robot or robot-obstacle collision avoidance can be defined as:

$$h^{\text{s}}_{i,j}(\mathbf{x}) = ||\mathbf{x}_i - \mathbf{x}_j||^2 - R_{\text{s}}^2, \forall i > j, \mathcal{H}^{\text{s}}_{i,j} = \{\mathbf{x} \in \mathbb{R}^{dN} : h^{\text{s}}_{i,j}(\mathbf{x}) \geq 0\} \tag{1}$$

$$\begin{aligned} h^{\text{obs}}_{i,o}(\mathbf{x}, \mathbf{x}^{\text{obs}}) = ||\mathbf{x}_i - \mathbf{x}_o^{\text{obs}}||^2 - R_{\text{obs}}^2, \forall i, o, \\ \mathcal{H}^{\text{obs}}_{i,o} = \{\mathbf{x} \in \mathbb{R}^{dN} : h^{\text{obs}}_{i,o}(\mathbf{x}, \mathbf{x}_o^{\text{obs}}) \geq 0\} \end{aligned} \tag{2}$$

And the desired set on $\mathbf{x}$ for any pairwise robots i and j satisfying connectivity constraint can be defined as:

$$h^{\text{c}}_{i,j}(\mathbf{x}) = R_{\text{c}}^2 - ||\mathbf{x}_i - \mathbf{x}_j||^2, \ \forall (v_i, v_j), \ \mathcal{H}^{\text{c}}_{i,j} = \{\mathbf{x} \in \mathbb{R}^{dN} : h^{\text{c}}_{i,j}(\mathbf{x}) \geq 0\} \tag{3}$$

Then for the entire team with any given connectivity spanning graph $\mathcal{G}^{\text{c}} = (\mathcal{V}, \mathcal{E}^{\text{c}}) \subseteq \mathcal{G}$ to enforce, the desired set for safety and required connectivity are thus defined as:

$$\begin{aligned} \mathcal{H}^{\text{s}} = \bigcap_{\{v_i, v_j \in \mathcal{V} : i > j\}} \mathcal{H}^{\text{s}}_{i,j}, \ \mathcal{H}^{\text{obs}} = \bigcap_{\{\forall i, o\}} \mathcal{H}^{\text{obs}}_{i,o}, \\ \mathcal{H}^{\text{c}}(\mathcal{G}^{\text{c}}) = \bigcap_{(v_i, v_j) \in \mathcal{E}^{\text{c}}} \mathcal{H}^{\text{c}}_{i,j} \end{aligned} \tag{4}$$

Control Barrier Functions (CBF) [21] are often used to define an admissible control space for robots rendering the desired set *forward invariant*. The results are below.

Lemma 1. *[summarized from [21]] Given a dynamical system affine in control and a desired set $\mathcal{H}$ as the 0-superlevel set of a continuous differentiable function $h(\mathbf{x}) : \mathcal{X} \rightarrow \mathbb{R}$, the function h is called a control barrier function, if there exists an extended class-$\mathcal{K}$ function $\kappa(\cdot)$ such that $\sup_{\mathbf{u} \in \mathcal{U}}\{\dot{h}(\mathbf{x}, \mathbf{u}) + \kappa(h(\mathbf{x}))\} \geq 0$ for all $\mathbf{x}$. Any Lipschitz continuous controller $\mathbf{u}$ in the admissible control space $\mathcal{B}(\mathbf{x})$ rendering $\mathcal{H}$ forward invariant (i.e. keeping the system state $\mathbf{x}$ staying in $\mathcal{H}$ over time) thus becomes:*

$$\mathcal{B}(\mathbf{x}) = \{\mathbf{u} \in \mathcal{U} | \dot{h}(\mathbf{x}, \mathbf{u}) + \kappa(h(\mathbf{x})) \geq 0\} \tag{5}$$

With this, the admissible control space for robots to stay collision-free and connected with maintained edges in a specified graph $\mathcal{G}^{\text{c}}$ (i.e. $\mathbf{x} \in \mathcal{H}^{\text{c}}(\mathcal{G}^{\text{c}})$) in (4) are introduced by [24], [25] as follows (also referred as safety barrier certificates (SBC) and connectivity barrier certificates (CBC)):

$$\mathcal{B}^{\text{s}}(\mathbf{x}) = \{\mathbf{u} \in \mathbb{R}^{qN} : \dot{h}^{\text{s}}_{i,j}(\mathbf{x}, \mathbf{u}) + \gamma h^{\text{s}}_{i,j}(\mathbf{x}) \geq 0, \forall i > j\} \tag{6}$$

$$\begin{aligned} \mathcal{B}^{\text{obs}}(\mathbf{x}, \mathbf{x}^{\text{obs}}) = \{\mathbf{u} \in \mathbb{R}^{qN} : \dot{h}^{\text{obs}}_{i,o}(\mathbf{x}, \mathbf{x}^{\text{obs}}, \mathbf{u}) + \gamma h^{\text{obs}}_{i,o}(\mathbf{x}, \mathbf{x}^{\text{obs}}) \geq 0, \\ \forall i, o\} \end{aligned} \tag{7}$$

$$\mathcal{B}^{\text{c}}(\mathbf{x}, \mathcal{G}^{\text{c}}) = \{\mathbf{u} \in \mathbb{R}^{qN} : \dot{h}^{\text{c}}_{i,j}(\mathbf{x}, \mathbf{u}) + \gamma h^{\text{c}}_{i,j}(\mathbf{x}) \geq 0, \forall (v_i, v_j) \in \mathcal{E}^{\text{c}}\} \tag{8}$$

where γ is a user-defined parameter in the particular choice of $\kappa(h(\mathbf{x})) = \gamma h(\mathbf{x})$ as in [15]. It is proven in [24], [25] that the forward invariance of the safety set $\mathcal{H}^{\text{s}}, \mathcal{H}^{\text{obs}}$ and the connectivity set $\mathcal{H}^{\text{c}}$ is ensured as long as the joint control input $\mathbf{u}$ stays in set $\mathcal{B}^{\text{s}}(\mathbf{x})$, $\mathcal{B}^{\text{obs}}(\mathbf{x}, \mathbf{x}^{\text{obs}})$, and $\mathcal{B}^{\text{c}}(\mathbf{x}, \mathcal{G}^{\text{c}})$.

B. Line-of-Sight (LOS) Connectivity Constraints

Similarly, the desired set with pairwise LOS connectivity for robots i, j can be described as follows.

$$\begin{aligned} \mathcal{H}^{\text{los}}_{i,j} = & \{\mathbf{x} \in \mathbb{R}^{dN} : h^{\text{los}}_{i,j}(\mathbf{x}, \mathcal{C}_{\text{obs}}) \geq 0\} = \\ & \{\mathbf{x} \in \mathbb{R}^{dN} : \mathbf{x}_i(1-\beta) + \mathbf{x}_j\beta \notin \mathcal{C}_{\text{obs}}, \ \forall \beta \in [0, 1]\} \end{aligned} \tag{9}$$

Note that it is non-trivial to define an analytical form of $h^{\text{los}}_{i,j}$ w.r.t. $\mathbf{x}, \mathcal{C}_{\text{obs}}$. In Section IV-A, we will reformulate (9) and propose an explicit form of the continuously differentiable function $h^{\text{los}}_{i,j}$ for $\mathcal{H}^{\text{los}}_{i,j}$. Then for any LOS connectivity spanning graph $\mathcal{G}^{\text{slos}} = (\mathcal{V}, \mathcal{E}^{\text{slos}}) \subseteq \mathcal{G}^{\text{los}}$ to enforce, the desired set $\mathbf{x}$ with the required LOS connectivity becomes:

$$\mathcal{H}^{\text{los}}(\mathcal{G}^{\text{slos}}) = \Big(\bigcap_{\{v_i, v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}^{\text{slos}}\}} \mathcal{H}^{\text{los}}_{i,j} \Big) \bigcap \mathcal{H}^{\text{c}}(\mathcal{G}^{\text{slos}}) \tag{10}$$

Following Lemma 1, we develop the Line-of-Sight connectivity barrier certificates (LOS-CBC) $\mathcal{B}^{\text{los}}(\mathbf{x}, \mathcal{C}_{\text{obs}}, \mathcal{G}^{\text{slos}})$ as follows (See Section IV-A for detailed discussion) to characterize the admissible control space rendering $\mathcal{H}^{\text{los}}(\mathcal{G}^{\text{slos}})$ in (10) forward invariant, i.e. satisfying both connectivity constraints with maintained edges $\mathcal{E}^{\text{slos}}$ in $\mathcal{G}^{\text{slos}}$ and the occlusion-free condition:

$$\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}})=\mathcal{B}^{c}(\mathbf{x},\mathcal{G}^{\text{slos}})\bigcap$$
$$\{\mathbf{u}\in\mathbb{R}^{qN}:\dot{h}^{\text{los}}_{i,j}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathbf{u})+\gamma h^{\text{los}}_{i,j}(\mathbf{x},\mathcal{C}_{\text{obs}})\geq 0,\forall(v_i,v_j)\in\mathcal{E}^{\text{slos}}\} \tag{11}$$

III. PROBLEM STATEMENT

To simplify the discussion, for the rest of the paper we consider the single-integrator robot dynamics $\dot{\mathbf{x}}_i=\mathbf{u}_i$ with $\mathbf{u}_i\in\mathbb{R}^d$ (i.e. $q=d$) as commonly used in [6], [26]. We assume the robotic team $\mathcal{S}$ with its real-time LOS communication graph $\mathcal{G}^{\text{los}}$ has been assigned M simultaneous tasks ($M\leq N$) with M divided sub-groups $\mathcal{S}=\{\mathcal{S}_1,\dots,\mathcal{S}_M\}$, where each robot i has already been tasked to a sub-group $\mathcal{S}_m$ with a nominal task-related controller $\mathbf{u}_i=\hat{\mathbf{u}}_i\in\mathbb{R}^d$.

Global LOS Connectivity: A graph $\mathcal{G}^{\text{los}}$ is said to be *LOS connected* if there is at least one occlusion-free path between every pair of vertices on the graph.

Subgroup LOS Connectivity: A graph $\mathcal{G}^{\text{los}}$ is said to be *Subgroup LOS connected* if there is at least one occlusion-free path between every pair of vertices in each induced LOS subgroup graph $\mathcal{G}^{\text{los}}_m=\mathcal{G}^{\text{los}}[\mathcal{V}_m]\subseteq\mathcal{G}^{\text{los}},\forall m\in[\![1,M]\!]$, where $\mathcal{V}_m\subseteq\mathcal{V}$ contains all robots within the same subgroup.

The LOS communication graph $\mathcal{G}^{\text{los}}$ for the entire team should satisfy both global and subgroup LOS connectivity at all time. We assume the global and subgroup connectivity of the LOS communication graph $\mathcal{G}^{\text{los}}$ are satisfied initially. The step-wise optimization problem is defined as follows:

$$\mathbf{u}^*=\underset{\mathcal{G}^{\text{slos}},\mathbf{u}}{\arg\min}\sum_{i=1}^{N}||\mathbf{u}_i-\hat{\mathbf{u}}_i||^2 \tag{12}$$
$$\text{s.t.}\quad \mathcal{G}^{\text{slos}}=(\mathcal{V},\mathcal{E}^{\text{slos}})\subseteq\mathcal{G}^{\text{los}}\quad\text{is LOS connected} \tag{13}$$
$$\mathcal{G}^{\text{slos}}_m=\mathcal{G}^{\text{slos}}[\mathcal{V}_m]\quad\text{is LOS connected,}\quad\forall m=1,\dots,M \tag{14}$$
$$\mathbf{u}\in\mathcal{B}^{s}(\mathbf{x})\bigcap\mathcal{B}^{\text{obs}}(\mathbf{x},\mathbf{x}^{\text{obs}})\bigcap\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}}), \tag{15}$$
$$||\mathbf{u}_i||\leq u_{\max},\forall i=1,\dots,N$$

The optimization problem (12) seeks to minimally modify the given nominal task-related controller $\hat{\mathbf{u}}_i$ for each robot i at each time step, while respecting the required LOS connectivity constraints determined by a satisfying $\mathcal{G}^{\text{slos}}\subseteq\mathcal{G}^{\text{los}}$. Note that the LOS communication graph $\mathcal{G}^{\text{los}}$ at each time step could have multiple subgraphs $\{\mathcal{G}^{\text{slos}}\}$ that satisfy global and subgroup LOS connectivity, each of which defines a specific set of control constraints by $\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}})$ in (15). Thus it is beneficial to select an optimal subgraph $\mathcal{G}^{\text{slos}*}\subseteq\mathcal{G}^{\text{los}}$ so that (a) it specifies the least number of edges $|\mathcal{E}^{\text{slos}*}|$ to maintain (i.e. less LOS constraints needed to ensure global and subgroup LOS connectivity), and (b) enforcing constraints $\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}*})$ would introduce minimum control deviation from $\hat{\mathbf{u}}_i$ for all i (i.e. least constraining $\hat{\mathbf{u}}_i$ compared to maintaining other $\mathcal{G}^{\text{slos}}$). Therefore, the problem can be considered as a bilevel optimization process: (i) find the optimal spanning subgraph $\mathcal{G}^{\text{slos}*}\subseteq\mathcal{G}^{\text{los}}$ to preserve at each time step, and (ii) use $\mathcal{G}^{\text{slos}*}$ to define control constraints $\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}*})$ and solve for step-wise control $\mathbf{u}^*\in\mathbb{R}^{dN}$. As (12) is repeatedly solved at every time step, the selected $\mathcal{G}^{\text{slos}*}\subseteq\mathcal{G}^{\text{los}}$ will be re-computed as $\mathcal{G}^{\text{los}}$ changes due to updated robot positions, thus enabling flexible coordination with dynamic LOS connectivity maintenance.

IV. METHOD

A. Line-of-Sight Connectivity Barrier Certificates

Since it is non-trivial to derive an analytical form of $h^{\text{los}}_{i,j}(\mathbf{x},\mathcal{C}_{\text{obs}})$ based on the occlusion-free condition in (9), here we propose to use an approximation method such as an ellipsoid to represent a LOS communication edge $(v_i,v_j)\in\mathcal{E}^{\text{los}}$ and to analytically determine whether any obstacle is intersecting with this edge. To prevent overly conservative approximation, we formulate the ellipsoidal approximation as a Minimum Volume Enclosing Ellipsoid (MVEE) problem [27] covering the edge $(v_i,v_j)\in\mathcal{E}^{\text{los}}$.

As proved in [28], a set of $2d$ points in space $\mathbb{R}^d$ suffice to determine a MVEE that tightly covers those points. To ensure a good approximation, for each edge $(v_i,v_j)\in\mathcal{E}^{\text{los}}$ we use a particular choice of $2d$ points analytically defined as $\mathcal{P}_{i,j}=\{\mathbf{p}^1_{i,j},...,\mathbf{p}^{2d}_{i,j}\}$ to reconstruct the corresponding MVEE covering the line segment $\mathbf{x}_i(1-\beta)+\mathbf{x}_j\beta,\ \ \forall\beta\in[0,1]$. $\mathbf{p}^1_{i,j}=\mathbf{x}_i,\mathbf{p}^2_{i,j}=\mathbf{x}_j$ are two vertices on the major principle axis of the MVEE and $\{\mathbf{p}^3_{i,j},...,\mathbf{p}^{2d}_{i,j}\}$ are the other vertices on the non-major principle axis of the MVEE centered at the middle point $\mathbf{p}^0_{i,j}=\frac{\mathbf{x}_i+\mathbf{x}_j}{2}$ with the same semi-axis length as $||\mathbf{p}^l_{i,j}-\mathbf{p}^0_{i,j}||=\delta,\forall l=3,\dots,2d$, a small value determined beforehand to reflect the "thickness" of the ellipsoid. With that, the corresponding MVEE [28] for edge $(v_i,v_j)\in\mathcal{E}^{\text{los}}$ is determined by a positive-definite matrix $Q_{i,j}\in\mathbb{R}^{d\times d}$ and centered at $\mathbf{p}^0_{i,j}$, where $Q_{i,j}\leftarrow\arg\min\det(Q^{-1}_{i,j})$ and $(\mathbf{p}^r_{i,j}-\mathbf{p}^0_{i,j})^TQ_{i,j}(\mathbf{p}^r_{i,j}-\mathbf{p}^0_{i,j})\leq 1,r=1,...,2d$. Note that the approximated ellipsoid characterized by $Q_{i,j}$ will update as robots i,j move over time. Then the function of $h^{\text{los}}_{i,j}$ for occlusion-free condition and its 0-superlevel set $\mathcal{H}^{\text{los}}_{i,j}$ in (9) are analytically re-defined as, $\forall o$:

$$h^{\text{los}}_{i,j,o}(\mathbf{x},\mathbf{x}^{\text{obs}})=(\mathbf{x}^{\text{obs}}_o-\mathbf{p}^0_{i,j})^TQ_{i,j}(\mathbf{x}^{\text{obs}}_o-\mathbf{p}^0_{i,j})-1,\forall(v_i,v_j)\in\mathcal{E}^{\text{los}}$$
$$\mathcal{H}^{\text{los}}_{i,j,o}=\{\mathbf{x}\in\mathbb{R}^{dN}:h^{\text{los}}_{i,j,o}(\mathbf{x},\mathbf{x}^{\text{obs}})\geq 0\ ,\ \forall(v_i,v_j)\in\mathcal{E}^{\text{los}},\forall o\}$$
$$\mathcal{H}^{\text{los}}_{i,j}=\bigcap_{\forall o}\mathcal{H}^{\text{los}}_{i,j,o} \tag{16}$$

Following Lemma 1 with control barrier functions, we now formally define the Line-of-Sight Connectivity Barrier Certificates (LOS-CBC) as follows.

Lemma 2. ***Line-of-Sight Connectivity Barrier Certificates(LOS-CBC):*** *Given a LOS communication spanning graph $\mathcal{G}^{\text{slos}}=(\mathcal{V},\mathcal{E}^{\text{slos}})\subseteq\mathcal{G}^{\text{los}}$ and a desired set $\mathcal{H}^{\text{los}}(\mathcal{G}^{\text{slos}})$ in (10) with $h^{\text{los}}_{i,j,o}$ from (16), for any Lipschitz continuous controller $\mathbf{u}$, the Line-of-Sight connectivity barrier certificates (LOS-CBC) as the admissible control space $\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}})$ defined below renders $\mathcal{H}^{\text{los}}(\mathcal{G}^{\text{slos}})$ forward invariant (i.e keeping joint robot state staying in $\mathcal{H}^{\text{los}}(\mathcal{G}^{\text{slos}})$):*

$$\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}})=\mathcal{B}^{c}(\mathbf{x},\mathcal{G}^{\text{slos}})\bigcap\{\mathbf{u}\in\mathbb{R}^{dN}:$$
$$\dot{h}^{\text{los}}_{i,j,o}(\mathbf{x},\mathbf{x}^{\text{obs}},\mathbf{u})+\gamma h^{\text{los}}_{i,j,o}(\mathbf{x},\mathbf{x}^{\text{obs}})\geq 0,\forall(v_i,v_j)\in\mathcal{E}^{\text{slos}},\forall o\} \tag{17}$$

where $\dot{h}^{\text{los}}_{i,j,o}(\mathbf{x},\mathbf{x}^{\text{obs}},\mathbf{u})=-(\mathbf{x}^{\text{obs}}_o-\frac{\mathbf{x}_i+\mathbf{x}_j}{2})^TQ_{i,j}(\mathbf{u}_i+\mathbf{u}_j)$.

$\mathcal{B}^{\text{los}}(\mathbf{x},\mathcal{C}_{\text{obs}},\mathcal{G}^{\text{slos}})$ relies on the composition of $h^{c}_{i,j}$ and $h^{\text{los}}_{i,j,o}$. Proof of Lemma 2 is provided in Section VII-B [29].

B. Minimum Line of Sight Connectivity Constraint Spanning Tree (MLCCST)

As each edge $(v_i, v_j) \in \mathcal{E}^{\mathrm{slos}}$ in a candidate graph $\mathcal{G}^{\mathrm{slos}}$ enforces one particular LOS connectivity requirement restricting the motion of robot i and j, the desired graph $\mathcal{G}^{\mathrm{slos}*}$ whose edges define the minimum LOS connectivity constraints (i.e. most *unlikely* to be violated if following nominal control $\hat{\mathbf{u}}_i, \hat{\mathbf{u}}_j$) must exist among the set of all spanning trees $\mathcal{T}^{\mathrm{los}}$ of $\mathcal{G}^{\mathrm{los}}$ that have the minimum number of edges (i.e. $N-1$) for $\mathcal{G}^{\mathrm{slos}*}$ to stay LOS connected. Hence, the first sub-problem boils down to finding the optimal spanning tree $\mathcal{G}^{\mathrm{slos}*} = \mathcal{T}^{\mathrm{los}*} \subseteq \mathcal{G}^{\mathrm{los}}$ whose edges invoke the minimum LOS connectivity constraints in the form of (17) over the nominal robots' controller. Below we introduce the connectivity weight $w^{\mathrm{d}}_{i,j}$, line of sight weight $w^{\mathrm{los}}_{i,j}$, and the edge weight $w^{\mathrm{d+los}}_{i,j}$ as the sum of the two to heuristically quantify how *unlikely* the pairwise LOS constraints are to be violated under the nominal controller û, $\forall (v_i, v_j) \in \mathcal{E}^{\mathrm{los}}$:

$$w^{\mathrm{d}}_{i,j} = \dot{h}^{\mathrm{c}}_{i,j}(\mathbf{x}, \hat{\mathbf{u}}) + \gamma h^{\mathrm{c}}_{i,j}(\mathbf{x}) \tag{18}$$

$$w^{\mathrm{los}}_{i,j} = \frac{1}{F}\sum_{o=1}^{F}(\dot{h}^{\mathrm{los}}_{i,j,o}(\mathbf{x}, \mathbf{x}^{\mathrm{obs}}, \hat{\mathbf{u}}) + \gamma h^{\mathrm{los}}_{i,j,o}(\mathbf{x}, \mathbf{x}^{\mathrm{obs}})) \tag{19}$$

$$w^{\mathrm{d+los}}_{i,j} = \begin{cases} \quad w^{\mathrm{los}}_{i,j} + w^{\mathrm{d}}_{i,j} \quad \text{if} \quad \forall o, \quad h^{\mathrm{los}}_{i,j,o}(\mathbf{x}, \mathbf{x}^{\mathrm{obs}}) \geq 0 \\ \epsilon \quad \text{otherwise, namely} \quad \exists o, \quad h^{\mathrm{los}}_{i,j,o}(\mathbf{x}, \mathbf{x}^{\mathrm{obs}}) < 0, \end{cases} \tag{20}$$

where $\epsilon \in \{\epsilon \ll 0 : \epsilon \ll w^{\mathrm{d+los}}_{i,j}, \forall (v_i, v_j) \in \mathcal{E}\}$ is a user-defined constant for the entire graph $\mathcal{G}^{\mathrm{los}}$. $w^{\mathrm{d}}_{i,j}, w^{\mathrm{los}}_{i,j}$ indicate the level of violation of the two constraints between robots i, j under the nominal controller $\hat{\mathbf{u}}_i, \hat{\mathbf{u}}_j$ from $\hat{\mathbf{u}}$, with the higher value of $w^{\mathrm{d}}_{i,j}, w^{\mathrm{los}}_{i,j}$ the less violated the constraints are. In particular, $w^{\mathrm{los}}_{i,j}$ defined in (19) reflects preference over LOS edges that are unlikely to be violated on average w.r.t. all of the obstacles, thus implying a larger slackness in general when being preserved. By introducing the penalization term of ϵ in (20), the Line-of-Sight connectivity weight $w^{\mathrm{d+los}}_{i,j}$ naturally reveals those LOS connectivity edges $(v_i, v_j) \in \mathcal{E}^{\mathrm{los}}$ satisfying our redefined occlusion-free condition (16). Note that to further reduce computation burden in (19) with a large number of F discretized obstacles, tools such as efficient nearest neighbor searches [30] could be employed to only consider a smaller number of obstacles closer to the robots i, j, which are beyond the scope of this paper. With that, each candidate spanning tree $\mathcal{T}^{\mathrm{los}} \subseteq \mathcal{G}^{\mathrm{los}}$ is redefined as a weighted spanning tree $\mathcal{T}^{\mathrm{los}}_w = (\mathcal{V}, \mathcal{E}^T, \mathcal{W}^T)$ where $\mathcal{E}^T \subseteq \mathcal{E}^{\mathrm{los}}$ with weight $\mathcal{W}^T = \{-w^{\mathrm{d+los}}_{i,j}\}$. Hence the optimal LOS connectivity graph $\mathcal{G}^{\mathrm{slos}*}$ with constraints in (13),(14) can be defined as follows.

$$\begin{aligned} \mathcal{G}^{\mathrm{slos}*} = & \arg\max_{\{\mathcal{T}^{\mathrm{los}}_w\}} \sum_{(v_i,v_j)\in\mathcal{E}^T} w^{\mathrm{d+los}}_{i,j} = \arg\min_{\{\mathcal{T}^{\mathrm{los}}_w\}} \sum_{(v_i,v_j)\in\mathcal{E}^T} -w^{\mathrm{d+los}}_{i,j} \\ \text{s.t.} \quad & \mathcal{T}^{\mathrm{los}}_m = \mathcal{T}^{\mathrm{los}}_w[\mathcal{V}_m] \quad \text{is LOS connected,} \quad \forall m = 1, \dots, M \end{aligned} \tag{21}$$

The optimal solution of (21) is the Minimal Spanning Tree (MST) of the LOS communication graph $\mathcal{G}^{\mathrm{los}}$ weighted by $\{-w^{\mathrm{d+los}}_{i,j}\}$ with additional subgroup LOS connectivity constraints. Now we define another class of spanning trees as follows and relate its unconstrained MST to the solution of the subgroup LOS connectivity constrained MST in (21).

Definition 3. ***Line of Sight Connectivity Constraint Spanning Tree(LCCST):*** *Given a LOS communication graph* $\mathcal{G}^{\mathrm{los}} = (\mathcal{V}, \mathcal{E}^{\mathrm{los}})$ *and for all edges* $(v_i, v_j) \in \mathcal{E}^{\mathrm{los}}$ *on* $\mathcal{G}^{\mathrm{los}}$*, redefine their weights by the following.*

$$w'_{i,j} = \begin{cases} \lambda \cdot w^{\mathrm{d+los}}_{i,j}, & \textit{if} \quad v_i \textit{ and } v_j \textit{ are in the same sub-group} \\ w^{\mathrm{d+los}}_{i,j}, & \textit{if} \quad v_i \textit{ and } v_j \textit{ are in different sub-groups} \end{cases} \tag{22}$$

where $\lambda \in \{\lambda \gg 1 : \lambda \cdot w^{\mathrm{d+los}}_{i,j} \gg w^{\mathrm{d+los}}_{i',j'}, \forall v_i, v'_i, v_j, v'_j \in \mathcal{V}\}$ *is a unique user-defined constant for the entire graph* $\mathcal{G}^{\mathrm{los}}$*. The weight-modified graph from* $\mathcal{G}^{\mathrm{los}}$ *is thus denoted as* $\mathcal{G}^{\mathrm{los}'} = (\mathcal{V}, \mathcal{E}^{los}, \mathcal{W}')$ *with* $\mathcal{W}' = \{-w'_{i,j}\}$*. Then we call the redefined LOS spanning tree* $\mathcal{T}^{\mathrm{los}'}_w = (\mathcal{V}, \mathcal{E}^T, \mathcal{W}^{T'}) \subseteq \mathcal{G}^{\mathrm{los}'}$ *as the Line of Sight Connectivity Constraint Spanning Tree(LCCST).*

Considering all those LOS edges satisfying our redefined occlusion-free condition (16), the designed parameter λ in (22) ensures that the new weight $\{-w'_{i,j}\}$ over those edges connecting different subgroups are always larger than any edges within the same subgroup. In this way, we present Theorem 4 to transform the constrained MST problem in (21) into an unconstrained MST problem with the same optimally guarantee, ensuring the optimal MST computed from $\{\mathcal{T}^{\mathrm{los}'}_w\}$ contains the MST of each subgroup as well.

Theorem 4. *Given the redefined Line of Sight Connectivity Constraint Spanning Tree (LCCST)* $\mathcal{T}^{\mathrm{los}'}_w = (\mathcal{V}, \mathcal{E}^T, \mathcal{W}^{T'})$ *in Definition 3 and denote minimum weighted LCCST as* $\bar{\mathcal{T}}^{\mathrm{los}'}_w = \arg\min_{\{\mathcal{T}^{\mathrm{los}'}_w\}} \sum_{(v_i,v_j)\in\mathcal{E}^T}\{-w'_{i,j}\}$*, then we have:* $\mathcal{G}^{\mathrm{slos}*} = \bar{\mathcal{T}}^{\mathrm{los}'}_w$ *in (21). Namely, the Minimum Spanning Tree* $\bar{\mathcal{T}}^{\mathrm{los}'}_w$ *of* $\mathcal{G}^{\mathrm{los}'}$ *is the optimal solution of* $\mathcal{G}^{\mathrm{slos}*}$ *in (21) and we call the graph* $\bar{\mathcal{T}}^{\mathrm{los}'}_w$ *as Minimum Line-of-Sight Connectivity Constraint Spanning Tree (MLCCST) of the original LOS communication graph* $\mathcal{G}^{\mathrm{los}}$*.*

The detailed proof is presented in Section VII-D [29]. Note that the MLCCST $\bar{\mathcal{T}}^{\mathrm{los}'}_w$ could be updated over time due to dynamically changing $\mathcal{G}^{\mathrm{los}}$ between each time step.

Finally, our proposed MLCCST algorithm for computing the MLCCST $\bar{\mathcal{T}}^{\mathrm{los}'}_w$ and the revised multi-robot controllers is summarized in Algorithm 1. As shown by Theorem 4, the derived $\mathcal{G}^{\mathrm{slos}*} = \bar{\mathcal{T}}^{\mathrm{los}'}_w \subseteq \mathcal{G}^{\mathrm{los}}$ satisfies the LOS conditions in (13) and (14). Recalling the weight definition in (20), the *minimally weighted* nature of the global and subgroup LOS connected $\bar{\mathcal{T}}^{\mathrm{los}'}_w$ thus indicates the resultant *least constraining* LOS constraints $\mathcal{B}^{\mathrm{los}}(\mathrm{x}, \mathcal{C}_{\mathrm{obs}}, \bar{\mathcal{T}}^{\mathrm{los}'}_w)$ given the task-related nominal control, and so to provide the greatest flexibility for the team. Hence, the computed $\mathbf{u}^*$ by Algorithm 1 is a desired solution for problem (12). Note that the control constraints $\mathbf{u} \in \mathcal{B}^{\mathrm{s}}(\mathbf{x}) \bigcap \mathcal{B}^{\mathrm{los}}(\mathrm{x}, \mathcal{C}_{\mathrm{obs}}, \bar{\mathcal{T}}^{\mathrm{los}'}_w) \bigcap \mathcal{B}^{\mathrm{obs}}(\mathbf{x}, \mathbf{x}^{\mathrm{obs}})$ on Line 7 (Algorithm 1) are linear w.r.t. $\mathbf{u}$, therefore making it a standard step-wise Quadratic Programming (QP) that could be efficiently solved in real-time.

C. Theoretical Analysis

In this section, we provide discussions on the validity and feasibility of the proposed LOS-CBC in Lemma 2, the feasibility of the problem (12), and how the global and subgroup LOS connectivity is guaranteed over time. Detailed proofs can be found in Section VII-A and Section VII-E in [29], and here we only summarize the main results.

Algorithm 1 MLCCST Algorithm

```
Input: $\mathbf{x}$-the current states (positions) of the robots, $\hat{\mathbf{u}}$-the nominal task-related multi-robot controller, $\mathcal{C}_{\text{obs}}$ the occupied space of the obstacles
Output: The desired minimally modified controller $\mathbf{u}^* \in \mathbb{R}^{dN}$ from (12)
function MLCCST($\mathbf{x}$, $\hat{\mathbf{u}}$, $\mathcal{C}_{\text{obs}}$)
    for Each Time Step do
        for All Edges $(v_i, v_j) \in \mathcal{E}^{\text{los}}$ of current LOS communication Graph $\mathcal{G}^{\text{los}} = (\mathcal{V}, \mathcal{E}^{\text{los}})$ do
            Weight assignment: $\mathcal{W}'_{i,j} \leftarrow -w'_{i,j}$ using (20) and (22)
        Get new weighted graph $\mathcal{G}^{\text{los}'} = (\mathcal{V}, \mathcal{E}^{\text{los}'}, \mathcal{W}')$
        Solve $\bar{\mathcal{T}}_w^{\text{los}'} = \arg\min_{\{\mathcal{T}_w^{\text{los}'}\}} \sum_{(v_i,v_j)\in\mathcal{E}^T} -w'_{i,j}$ by standard MST algorithm: $\bar{\mathcal{T}}_w^{\text{los}'} \leftarrow \text{MST}(\mathcal{G}^{\text{los}'})$
        return $\mathbf{u}^* = \arg\min_{\mathbf{u}} \sum_{i=1}^N ||\mathbf{u}_i - \hat{\mathbf{u}}_i||^2$ where $\mathbf{u} \in \mathcal{B}^{\text{s}}(\mathbf{x}) \bigcap \mathcal{B}^{\text{obs}}(\mathbf{x}, \mathbf{x}^{\text{obs}}) \bigcap \mathcal{B}^{\text{los}}(\mathbf{x}, \mathcal{C}_{\text{obs}}, \bar{\mathcal{T}}_w^{\text{los}'}), ||\mathbf{u}_i|| \le u_{\max}, \forall i = 1, \dots, N$
```

Lemma 5. *Function $h_{i,j,o}^{\text{los}}(\mathbf{x}, \mathbf{x}^{\text{obs}})$ in (16) is a valid CBF and the admissible control space constrained by (17) is always non-empty.*

With Lemma 5, assuming the robotic team is initially LOS connected, then Lemma 2 defines a non-empty (feasible) control space that enforces robots on staying LOS connected.

In this paper, the composition of different valid control barrier functions can be defined as:

$$\begin{aligned} h_{i,j,o}^{\text{sys}} &= h_{i,j,o}^{\text{los}} \wedge h_{i,j}^{\text{c}} \wedge h_{i,j}^{\text{s}} \wedge h_{i,o}^{\text{obs}} \\ &= \min\{\min\{h_{i,j,o}^{\text{los}}, h_{i,j}^{\text{c}}\}, \min\{h_{i,j}^{\text{s}}, h_{i,o}^{\text{obs}}\}\} \end{aligned} \tag{23}$$

The composition of CBFs is studied in [6], [31], [32] and feasibility analysis under bounded control inputs with mitigation strategies are given in [33]. Readers are referred to our appendix in Section VII-C [29] for detailed discussion. In summary, if the team of robots satisfy the safety and LOS connectivity condition initially, then the presented QP problem (12) can always be made feasible that ensures LOS connectivity requirement and safety at all times, e.g. by robots decelerating to zero velocities so that safety and current LOS connectivity graph is preserved even in extreme cases similar to [25].

Proposition 6. *Assume $\mathcal{G}^{\text{los}}$ is initially both global and subgroup LOS connected. By following the process in Algorithm 1 at each time step, it is guaranteed that the resulting communication graph $\mathcal{G}^{\text{los}}$ in the next time step is always global and subgroup LOS connected (See proof in [29]).*

V. RESULT

A. Simulation Example

The first set of experiments performed on a team of $N = 40$ with unicycle dynamics are shown in Figure 1 and Figure 2. We apply the minimally revised controllers from (12) to the robots with unicycle dynamics using kinematics mapping from [25]. To demonstrate 1) the effectiveness of the proposed LOS-CBC, and 2) the importance of updating the LOS connectivity graph dynamically over time, two baseline methods are implemented for performance comparison: Minimum Connectivity Constraint Spanning Tree (MCCST) [15] which only considers the regular connectivity constraints, and Fixed MLCCST, which preserves the initial MLCCST computed from the first time step with our method but without dynamically updating over time. For MCCST, without considering the line-of-sight connectivity constraints, the robots easily lose inter-robot LOS communications due to obstacle occlusions. In Figure 2, it is observed that without dynamically updating the LOS connectivity graph as robots move, the task performance is significantly affected.

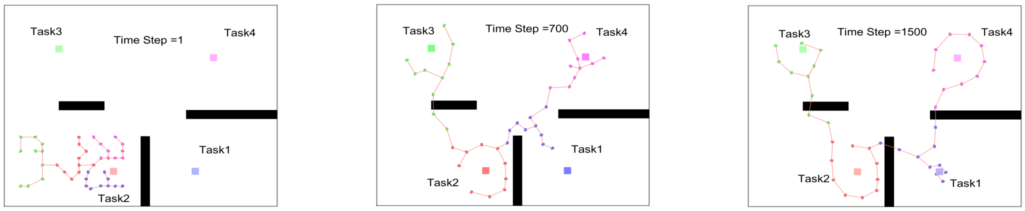
Figure 3(a) shows that all three algorithms satisfy the safety requirement with no collision happening. Figure 3(b) indicates that our proposed MLCCST method achieves almost the same high task efficiency as MCCST does. Note that MCCST performs slightly better than our proposed method as MCCST does not have Line-of-Sight connectivity constraints and therefore robot motion is less restrictive. Without dynamically updating the connectivity graph, the performance of Fixed MLCCST shows that the fixed connectivity topology graph will limit the task performance significantly. Figure 3(c) shows that both our proposed MLCCST and Fixed MLCCST ensure the Line-of-Sight connectivity during the task, but robots using MCCST becomes disconnected in terms of Line-of-Sight ($\lambda_2 = 0$). Figure 3(d) shows that Fixed MLCCST introduced the most average control perturbation, while our MLCCST method achieves the least average control perturbation which reflects the task efficiency due to our minimally constraining design. In summary, based on the four selected evaluation metrics, our proposed MLCCST method performs the best among all three algorithms.

B. Quantitative Results

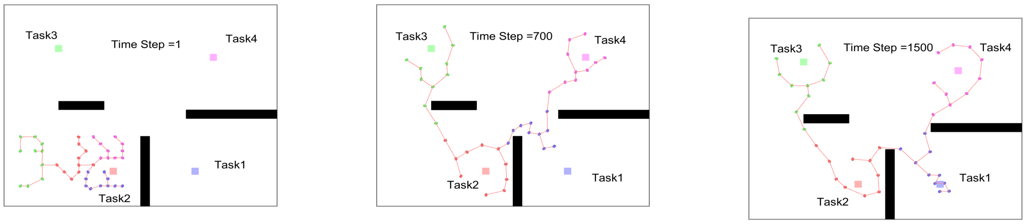
To validate the computation efficiency and scalability of our algorithm, we run experiments with up to 64 robots and 4 parallel behaviors. We conducted 10 trials for each batch of robots with varying sizes. Figure 4(a) shows the average computation time with our MLCCST and indicates its good computation efficiency for real-time application. Figure 4(b)-(c) demonstrate that collision-free motion is achieved for all three methods and our MLCCST method achieves good task efficiency in terms of reducing distance to target region. Figure 4(d) shows that both Fixed MLCCST and our proposed MLCCST maintain the satisfying LOS connectivity, while MCCST fails to preserve it ($\lambda_2 = 0$) as it does not consider LOS connectivity. Figure 4(e) indicate that both MCCST and our proposed MLCCST achieve comparable average control perturbance much less than Fixed MLCCST, inferring more flexibility on the robot motion by our MLCCST despite the LOS connectivity constraints.

VI. CONCLUSION

In this work, we propose a novel notion of Line-of-sight Connectivity Barrier Certificate (LOS-CBC) to define admissible control space that guarantees LOS connectivity between pairwise robots over time. By combining LOS-CBC and a graph theoretic approach, we formulate the global and subgroup LOS-aware multi-robot coordination problem as a bilevel optimization, and the Minimum LOS Connectivity Constraint Spanning Tree (MLCCST) algorithm is proposed to realize the minimal deviation control subject to safety and optimized LOS connectivity constraints. Future work includes to fully decentralize the MLCCST algorithm so that it could run on distributed multi-robot systems.

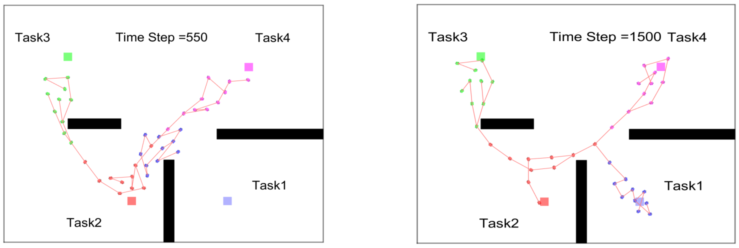


(a) t = 1 (MCCST [15]) (b) t=700 (MCCST [15]) (c) t = 1500 (MCCST [15], Converged)



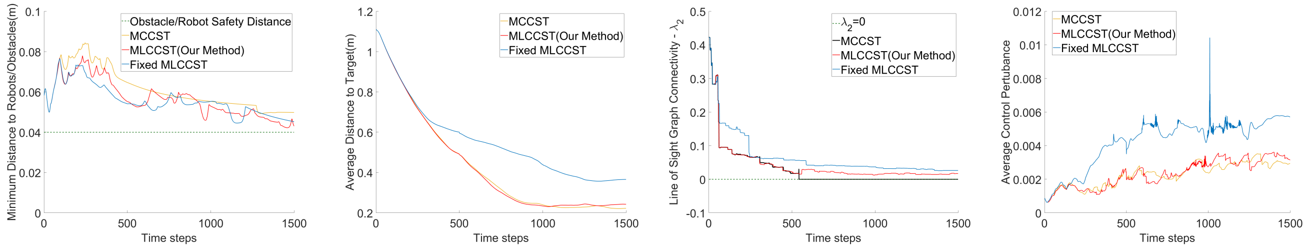
(d) t = 1 (MLCCST) (e) t=700 (MLCCST) (f) t=1500 (MLCCST, Converged)

Fig. 1: Comparison of our proposed MLCCST (second row) with MCCST [15] (first row). Our proposed MLCCST with the Line-of-Sight Connectivity Barrier Certificate can ensure the LOS connectivity over time. However, the MCCST can't ensure the LOS connectivity (The communication edges are cutoff by obstacles in Figure (b) and (c)). t is time step. The red lines in this figure denote the currently active line of sight connectivity graph. The black boxes represent the obstacles. The robot team is divided into $M = 4$ subgroup with different colors and is tasked with 4 parallel behaviors. In the figures, robots in blue subgroup 1 execute biased rendezvous behaviors towards the blue task site 1, while robots in red subgroup 2, green subgroup 3, and magenta subgroup 4 perform circle formation behaviors around the red task site 2, green task site 3 and magenta task site 4 respectively.



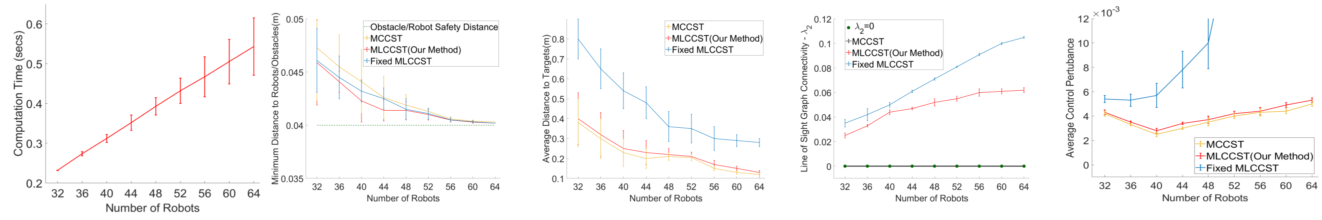
(a) t=550 (Fixed MLCCST) (b) t=1500 (Fixed MLCCST, Converged)

Fig. 2: Performance of Fixed MLCCST (our proposed method without dynamically updated LOS connectivity graph in runtime). With the fixed initial MLCCST, the team of robots could ensure the LOS connectivity, but the task performance will be perturbed greatly.



(a) D_{min} to robots/obstacles (b) D_{ave} to target region (c) Algebraic LOS connectivity (d) Average control perturbation

Fig. 3: Performance comparison of the proposed MLCCST and two baseline methods on the simulation example demonstrated in Figure 1 and Figure 2 with respect to four selected metrics: (a) Minimum distance to robots/obstacles (verify safety constraint satisfaction and degree of conservativeness of the system) (safety distance is 0.04m), (b) Average distance between robots to tasked region (indicate the overall task efficiency), (c) Average algebraic LOS connectivity (indicate whether the LOS graph is LOS connected ($\lambda_2 > 0$) or not ($\lambda_2 = 0$), λ_2 is the second smallest eigenvalue of the Line-of-Sight laplacian matrix calculated from the Line-of-Sight adjacency matrix. The elements in the Line-of-Sight adjacency matrix indicate whether the pairwise robots are LOS connected), (d) Average Control perturbation (computed by $\frac{1}{N}\sum_{i=1}^{N}||\mathbf{u}_i - \hat{\mathbf{u}}_i||^2$ to measure the accumulated deviation from nominal controllers).



(a) Average computation time (b) robots/obstacles D_{min} (c) D_{ave} to target region (d) Algebraic LOS connectivity (e) Average control perturbation

Fig. 4: Quantitative results of performance on different sizes of robot team with respect to five selected metrics. For all figures, the error bar shows the standard deviation.

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