

Global Terminal Sliding Mode Control of Tethered Satellites Formation with Chattering Reduction via PID Laws

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Abstract—This paper researches a novel global terminal sliding mode control(GTSMC) on a tethered satellites system(TSS) under outer disturbances, and the effect of PI/PD compensation in restraining chattering on sliding surface is appended. By taking advantage of the finite-time convergence of traditional terminal sliding surface, the sliding surface with global and terminal sliding motion is proposed, and the convergent time by GTSMC is qualitatively evaluated by the sliding surface. Then the integral/derivative function of the low-pass filtered switching control is appended in GTSMC. By virtue of the accuracy of integral and the damping of derivative, respectively, the persisting on sliding surface is eliminated, such that the chattering effect of the controlled system on the surface is restrained consequently. Finally, simulations of the proposed control on TSS is shown to validate the theoretical analyses.

Index Terms—global terminal sliding mode, chattering reduction, tethered satellites system, integral/derivative function

I. INTRODUCTION

On introducing a terminal attractor which results in finite-time convergence[1], terminal sliding mode control(TSMC) was proposed in a second-order system in 1992. Then the extended studies for a class of higher-order systems of single-input single-output[2] and multi-input multi-output[3], [4] were made afterwards, which occupy less disturbance sensibility and faster convergence rate. To improve the performance of TSMC, novel design methods are proposed. In terms of the singularity arisen in controller, integral TSMC[5], [6], non-singular TSMC[7] and linear auxiliary SMC technique[8], [9] are proposed to compensate for the singular domain. In view of the situations when the available system knowledge is limited, adaptive TSMC is developed to deal with the uncertainties without retarding the control performance[9], [10].

However, despite the superiority of TSMC, restricting the influence of uncertainties and disturbances of controlled system is still a tough problem as the susceptible divergence in reaching phase and the inevitable chattering on sliding surface(SS). Recently, [11] proposes an adaptive sliding mode disturbance observer to obtain the estimate of disturbance in system, such that the responses of closed-loop system on sliding mode are stable. In [12] a high-order sliding mode observer(SMO) is designed to estimate the time-varying

disturbances in a flexible joint robot system. [13] designs a cascaded finite-time SMO to estimate unknown states and lumped disturbances where the upper bound of the convergence time is obtained as well. On the other hand, in [14] the TSMC is designed for a second-order system with fixed convergence time, and the bounded mismatched disturbances are suppressed by the designed SS. Furthermore, a robust model-free controller is incorporated with adaptive TSMC to obtain the finite-time convergence of system in the presence of uncertainties in[10].

On the other hand, in virtue of global sliding-mode technique, the robustness of TSMC is improved by keeping the system on sliding mode all the time to obviate disturbances in reaching phase in [15], where the integral GTSMC is proposed in missile motion control such that the controllable state is tracked to reference value in finite time when the derivative of SS is kept zero. Compared with the methods on suppressing the disturbances by disturbance observer system, simply using PID tuning with specifically designed switching control meets the disturbance rejection as well, while the complexity of control is reduced considerably. Moreover, because of the susceptibility to noises of derivatives[16], which in turn deteriorates the control performance, to compose global SS with normal terminal sliding mode is beneficial.

In this study, we research the GTSMC of an affine nonlinear TSS with outer disturbances based on the terminal sliding surface, which is stabilized in finite time, while a novel function with derivation is composed in SS which results in global sliding mode. Then PI/PD based GTSMC is designed to eliminate the chattering phenomena on SS. Specifically, by adding the integral of the low-pass filtered switching function, the order of system via PI-GTSMC is increased, thus the steady error is driven to zero, and by adding the derivative of switching function, the damping effect on controlled system by PD-GTSMC is incurred and the switching at steady state is restrained as well. The main contribution of this paper is summarized as follows:

- 1) A novel global sliding mode surface for TSS is proposed, where the system is moved on the terminal sliding mode in global time. Compared with [15] where the convergence of system is met as the derivative of SS is controlled to zero, the proposed global SS takes advantage of the terminal sliding mode to compose the surface, thus the system stabilization is proven effect as soon as the system is on surface;
- 2) To restrain the chattering phenomena on the SS, PI/PD based GTSMC is designed, to eliminate the consistent switching of SS where the order increase of system by

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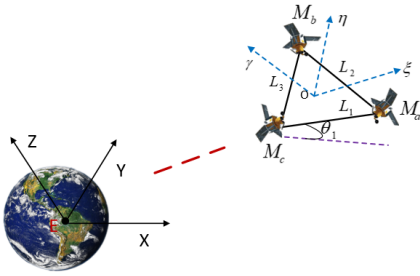


Fig. 1: Schematic Diagram of tethered satellites with coordinate system

integral and damping effect of derivative are resorted, respectively.

This paper is organized as follows. In Section 2, the preliminary results on system modeling and SS design are presented. In Section 3, the main results on GTSMC, PI-GTSMC and PD-GTSMC are given. Numerical simulation is given in Section 4. Finally, the conclusion remarks are made in Section 5.

II. PRELIMINARY RESULTS

Notation. Throughout this paper, the bold font x and normal font x denote the vector and scalar unless explicitly stated, respectively. \dot{x} denotes the time derivative of variable x . \mathbb{R} denotes the set of real numbers. \mathbb{R}^m denotes the set of real-valued vectors in dimension m . e denotes the constant such that $\ln(e) = 1$. $|x|$ denotes the absolute value of variable x . $\int f(t)dt$ denotes indefinite integral of $f(t)$.

A. Tethered Satellites System

Consider the triangular TSS as shown in Fig 1, where the reference frames $E - XYZ$ and $o - \xi\eta\gamma$ denote earth-centered inertial frame and system centroid centered orbital frame, respectively, where axis ξ of orbital frame directs to formation centroid from earth-center, axis η directs along the forward tangential line of the orbit, and axis γ is determined by right-hand triad. To facilitate the theoretic derivation, the following assumptions in the system modeling are proposed in view of the practical system configuration

- The vertice satellites are assumed to be mass points with equal mass such that $\sum_{i=1}^3 m_i = m$.
- The tethers connecting satellites are massless and elastically strained in the longitudinal direction.
- The satellites formation is serviced on a circular Kepler orbit around earth with angular velocity ω .

By deriving the kinetic energy \mathcal{T} and gravitational potential energy \mathcal{V}_g of satellites as well as the elastic potential energy \mathcal{V}_k of tethers, the Lagrangian variable is constituted as $\mathcal{L} = \mathcal{T} - \mathcal{V}_g - \mathcal{V}_k$. By taking \mathcal{L} into the following Lagrangian equation and further expanding the equation

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = \mathcal{Q}$$

where q denotes generalized coordinate and \mathcal{Q} denotes generalized force acting on the system, the dynamical equation

of TSS is obtained as follows[17]

$$\mathbb{S}_i : \begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = F_i + d_i + g_i u_i, \end{cases} \quad i \in \{1, \dots, 4\}, \quad (1)$$

where $[x_{11} \ \dots \ x_{41}] = \left[\frac{l_1}{l_0} \ \frac{l_2}{l_0} \ \theta_1 \ \theta_2 \right]$ which is non-dimensional denotes the state vector of tether length of L_1, L_2 and in-plane angles between L_1, L_2 and axis ξ ,

$$F_1 = x_{11}x_{32}(x_{32} + 2) + 3x_{11}\cos^2x_{31} + \kappa \left[-6\xi_1(x_{11} - 1) + 3\xi_2\cos\theta(x_{21} - 1) - 3\xi_3\frac{\ell_3 - 1}{\ell_3}(x_{11} + x_{21}\cos\theta) \right],$$

$$F_2 = x_{21}x_{42}(x_{42} + 2) + 3x_{21}\cos^2x_{41} + \kappa \left[-6\xi_2(x_{21} - 1) + 3\xi_1\cos\theta(x_{11} - 1) - 3\xi_3\frac{\ell_3 - 1}{\ell_3}(x_{21} + x_{11}\cos\theta) \right],$$

$$F_3 = -2\frac{x_{12}}{x_{11}}(x_{32} + 1) - 3\sin x_{31}\cos x_{31} + \kappa \left[-3\xi_2\sin\theta\frac{x_{21} - 1}{x_{11}} + 3\xi_3\sin\theta\frac{x_{21}(\ell_3 - 1)}{x_{11}\ell_3} \right],$$

$$F_4 = -2\frac{x_{22}}{x_{21}}(x_{42} + 1) - 3\sin x_{41}\cos x_{41} + \kappa \left[3\xi_1\sin\theta\frac{x_{11} - 1}{x_{21}} - 3\xi_3\sin\theta\frac{x_{11}(\ell_3 - 1)}{x_{21}\ell_3} \right],$$

in which, $\theta = x_{31} - x_{41}$, $\kappa = \frac{EA}{m\omega^2l_0}$ where E, A denote elastic modulus and cross sectional area of tethers, $\ell_3 = \sqrt{x_{11}^2 + x_{21}^2 + 2x_{11}x_{21}\cos\theta}$, and

$$\xi_i = \begin{cases} 1 & l_i \geq l_0 \\ 0 & l_i < l_0. \end{cases} \quad (2)$$

Moreover, d_i denotes unknown external disturbance, and u_i denotes the control input to \mathbb{S}_i with nonzero coefficient g_i .

Without loss of generality, in the later control design, we discuss on any single system \mathbb{S}_i with input u_i , such that the designed control fits for the other subsystems. In a more compact form, the objective system is rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= F + d + gu, \end{aligned} \quad (3)$$

which is actually a second-order affine nonlinear system with uncertainty d .

Asumption. Disturbance d is upper bounded in magnitude satisfying $|d| < \delta$, where δ is a positive real.

B. Sliding Surface Design

Let us recall the TSMC for system (3), the sliding surface is constructed as[9]

$$S_0 = x_2 + c_1x_1 + c_2x_1^r, \quad r = \frac{q}{p}, \quad (4)$$

where p, q are odd integers satisfying $p > q$, i.e. $0 < r < 1$, and c_1, c_2 are positive reals. When the system is on sliding surface S_0 , the state equation satisfies

$$\frac{dx_1}{c_1x_1 + c_2x_1^r} = -dt,$$

where the solution of x_1 is solved as follows

$$x_1(t_0) = \left[\frac{(e^{c_1(1-r)(t_*-t_0)} - 1)c_2}{c_1} \right]^{\frac{1}{1-r}}, \quad (5)$$

in which t_0 denotes the initial time of the response, and t_* denotes the time as x_1 is stabilized. We denote $x_1(t_0), x_2(t_0)$ as x_1^0, x_2^0 , and inspired from (5), consider the following function

$$\alpha_1(t) = \left[\frac{(e^{c_1(1-r)(t_*-t)} - 1)c_2}{c_1} \right]^{\frac{1}{1-r}}, \quad (6)$$

whose time derivative is

$$\dot{\alpha}_1(t) = -c_2 \left[\frac{(e^{c_1(1-r)(t_*-t)} - 1)c_2}{c_1} \right]^{\frac{r}{1-r}} \times e^{c_1(1-r)(t_*-t)},$$

then another function is straight obtained as follows

$$\alpha_2(t) = \frac{x_2^0}{-c_1 x_1^0 - c_2 (x_1^0)^r} \dot{\alpha}_1(t). \quad (7)$$

The proposed functions $\alpha_1(t)$ and $\alpha_2(t)$ satisfy

$$\begin{aligned} \alpha_1(t_0) &= x_1^0, & \alpha_1(t_*) &= 0; \\ \alpha_2(t_0) &= x_2^0, & \alpha_2(t_*) &= 0. \end{aligned} \quad (8)$$

Accordingly, with the functions (6),(7) obtained, the following sliding surface is proposed

$$\begin{aligned} \mathcal{S} &= (x_2 + c_1 x_1 + c_2 x_1^r) \\ &\quad - (\alpha_2 + c_1 \alpha_1 + c_2 \alpha_1^r) U(t_* - t), \end{aligned} \quad (9)$$

where step function

$$U(t_* - t) = \begin{cases} 1 & t \leq t_* \\ 0 & t > t_* \end{cases}$$

If we let

$$\mathcal{F}(t) = (\alpha_2 + c_1 \alpha_1 + c_2 \alpha_1^r) U(t_* - t), \quad (10)$$

evidently, $\mathcal{F}(t)$ meets following three principles:

- $\mathcal{F}(t_0) = \alpha_2 + c_1 \alpha_1 + c_2 \alpha_1^r$;
- $\mathcal{F}(t_*) = 0$;
- $\mathcal{F}(t)$ has bounded first time derivative in $(0, t_*)$.

These principles look quite familiar as in the functions designed in global SMC[18]. Actually, as proved in the main results, the proposed sliding surface \mathcal{S} has global terminal sliding mode.

On the other hand, from the terminal SS in (4), by solving a differential equation, the finite time $t_* - t_0$ of stabilization is obtained as follows

$$\psi(x_1) = \frac{1}{c_1(1-r)} \ln \left| 1 + \frac{c_1}{c_2} x_1^{1-r} \right|,$$

which can be also found in [19].

Note that in (1), the variables x_{11}, x_{21} appear in the denominators of F_3 and F_4 , which indicates that the initial state of x_{11} should be nonzero. Thus when applied to TSS (1), the stability of control is not strictly global. By leaving

out the singular point $x_{11} = 0, x_{21} = 0$, namely, on the nearly global set of $\mathbf{x} = [x_1 \ x_2]$ for $\mathbb{S}_1, \mathbb{S}_2$

$$\mathcal{R} = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1 \neq 0\}, \quad (11)$$

the finite-time stability of the closed-loop system of $\{\mathbb{S}_1, \dots, \mathbb{S}_4\}$ is still held.

III. MAIN RESULTS

In this section, GTSMC with global sliding mode is devised to improve the traditional TSMC as the reaching phase to sliding mode is obviated. Then to alleviate the chattering phenomenon on SS, a PI based GTSMC and PD based GTSMC are devised respectively by appending the integral of switch function to increase the system order as well as by appending the derivative of switch function to augment the damping effect.

In the following, we propose a general GTSMC by virtue of sliding surface \mathcal{S} in (9), and prove that closed-loop system owns global sliding mode and stabilizes in finite time slightly over $\psi(x_1^0)$.

Theorem 1. Consider the affine nonlinear system (3) and SS designed in (9). If the following control is applied

$$\mathcal{U}^1 = g^{-1}(-F - c_1 x_2 - c_2 r x_1^{r-1} x_2 + \dot{\mathcal{F}} - k_1 \text{sgn}(\mathcal{S})), \quad (12)$$

where real coefficient $k_1 > \delta$, then the closed-loop system shows global sliding mode on surface \mathcal{S} , and stabilizes in $t = \psi(x_1^0) + \Delta_t$, where Δ_t is a sufficiently small time interval related to initial states of system.

Proof. According to the principles \mathcal{F} holds, we have

$$\mathcal{S}(0) = x_2^0 + c_1 x_1^0 + c_2 (x_1^0)^r - \mathcal{F}(0) = 0. \quad (13)$$

Let $\mathcal{V} = \frac{1}{2} \mathcal{S}^2$, and $u = \mathcal{U}^1$ in \dot{x}_2 , the expression of $\dot{\mathcal{V}}$ is expanded as follows

$$\begin{aligned} \dot{\mathcal{V}} &= \mathcal{S}(F + d + g\mathcal{U}^1 + c_1 x_2 + c_2 r x_2 x_1^{r-1} - \dot{\mathcal{F}}) \\ &= -k_1 |\mathcal{S}| + d\mathcal{S} \\ &\leq -(k_1 - \delta) |\mathcal{S}|. \end{aligned}$$

Note that

$$\alpha_2(t_*) + c_1 \alpha_1(t_*) + c_2 \alpha_1^r(t_*) = 0,$$

and

$$\dot{\mathcal{F}}(t) = \dot{\Psi} U(t_* - t) - \Psi \delta(t_* - t),$$

where $\Psi = \alpha_2 + c_1 \alpha_1 + c_2 \alpha_1^r$, and δ denotes Dirac function, therefore, $\dot{\mathcal{F}}(t_*) = \dot{\Psi}(t_*)$ is bounded, and \mathcal{U}^1 is continuous consequently. Namely, $\mathcal{V} < 0$ such that \mathcal{V} is a Lyapunov function of \mathcal{S} . Considered (13), one concludes that the closed-loop system owns global sliding mode on \mathcal{S} by the effect of \mathcal{U}^1 .

Moreover, because the designed $\alpha_1(t)$ and $\dot{\alpha}_1(t)$ meet the sliding mode of \mathcal{S}_0 , i.e.

$$\dot{\alpha}_1 + c_1 \alpha_1 + c_2 (\alpha_1)^r = 0,$$

and $\alpha_2(t)$ is amended on $\dot{\alpha}_1$ to meet the initial condition of x_2 , which means by constraining \mathcal{S} on zero,

$$\Gamma = x_2 + c_1 x_1 + c_2 x_1^r$$

is evolved along the sliding mode S_0 .

On one hand, as $\mathcal{F}(t_*) = 0$, Γ approaches zero at least in

$$t = t_* = \psi(x_1^0).$$

On the other hand, $x_1(t)$ is stabilized at $t = \psi(x_1^0)$ when the system moves on S_0 , then at $t = \psi(x_1^0)$ system (3) via control (12) may move to a small neighborhood around zero, say $O(\Delta_x)$, which is reflected on initial states x_1^0, x_2^0 .

Finally, since $\mathcal{F}(t)$ vanishes as $t > \psi(x_1^0)$, the closed-loop system will stabilize from $\mathbf{x} \in O(\Delta_x)$ along S_0 consequently, which takes small time Δ_t that is tuned by Δ_x . \square

As shown in the proof, the addition of auxiliary function $\mathcal{F}(t)$ engages global finite-time convergence to the system on \mathcal{S} , which is beneficial to the stability of system.

Remark 1. If the initial states x_1^0, x_2^0 meet sliding condition

$$x_2^0 + c_1 x_1^0 + c_2 (x_1^0)^r = 0,$$

then $\mathcal{F}(t) \equiv 0$, and system stabilizes in exactly $t = \psi(x_1^0)$. On the other hand, because $\alpha_1(t)$ in $\mathcal{F}(t)$ is predetermined by sliding mode $S_0 = 0$, thus the system is on the motion of $S_0 = 0$, even $\alpha_2(t)$ is amended to meet the global sliding conditions as in (8).

Next, to reduce the chattering effect of GTSMC, the integral of $k_1 \text{sgn}(\mathcal{S})$ is designed and compensated in the control.

Firstly, since $U_d = k_1 \text{sgn}(\mathcal{S})$ switches frequently, to maintain the continuity of integrand, by passing U_d into a low pass filter, its average value is expressed as

$$\tau \dot{U}_{ed} + U_{ed} = U_d, \quad (14)$$

where τ is the positive filter time constant. In control (12), the presence of $k_1 \text{sgn}(\mathcal{S})$ will bring unexpected chattering on SS, despite the persisting on $|\mathcal{S}| = 0$. Therefore, the switching function is redesigned as

$$U_d = -(k_2 |U_{ed}| + k_s |\mathcal{S}|) \text{sgn}(\mathcal{S}), \quad (15)$$

where the coefficient of $\text{sgn}(\mathcal{S})$ is replaced by $k_2 |U_{ed}| + k_s |\mathcal{S}|$, thus the magnitude of U_d will converge to zero as long as $|U_{ed}|$ is regulated to zero on $\mathcal{S} = 0$ [20], and this is finished by the integral compensation, say Φ_i , which is

$$\Phi_i = k_i \int U_{ed} dt, \quad (16)$$

with integral gain $k_i > 0$, where the integrand is chosen as U_{ed} to circumvent discontinuity of U_d . The effect of integral compensation is demonstrated in the following theorem.

Theorem 2. Consider the affine nonlinear system (3) and SS designed in (9). If the following control is applied

$$U^2 = g^{-1}(-F - c_1 x_2 - c_2 r x_1^{r-1} x_2 + \dot{\mathcal{F}} + U_d + \Phi_i), \quad (17)$$

in which, U_d, Φ_i are given in (15) and (16), respectively, where real numbers $k_2 > \delta, k_s, k_i > 0$, then the closed-loop system stabilizes in $t = \psi(x_1^0) + \Delta_t$, and the switching function U_d is eliminated on $\mathcal{S} = 0$.

Proof. According to the proof in Theorem 1, the global sliding mode and finite time stability is obtained with the assistance of SS (9), as long as a Lyapunov function for \mathcal{S} is found. Hence we first prove that there is a Lyapunov function for closed-loop system by (17).

Let $\mathcal{V} = \frac{1}{2} \mathcal{S}^2$, take its time derivative on system (3), and by considering (17), we have

$$\begin{aligned} \dot{\mathcal{V}} &= \mathcal{S}(F + d + g\mathcal{U}^2 + c_1 x_2 + c_2 r x_2 x_1^{r-1} - \dot{\mathcal{F}}) \\ &= -k_2 |U_{ed}| |\mathcal{S}| + k_i \int U_{ed} dt \mathcal{S} - k_s \mathcal{S}^2 + d\mathcal{S}. \end{aligned} \quad (18)$$

Since U_{ed} is equal to $\pm |U_d|$ when \mathcal{S} is not zero, we have

$$\int U_{ed} dt \approx 0, \quad \text{as } \mathcal{S} \neq 0,$$

then (18) is rewritten as

$$\dot{\mathcal{V}} \leq -(k_2 |U_{ed}| - \delta) |\mathcal{S}| - k_s \mathcal{S}^2. \quad (19)$$

If an abrupt increase of $|\mathcal{S}|$ is happened such that $|U_d| > 1$, so as to $|U_{ed}| > 1$, then by $k_2 > \delta$ we have

$$\dot{\mathcal{V}} \leq 0,$$

such that $|\mathcal{S}|$ is convergent to zero. On the other hand, historical $|U_{ed}|$ will keep the magnitude of U_d despite the reduction in $|\mathcal{S}|$, thus $k_2 U_{ed} - \delta > 0$ is held. Therefore, by switching function U_d in (15), the negative definiteness of $\dot{\mathcal{V}}$ is held, thus \mathcal{V} is a Lyapunov function of \mathcal{S} .

Furthermore, as $\mathcal{S}, \dot{\mathcal{S}}$ is settled down on zero, from (18) we have

$$-k_2 |U_{ed}| \text{sgn}(\mathcal{S}) + k_i \int U_{ed} dt + d = 0. \quad (20)$$

Because $\text{sgn}(U_d) = -\text{sgn}(\mathcal{S})$, and $k_2 |U_{ed}| > \delta$, by overwhelming d we have

$$k_2 U_{ed} + k_i \int U_{ed} dt = 0, \quad (21)$$

which is a stable second-order polynomial of $\int U_{ed} dt$, namely, via the integral compensation, U_{ed} is convergent to zero at $\mathcal{S} = 0$. This completes the proof. \square

Instead of using the integral of U_{ed} , considered the damping effect by $\frac{d}{dt} U_{ed}$ on $|\mathcal{S}|$, the PD based GTSMC on system (3) is proposed, which tends to eliminate $|U_{ed}|$ by differential regulation. Specifically, switch function U_d is the same as in (15), but the PD regulation is varied to

$$\Phi_d = k_d \dot{U}_{ed}, \quad (22)$$

the control and the proof are given as follows.

Theorem 3. Consider the affine nonlinear system (3) and SS designed in (9). If the following control is applied

$$U^3 = g^{-1}(-F - c_1 x_2 - c_2 r x_1^{r-1} x_2 + \dot{\mathcal{F}} + U_d + \Phi_d), \quad (23)$$

where U_d, Φ_d are shown in (15), (22), respectively, and real coefficients $k_d > 0$, then the closed-loop system stabilizes in $t = \psi(x_1^0) + \Delta_t$, and the switching function U_d is eliminated on $\mathcal{S} = 0$.

Proof. By $\mathcal{V} = \frac{1}{2}\mathcal{S}^2$, and considered control \mathcal{U}^3 in system (3), the derivative of \mathcal{V} along the system is

$$\begin{aligned}\dot{\mathcal{V}} &= \mathcal{S}(F + d + g\mathcal{U}^3 + c_1x_2 + c_2rx_2x_1^{r-1} - \dot{\mathcal{F}}) \\ &= -k_2|U_{ed}||\mathcal{S}| + (k_d\dot{U}_{ed} + d)\mathcal{S} - k_s\mathcal{S}^2 \\ &\leq -(k_2|U_{ed}| - k_d|\dot{U}_{ed}| - \delta)|\mathcal{S}| - k_s\mathcal{S}^2.\end{aligned}\quad (24)$$

Since U_{ed} is filtered through low frequency, $|\dot{U}_{ed}|$ is negligible compared with $|U_{ed}|$, moreover, by $k_2 > \delta$, we have

$$k_2|U_{ed}| - k_d|\dot{U}_{ed}| - \delta > 0$$

is held by sufficiently large k_2 chosen, such that $\dot{\mathcal{V}} < 0$, namely, \mathcal{V} is a Lyapunov function of \mathcal{S} , thus the global sliding mode and the finite-time convergence of closed-loop system is obtained.

On the other hand, as $\mathcal{S}, \dot{\mathcal{S}}$ is stabilized, we have

$$-k_2|U_{ed}|\text{sgn}(\mathcal{S}) + k_d\dot{U}_{ed} + d = 0, \quad (25)$$

Because the chattering of $|U_{ed}|$ and the well-chosen k_2 result in the neglect of d with regards to $k_2|U_{ed}|\text{sgn}(\mathcal{S})$, moreover, by $\text{sgn}(U_d) = -\text{sgn}(\mathcal{S})$, we have

$$k_d\dot{U}_{ed} + k_2U_{ed} = 0. \quad (26)$$

which is a second-order differential equation of U_{ed} , such that $\dot{U}_{ed} \rightarrow 0$, namely, the damping of U_{ed} is arisen, and $U_{ed} \rightarrow 0$ implying that the switching function U_d is eliminated on $\mathcal{S} = 0$. This completes the proof. \square

Compared with PI-GTSMC (17), the closed-loop system via \mathcal{U}^3 has higher damping around $\mathcal{S} = 0$, which means smaller settling time to steady state. On the other hand, the order of the system via (17) is higher, meaning better steady accuracy.

In this section, the global sliding mode of system (3) on the proposed surface (9) is shown, and the finite-time stability of the closed-loop system via GTSMC (12) is proved. Then PI-GTSMC and PD-GTSMC with the integral/derivative compensation in control are proposed, which devote to eliminating steady-state error on SS.

IV. NUMERICAL COMPUTING

In this section, the control method is tested on TSS system (1) to validate the performance of the proposed controller. The outer disturbances in system are chosen as $d_1(t) = M\sin(0.5t)$, $d_2(t) = M\cos(0.6t + 0.2)$ where $M = 2$. The parameters in sliding surface (9) are selected as $c_1 = 2$, $c_2 = 3$, $r = \frac{3}{5}$. The parameters in control (12) are $k_1 = 15$, and the parameters in control (17) and (23) are $k_2 = 15$, $k_s = 10$, $k_d = 15$, $k_i = 10$, $\tau = 1$. Moreover, TSMC is also tested to compare with GTSMC, whose parameters are coincident with GTSMC (12). The simulation is run on MATLAB, and the time range and step size is fixed to $[0 \ 20]$ and 0.001, respectively. The initial states of system (1) are chosen as $x_{11}^0 = x_{21}^0 = 16$, $x_{12}^0 = x_{22}^0 = 4$, $x_{31}^0 = x_{41}^0 = 5$, $x_{32}^0 = x_{42}^0 = 6$.

We show the response of subsystem \mathcal{S}_1 , whose steady state is $[1 \ 0]$, which corresponds to non-dimensional length 1

and changing rate 0. Furthermore, in order to fit the variables on sliding surface, the states are amended as

$$\bar{x} = [\bar{x}_1 \ \bar{x}_2] = [x_{11} - 1 \ x_{12}],$$

accordingly, the sliding surface is rewritten by replacing \bar{x}_1, \bar{x}_2 into (9). The responses via GTSMC (12), PI-GTSMC (17), PD-GTSMC (23) as well as TSMC are shown in Fig 2-4.

In Fig 2, the curve of \bar{x}_1 by PI-GTSMC shows the least steady error among all the controls, the curve of PD-GTSMC settles at second, and the the curves of TSMC & GTSMC hold the most error. Moreover, the response of \bar{x}_1 by TSMC shows the smallest converging rate as compared with the other three controls. In Fig 3, SS of TSMC linearly converges from initial point to zero after $t = 3$, while SS of the other three controls stay around zero all the time. Besides, the magnitude of SS for PI-GTSMC nearly holds on zero at $1E - 6$ scale, while the magnitudes for the other controls possess some steady errors. In Fig 4, the curves of control inputs via four controls are exhibited, where the curves of PI-GTSMC & PD-GTSMC lie in a band enveloped by U_{TSMC} and U_{GTSMC} despite some stick impulses shown in the response of PD-GTSMC. Namely, the energy costed by $U_{PI-GTSMC}$ and $U_{PD-GTSMC}$ is the least, which is coincident with the low steady error of PI/PD-GTSMC as shown in Fig 2, besides, the sticks which are caused by oscillations in differential operation are reflected in curves of Fig 3 as well. In general, the converging speed of GTSMC is superior to TSMC, and GTSMC by PI tuning shows least steady error, while GTSMC by PD tuning tends to reduce average steady error as well, but the extra oscillations are raised in the response.

Furthermore, to manifest the differences in the responses under different control parameters and disturbances, the plots of \bar{x}_1 by GTSMC via varied M and r are shown in Fig 5-6. As shown in Fig 5, the stability of state \bar{x}_1 are nearly kept with the increase of M as long as the condition $k_1 > M$ are held, despite some oscillation phenomena arise in the cases for higher M . In contrast, the influence of parameter r in the controller is more evident as shown in Fig 6, where the steady error gradually grows with r , namely, lower value of r is expected to confine steady error as small as possible. In summary, the rise of parameter r is adverse to decrease steady error, just as the effect of magnitude M of disturbance on the state responses.

V. CONCLUSION

This paper researches a novel GTSMC on an affine nonlinear TSS system under outer disturbances, and the effect of PI/PD compensation in restraining steady-state error. The designed global sliding surface takes advantage of the finite-time convergence of traditional terminal sliding surface, and the system is thus confined on sliding surface all the time and stabilizes in finite time. Then the integral of the low-pass filtered switching function is appended in GTSMC to eliminate the switching at steady state, and the chattering effect of the controlled system on sliding surface is

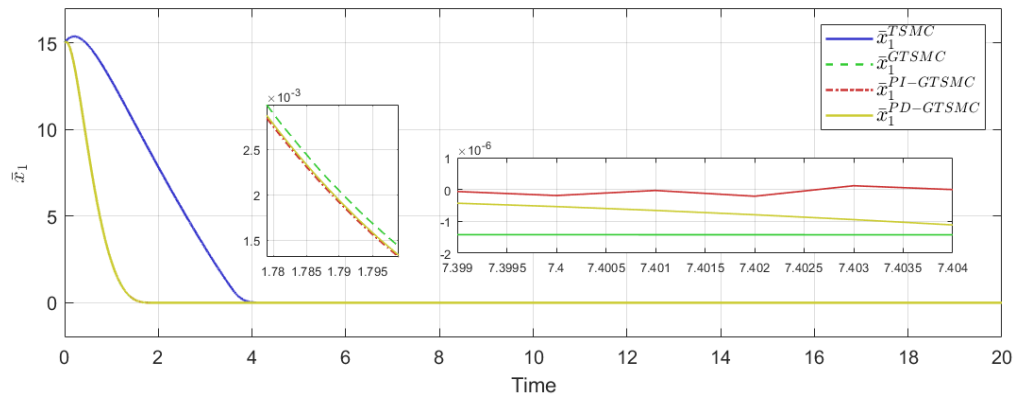


Fig. 2: Response of variable \bar{x}_1 by controls.

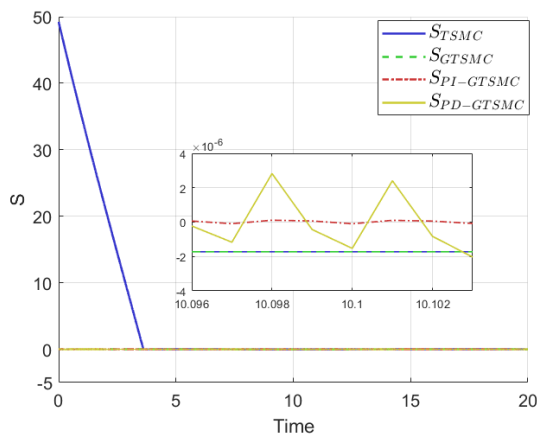


Fig. 3: Responses of sliding surface S via controls.

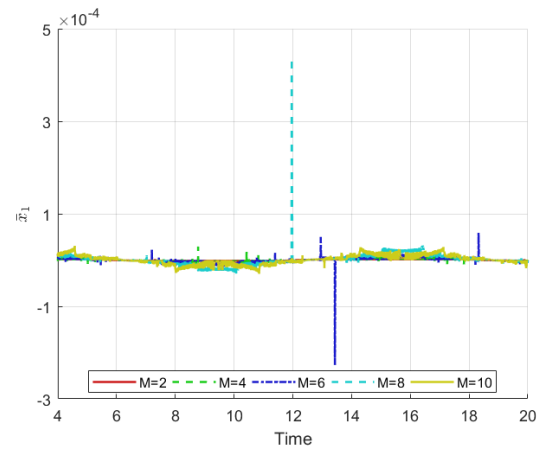


Fig. 5: Responses of variable \bar{x}_1 via GTSMC under varied M .

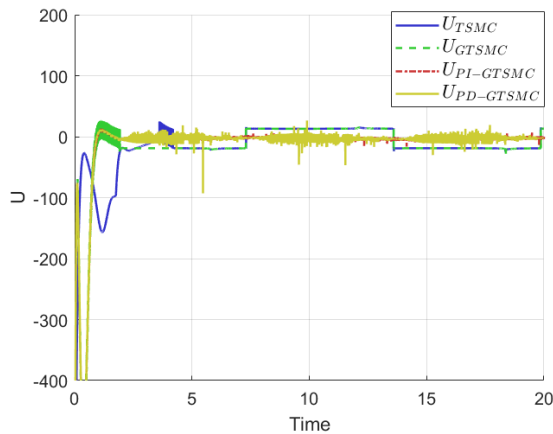


Fig. 4: Responses of control inputs.

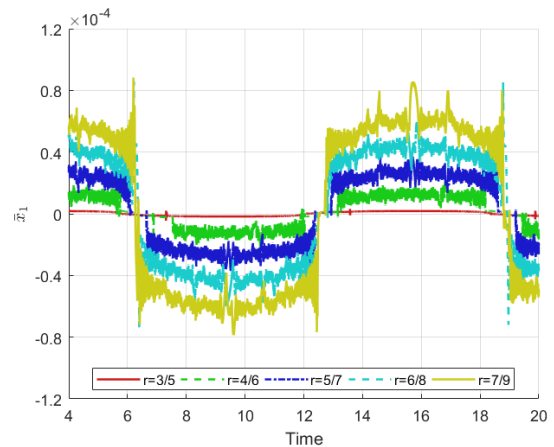


Fig. 6: Responses of variable \bar{x}_1 via GTSMC under varied r .

restrained. Moreover, by appending the derivative of filtered switching function, it is proved that the damping effect of controlled system is strengthened and the chattering effect is diminished as well. The future works should further improve the instantaneous performance of the system in sliding mode.

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