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Robust Balancing Control of Biped Robots for External Forces

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Abstract— **This paper develops a controller synthesis method for ensuring an admissible bound of external forces on biped robots in a desired level. We first introduce the authors' preceding results on the norm-based stability criterion for a biped walking constructed on its linear inverted pendulum model (LIPM). More precisely, an induced norm can be taken to formulate the fact that the balance for a biped robot is achieved if its zero moment point (ZMP) always stays in the supporting region at each step. Based on this norm-based criterion, we aim at making the maximum energy of external forces admissible for balancing the biped robot be a pregiven desired bound $\gamma(>0)$. To achieve this objective, a robust controller is designed through the linear matrix inequality (LMI)-based approach. More importantly, a necessary and sufficient condition for the existence of a robust controller leading to the desired bound is characterized by some LMI conditions. The effectiveness of the overall arguments is validated through some comparative simulation results of a biped walking robot with external forces.**

I. INTRODUCTION

Humanoids akin to human motions have been extensively studied for the last few decades [1]–[3], in which the walking balance is regarded as one of the most crucial issues. Notably, ensuring robustness against possible external forces is quite important since they could make humanoids off balance.

In connection with this issue, the concept of a capture point (CP) approach and the maximal output admissible (MOA) set approach are concerned with in [4], [5] and [6]–[8], respectively. The former indicates a point on the ground on which the biped robot steps foot to avoid falling, while the latter is for clarifying the initial condition of the biped robot to maintain a walking balance based on the fact that the zero moment point (ZMP) is required to be located inside the supporting region for stable walking [9], [10]. In comparisons between these two approaches, no consideration of the center of mass (CoM) is given in the CP approach, while the state variables in the MOA set approach involve the motion of the CoM. In this sense, the controller in [6] could also lead to a desired trajectory of the CoM.

Even though some effectiveness of taking the MOA set approach is discussed in [6]–[8], significant computational demands are often required in them. More precisely, various linear inequalities should be solved for describing supporting regions at each step, and thus, computing the MOA set does depend on the number of steps of the biped robot to be concerned with. This could make the number of linear inequalities to be solved for achieving the walking balance increase as that of the walking steps of the biped robot becomes larger.

To address this computational problem, the MOA set approach is significantly modified in [11], [12] by taking some induced norms adequately for types of disturbances. Taking this norm-based argument in those studies leads to a finite number of iterations for obtaining an exact MOA set, irrespective of the number of walking steps. Some admissible bounds of time-varying external forces are also derived in [11], [12] for ensuring the standing and walking balances, but these bounds do depend on pregiven controllers.

To solve this problem, we develop a synthesis method of a robust controller ensuring that the energy of external forces can be increased to a desired level without raising instability. More precisely, a necessary and sufficient condition for the existence of such a controller is characterized by some linear matrix inequality (LMI) conditions. Thus, we can also obtain an optimal controller leading to the maximum bound of admissible external forces for stable biped walking by taking the desired level larger until the LMI conditions become infeasible.

The remainder of this paper is structured as follows. We first introduce the trajectory tracking control for the lateral motion of biped walking robots in Section II. The main results, i.e., some LMI-based conditions for leading to the robust controller, are given in Section III. In Section IV, the effectiveness of the overall arguments is demonstrated through some simulation results of a biped walking robot. The concluding remarks are provided in Section V.

Notations: The notations $\mathbb{R}^\nu$ and $\mathbb{N}_0$ denote the set of ν-dimensional real vectors and the set of nonnegative integers, respectively. We use the notations $\|\cdot\|_2$ and $\|\cdot\|_\infty$ to mean the ℓ_2 and ℓ_∞ norms of a vector-valued sequence, respectively, i.e.,

$$\|x\|_2 := \left(\sum_{k=0}^{\infty} x^T[k]x[k]\right)^{1/2}, \ \|x\|_\infty := \max_i \sup_{k\in\mathbb{N}_0} |x_i[k]|.$$

A positive definite matrix $M = M^T$ is denoted $M \succ 0$.

II. TRAJECTORY TRACKING CONTROL FOR BIPED ROBOTS IN THEIR LIPM REPRESENTATION

This section introduces the linear inverted pendulum model (LIPM) representation [13] of biped robots and the corresponding trajectory tracking control for lateral motion.

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A. Description of Lateral Motion via LIPM Representation

Let us consider the biped robot walking on a flat ground in the presence of external forces as shown in Fig. 1. The corresponding LIPM representation is given by

$$m\ddot{\mathbf{c}} = \frac{mg}{h}(\mathbf{c} - \mathbf{p}) + \mathbf{f} \tag{1}$$

where m is the weight of the robot, g is the gravitational acceleration, $\mathbf{c} := \begin{bmatrix} c_{\mathbf{x}} & c_{\mathbf{y}} \end{bmatrix}^T \in \mathbb{R}^2$ is the lateral position of the center of mass (CoM), h is the height of the CoM, $p := \begin{bmatrix} p_{\mathbf{x}} & p_{\mathbf{y}} \end{bmatrix} \in \mathbb{R}^2$ is the zero moment point (ZMP) position on the ground, and $\mathbf{f} = \begin{bmatrix} f_{\mathbf{x}} & f_{\mathbf{y}} \end{bmatrix} \in \mathbb{R}^2$ is the external force. Here, we assume that $\mathbf{f}$ has a finite energy, i.e., $f_x(\cdot)$ and $f_y(\cdot)$ are elements of the $\mathcal{L}_2$ space, and the time derivative of h is ignorable and it can be regarded as a constant as the previous works [7], [8], [11]–[13]. This assumption on the external disturbance is meaningful for some practical situations, such as a collision with obstacles, stepping on jagged edges, and so on, which could be interpreted as instantaneous external forces on biped robots.

Next, defining u and w as $u := p_{\mathbf{x}}$ and $w := f_{\mathbf{x}}$ leads to the state-space representation for (1) with respect to the $\mathbf{x}$-axis given by

$$\dot{x} = Ax + B_w w + B_u u \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix}, \ B_w = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \ B_u = \begin{bmatrix} 0 \\ -\omega_0^2 \end{bmatrix} \tag{3}$$

with $x := \begin{bmatrix} c_{\mathbf{x}} & \dot{c}_{\mathbf{x}} \end{bmatrix}^T$ and $\omega_0^2 := \dfrac{g}{h}$. Note that the pair (A, B_u) is controllable since the controllability matrix $\begin{bmatrix} B_u & AB_u \end{bmatrix}$ has a full rank.

Remark 1: The state-space representation in $\mathbf{y}$-axis is the same as that in $\mathbf{x}$-axis with taking $x = \begin{bmatrix} c_{\mathbf{y}} & \dot{c}_{\mathbf{y}} \end{bmatrix}$ since (1) is decoupled in terms of $\mathbf{x}$ and $\mathbf{y}$ axes. In this sense, we only consider the case for the $\mathbf{x}$-axis in this paper, with which the parallel arguments can be applied to that for the $\mathbf{y}$-axis.

Similarly to the previous studies [6]–[8], [11]–[13], we derive discretized state-space representation of the LIPM (2) with taking a nonpathological sampling period T relative to A [14] given by

$$x[k+1] = A_d x[k] + B_{wd} w[k] + B_{ud} u[k] \tag{4}$$

where

$$A_d := \exp(AT),$$
$$\begin{bmatrix} B_{wd} & B_{ud} \end{bmatrix} := \int_0^T \exp(A\theta) d\theta \begin{bmatrix} B_w & B_u \end{bmatrix}$$

with $x[k] := x(kT)$, $w[k] := w(kT)$, and $u[k] := u(kT)$. Because of the nonpathological sampling period T, the discrete-time pair (A_d, B_{ud}) is also controllable.

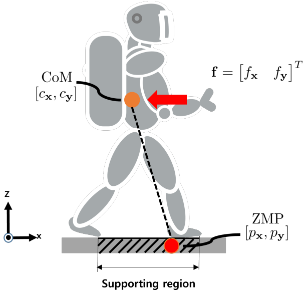


Fig. 1. LIPM representation with external forces.

B. Trajectory Tracking Controller

The reference trajectories for the CoM and ZMP are assumed to be generated by using the gait planning methods in [15], [16]. In other words, the trajectories are described by the equation

$$\phi[k+1] = A_d \phi[k] + B_{ud} \mu[k] \tag{5}$$

where $\phi[k] \in \mathbb{R}^2$ is the reference for $x[k]$ and $\mu[k] \in \mathbb{R}$ is that for $u[k]$. Furthermore, the reference ZMP should be predetermined to remain within the supporting region at every step.

With respect to this reference trajectory, the trajectory tracking controller is presented as

$$u[k] = -K(\phi[k] - x[k]) + \mu[k] \tag{6}$$

where K is a stabilizing gain for the corresponding closed-loop system. In other words, combining (4)–(6) leads to the closed-loop system

$$\begin{aligned} e[k+1] &:= \phi[k+1] - x[k+1] \\ &= A_d(\phi[k] - x[k]) - B_{wd} w[k] + B_{ud}(\mu[k] - u[k]) \\ &= (A_d + B_{ud} K) e[k] - B_{wd} w[k] \\ &= A_{2d} e[k] - B_{wd} w[k] \end{aligned} \tag{7}$$

where $e[k] := \phi[k] - x[k]$ is the CoM tracking error and K is determined to make A_{2d} be Schur stable. Such a K should exist for the controllability property of (A_d, B_{ud}).

Here, it is still unclear whether or not the biped robot is off balance during the walking regardless of the presence of an external force, although the stabilizing controller K leads to asymptotic stability for the closed-loop system when there is no external force. The following section is devoted to clarifying this issue by designing a robust balancing controller in terms of linear matrix inequalities (LMIs).

III. Controller Synthesis for Robust Balancing

This section establishes a controller synthesis method for ensuring the walking balance of biped robots robustly for external forces. More precisely, the roust controller leads to the walking balance when the maximum energy of external forces is given.

To this end, we first introduce the stability criterion [12] and characterize an admissible bound for the energy of external forces in terms of the stable walking balance.

A. Induced Norm-based Stability Criterion for Biped Walking

Based on the fact that a biped robot is off balance if its ZMP trajectory deviates from the corresponding supporting region, an admissible bound of external forces can be defined as their energy allowable for such a ZMP limitation.

To put it another way, the stability criterion in the authors' previous work [12] can be reinterpreted as follows.

Lemma 1: For pregiven trajectories of the reference ZMP and the supporting region, there should exist a positive constant σ, with which the inequality

$$\|\mu - u\|_\infty \le \sigma \tag{8}$$

ensures that the real ZMP u is always located in the supporting region at each step.

When the value of σ and the controller K are given, we can also compute the exact value of the maximum l_2 norm (or equivalently, the maximum energy) of w in terms of (8) by using the arguments in [12]. In other words, we can obtain the maximum defined as

$$\max_{\gamma} \sup_{\|w\|_2 \le \gamma} \|\mu - u\|_\infty \le \sigma. \tag{9}$$

As mentioned before, however, this maximum is determined after the controller K is given, and thus, it is still unclear how we can design a controller for ensuring an admissible energy of external forces to be bounded in a desired level γ. In this sense, the following subsection is devoted to deriving a controller K, leading to that

$$\sup_{\|w\|_2 \le \gamma} \|\mu - u\|_\infty \le \sigma \tag{10}$$

for given $\sigma, \gamma > 0$.

B. Robust Controller Synthesis for External Forces

As a preliminary step to derive a synthesis procedure for the aforementioned robust controller, we are led to the following proposition associated with characterizing the induced norm (10) in terms of the controller K, based on the arguments in Lemma 1.

Proposition 1: For given positive scalars σ and γ, the controller K ensures the induced norm-based performance (10) if and only if

$$(KXK^T)^{1/2} < \sigma_\gamma := \frac{\sigma}{\gamma} \tag{11}$$

where X is the solution to the Lyapunov equation

$$A_{2d} X A_{2d}^T - X + B_{wd} B_{wd}^T = 0. \tag{12}$$

Proof: We first note from (7) that the input/output relation is given by

$$e[k] = -\sum_{i=0}^{k-1} A_{2d}^{k-i-1} B_{wd} w[i]. \tag{13}$$

We next define the ZMP tracking error as

$$z[k] := \mu[k] - u[k] = K e[k]. \tag{14}$$

Then, combining (13) and (14) derives

$$\begin{aligned} \sup_{\|w\|_2 \le \gamma} |z[k]| &= \sup_{\|w\|_2 \le \gamma} \left| \sum_{i=0}^{k-1} K A_{2d}^{k-i-1} B_{wd} w[i] \right| \\ &\le \sup_{\|w\|_2 \le \gamma} \left(\sum_{i=0}^{k-1} |K A_{2d}^i B_{wd}|^2 \right)^{1/2} \cdot \left(\sum_{i=0}^{k-1} w^2[i] \right)^{1/2} \\ &\le \left(\sum_{i=0}^{k-1} |K A_{2d}^i B_{wd}|^2 \right)^{1/2} \cdot \gamma \end{aligned} \tag{15}$$

where the equality holds when

$$w[i] = \lambda (K A_{2d}^{k-i-1} B_{wd})^T \quad (\forall i = 0, \ldots, k-1) \tag{16}$$

for some λ. This clearly implies that

$$\begin{aligned} \sup_{\|w\|_2 \le \gamma} \|z\|_\infty &= \left(\sum_{i=0}^{\infty} |K A_{2d}^i B_{wd}|^2 \right)^{1/2} \cdot \gamma \\ &= \left(K \sum_{i=0}^{\infty} \left(A_{2d}^i B_{wd} B_{wd}^T (A_{2d}^T)^i \right) K^T \right)^{1/2} \cdot \gamma \\ &= (KXK^T)^{1/2} \cdot \gamma \end{aligned} \tag{17}$$

because

$$\sum_{i=0}^{\infty} \left(A_{2d}^i B_{wd} B_{wd}^T (A_{2d}^T)^i \right) = X. \tag{18}$$

This obviously completes the proof. ■

Proposition 1 clearly characterizes the necessary and sufficient condition for ensuring the induced norm-based performance (10) in terms of the controller K. With this result in mind, we provide a synthesis procedure for a robust controller K establishing (11) by deriving some LMI-based conditions.

Theorem 1: For given positive scalars σ and γ, there exists a controller K ensuring (11) if and only if there exist a positive definite matrix P and a matrix Q such that

$$\begin{bmatrix} \sigma_\gamma^2 & Q \\ Q^T & P \end{bmatrix} \succ 0, \tag{19}$$

$$\begin{bmatrix} P & A_d P + B_{ud} Q & B_{wd} \\ P A_d^T + Q^T B_{ud}^T & X & 0 \\ B_{wd}^T & 0 & 1 \end{bmatrix} \succ 0. \tag{20}$$

If these LMIs are feasible, the controller K is given by

$$K = QP^{-1}. \tag{21}$$

Proof: First show that there exists a $P \succ 0$ such that

$$KPK^T < \sigma_\gamma^2, \tag{22}$$

$$A_{2d} P A_{2d}^T - P + B_{wd} B_{wd}^T \prec 0 \tag{23}$$

if and only if the inequality (11) holds.

Sufficiency: Note that there should exists a $Y \succ 0$ such that $A_{2d} Y A_{2d}^T - Y \prec 0$ because of the stability of A_{2d}. Then, we can take an arbitrary small $\epsilon > 0$ such that

$$KXK^T < KPK^T < \sigma_\gamma^2 \tag{24}$$

with $P := X + \epsilon Y$. With this P, we can see that

$$A_{2d}PA_{2d}^T - P + B_{wd}B_{wd}^T$$
$$= A_{2d}XA_{2d}^T - X + B_{wd}B_{wd}^T + \epsilon(A_{2d}YA_{2d}^T - Y) \prec 0. \quad (25)$$

Necessity: In terms of (23), there exists a B_0 such that $B_0B_0^T \succ 0$ and

$$A_{2d}PA_{2d}^T - P + B_{wd}B_{wd}^T + B_0B_0^T = 0 \quad (26)$$

and this means that P is given by

$$P = \sum_{i=0}^{\infty} \left(A_{2d}^i B_{wd}B_{wd}^T (A_{2d}^T)^i + A_{2d}^i B_0 B_0^T (A_{2d}^T)^i\right). \quad (27)$$

Because X in (12) is given by

$$X = \sum_{i=0}^{\infty} A_{2d}^i B_{wd}B_{wd}^T (A_{2d}^T)^i, \quad (28)$$

it immediately follows that

$$KXK^T \leq KPK^T < \sigma_\gamma^2. \quad (29)$$

In terms of Schur complement Lemma, we next reinterpret the LMIs (22) and (23) respectively by

$$\begin{bmatrix} \sigma_\gamma^2 & KP \\ PK^T & P \end{bmatrix} \succ 0, \begin{bmatrix} P - B_{wd}B_{wd}^T & A_{2d}P \\ PA_{2d}^T & P \end{bmatrix} \succ 0. \quad (30)$$

From Schur complementary Lemma and $A_{2d} = A_d + B_{ud}K$, the second inequality in (30) further admits the representation

$$\begin{bmatrix} P & A_dP + B_{ud}KP & B_{wd} \\ PA_d^T + PK^TB_{ud}^T & P & 0 \\ B_{wd}^T & 0 & 1 \end{bmatrix} \succ 0. \quad (31)$$

Defining $Q := KP$ completes the proof. ■

Even though the arguments in Theorem 1 are assumed for a given $\gamma(> 0)$, we can readily obtain from this theorem an optimal controller K leading to the maximum value of γ establishing (10). This is because the assertions in Theorem 1 correspond to a necessary and sufficient condition for the existence of a controller in terms of (10). In this sense, such an optimal controller is determined by taking γ in Theorem 1 larger until the LMIs (19) and (20) become infeasible.

IV. Simulation Results

This section aims at demonstrating the theoretical validity and the practical effectiveness of the robust controller proposed in Theorem 1 through some simulation results.

To this end, we consider the biped robot model taken in the authors' previous study [12] for conducting simulations, and its detailed specifications are given in Table I. The sampling period is taken by $T = 0.001$ [s], with which the pair (A_d, B_{ud}) is still controllable, i.e., a nonpathological sampling period T. Here, the values of σ in (8) for each axis (i.e., $\sigma_{\mathbf{x}}$ and $\sigma_{\mathbf{y}}$) are determined based on the size of the foot. The reference CoM and ZMP trajectories (ϕ, μ) are obtained by using the arguments in [16], as shown in Fig. 2.

More precisely, we first verify the theoretical validity by showing that the ZMP trajectory always remains inside the supporting regions at each step when external forces whose

TABLE I

The Specifications for the Biped Robot [12]

Height: 1.35 [m]	Foot size: 0.14 [m] ×0.23 [m]
CoM Height: 0.697 [m]	Weight : 50 [kg]
$\sigma_{\mathbf{x}}$: 0.11 [m]	$\sigma_{\mathbf{y}}$: 0.06 [m]

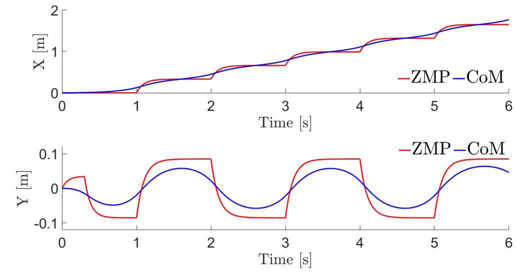


Fig. 2. Reference CoM and ZMP trajectories

energies are in their admissible bounds are inserted. We next demonstrate the practical superiority of the proposed robust controller over the conventional H_∞ optimal controller corresponding to time-varying external forces with finite energies.

A. Verification of Theoretical Validity

We solve the LMI conditions in Theorem 1 by taking γ larger until they are infeasible, and the maximum values of γ for ensuring the LMIs to be feasible are obtained by $\gamma_{\mathbf{x}} = 507.5$ and $\gamma_{\mathbf{y}} = 305.5$ for $\mathbf{x}$-axis and $\mathbf{y}$-axis, respectively. For these values, the resulting robust controllers for each axis are given respectively by

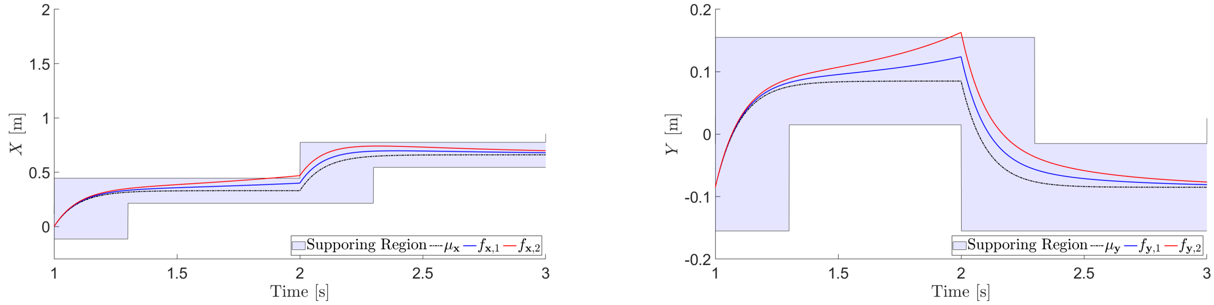
$$K_{\mathbf{x}} = \begin{bmatrix} 1.3457 & 0.3587 \end{bmatrix}, \quad K_{\mathbf{y}} = \begin{bmatrix} 1.5891 & 0.4236 \end{bmatrix}.$$

We consider two cases of external forces consisting of random sequences; (i) the forces are located in the above admissible bounds with $\|f_{\mathbf{x},1}\|_2 = 507.5$ and $\|f_{\mathbf{y},1}\|_2 = 305.5$, (ii) the energies of the forces exceed the above admissible bounds with $\|f_{\mathbf{x},2}\|_2 = 1015$ and $\|f_{\mathbf{y},2}\|_2 = 611$.

As a reference, one of the examples for each case with the worst disturbances is shown in Fig. 3, i.e., the real ZMP trajectories on the supporting regions for each case. We can observe from Fig. 3 that the real ZMP trajectories for both $\mathbf{x}$-axis and $\mathbf{y}$-axis are always located inside the supporting regions for each step when the energies of external forces are within the above admissible bounds. In contrast, the real ZMP trajectories deviate from the supporting regions when the energies of external forces are larger than such admissible bounds. These observations undoubtedly verify the theoretical validity derived in this paper.

B. Comparison to Conventional H_∞ Robust Controller

We evaluate the practical effectiveness of the proposed robust controller by comparing it to the conventional H_∞ robust controller [17]. This can be regarded as a fair comparison since the H_∞ control is also for dealing with disturbances in the ℓ_2-space; the objective of the H_∞ controller



(a) Supporting region and ZMP in the $\mathbf{x}$-axis

(b) Supporting region and ZMP in the $\mathbf{y}$-axis

Fig. 3. The reference and real ZMP for the worst-case disturbance

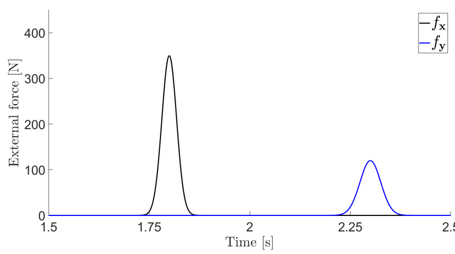


Fig. 4. The external force disturbances

TABLE II

ZMP AND CoM TRACKING ERRORS IN SIMULATIONS FOR $\mathbf{x}$-AXIS

$\mathbf{x}$-axis	$\|\tilde{c}_{\mathbf{x}}\|_\infty$	$\|\tilde{c}_{\mathbf{x}}\|_2$	$\|z_{\mathbf{x}}\|_\infty$
$K_{\mathbf{x}}$	0.0492	1.4649	0.1082
$K_{\infty,1}$	0.0003	0.0207	0.4968
$K_{\infty,2}$	0.0017	0.1030	0.4806
$K_{\infty,3}$	0.0179	1.0509	0.2388
$K_{\infty,4}$	0.0252	0.5075	0.1703

TABLE III

ZMP AND CoM TRACKING ERRORS IN SIMULATIONS FOR $\mathbf{y}$-AXIS

$\mathbf{y}$-axis	$\|\tilde{c}_{\mathbf{y}}\|_\infty$	$\|\tilde{c}_{\mathbf{y}}\|_2$	$\|z_{\mathbf{y}}\|_\infty$
$K_{\mathbf{y}}$	0.0193	0.4917	0.0572
$K_{\infty,1}$	0.0002	0.0094	0.1705
$K_{\infty,2}$	0.0008	0.0471	0.1677
$K_{\infty,3}$	0.0087	0.4833	0.1003
$K_{\infty,4}$	0.0122	0.2454	0.0768

is to suppress the ℓ_2 gain (i.e., the ℓ_2-induced norm) of a system below a given performance level in the presence of such disturbances.

The H_∞ controllers are obtained in terms of the regulated outputs defined as follows.

$$z_{\infty,1}[k] = e[k], \quad z_{\infty,2}[k] = \begin{bmatrix} e[k] \\ 0.1Ke[k] \end{bmatrix},$$
$$z_{\infty,3}[k] = \begin{bmatrix} e[k] \\ Ke[k] \end{bmatrix}, \quad z_{\infty,4}[k] = Ke[k].$$

Correspondingly, the H_∞ controllers are given by

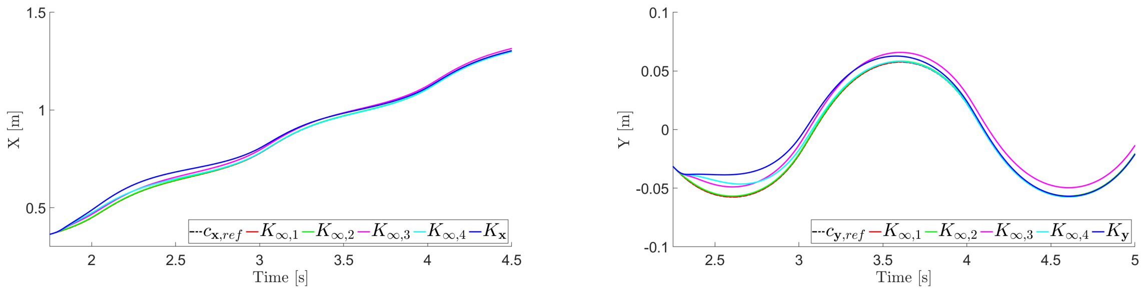
$$K_{\infty,1} = \begin{bmatrix} 1.0736 & 71.0503 \end{bmatrix}, \ K_{\infty,2} = \begin{bmatrix} 1.3368 & 13.5841 \end{bmatrix},$$
$$K_{\infty,3} = \begin{bmatrix} 1.0656 & 1.2533 \end{bmatrix}, \ K_{\infty,4} = \begin{bmatrix} 2.5021 & 0.6655 \end{bmatrix}.$$

On the other hand, the proposed robust controllers for each axis are taken to coincide with those of the previous subsection. The inserted external forces are as shown in Fig. 4, i.e., $f_{\mathbf{x}}, f_{\mathbf{y}} \in \ell_2$.

The simulation results for the reference and real CoM trajectories affected by the external forces shown in Fig. 4 are shown in Fig. 5. The simulation results for the reference and real ZMP trajectories affected by the external forces on the supporting regions are shown in Fig. 6. The results for $\|\tilde{c}\|_\infty$, $\|\tilde{c}\|_2$, and $\|z\|_\infty$ are provided in Tables II and III for $\mathbf{x}$-axis and $\mathbf{y}$-axis, respectively, where $\tilde{c}$ denotes the CoM tracking error.

First of all, it could be observed from Fig. 5 that the CoM tracking errors in the conventional H_∞ controllers seem smaller than those in the proposed robust controller. This supposition can be regarded as true if we note the results in Tables II and III since both the ℓ_2 and ℓ_∞ norms of the COM tracking errors for the H_∞ controllers are smaller than those for the proposed robust controller. From this point of view, one could argue that the conventional H_∞ controller is superior to the proposed robust controller for biped walking. However, the ZMP trajectories with the H_∞ controllers deviate from the supporting region as shown in Fig. 6, although the CoM tracking performance with the H_∞ controller is better than that with the proposed robust controller.

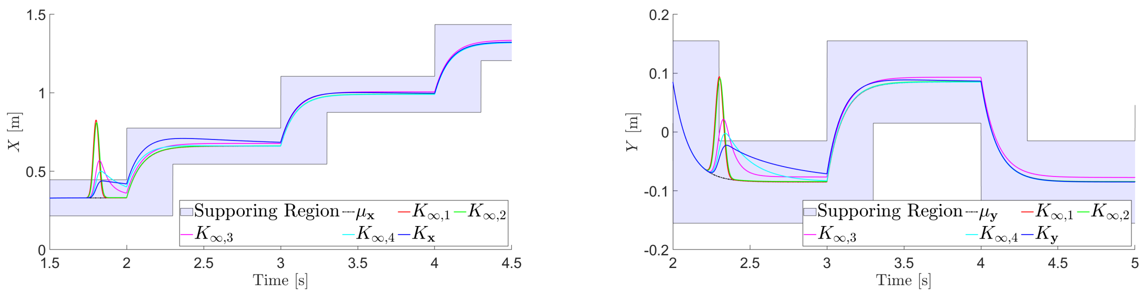
In contrast, the ZMP trajectories for both $\mathbf{x}$-axis and $\mathbf{y}$-axis with the proposed robust controller always stay inside the supporting regions at each step. This successful balance should be achieved by the intrinsic constraint to the proposed robust controller so that the ZMP trajectories are ensured to be located in the supporting regions. From these observations, we could conclude that it would be better to design tracking controllers of biped robots by considering the ZMP tracking performance rather than the CoM tracking performance for their successful walking balances. In this sense, the arguments presented in this paper might be interpreted as more practically effective for external forces than the conventional H_∞ controllers.



(a) CoM position in the **x**-axis

(b) CoM position in the **y**-axis

Fig. 5. The reference and real CoM positions



(a) Supporting region and ZMP in the **x**-axis

(b) Supporting region and ZMP in the **y**-axis

Fig. 6. The reference and real ZMP

V. CONCLUSIONS

We proposed a robust controller synthesis for ensuring the stability of biped walking in the presence of external forces. We first introduced a trajectory tracking control of the horizontal motion of biped robots based on their LIPM representation. We then considered the induced norm-based stability criterion in the authors' preceding study [12]. Based on this criterion, we derived a robust controller leading to stable biped walking when an upper bound on the energy of external forces is given. More precisely, a necessary and sufficient condition for the existence of such a robust controller is characterized in terms of some linear matrix inequalities (LMIs), by which we can also obtain the maximum energy bound of admissible external forces. Finally, some simulation results were given to demonstrate the effectiveness and validity of the proposed robust controller in comparison to the conventional H_∞ robust controller.

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