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Nullspace Adaptive Model-Based Trajectory-Tracking Control for a 6-DOF Underwater Vehicle with Unknown Plant and Actuator Parameters: Theory and Preliminary Simulation Evaluation

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***Abstract*— We report a novel model-based nullspace adaptive trajectory-tracking control (NS-ATTC) algorithm for fully-actuated 6-degree-of-freedom (DOF) underwater vehicles which estimates unknown plant and actuator model parameters simultaneously. We provide a stability and convergence analysis with proof of asymptotically stable tracking error convergence, as well as a preliminary simulation study demonstrating 6-DOF trajectory tracking. The NS-ATTC algorithm does not require acceleration instrumentation and provides a stable online parameter estimate, enabling robust model-based autonomy.**

I. Introduction

Accurate trajectory-tracking control (TTC) for underwater vehicles (UVs) enables missions such as high-precision seafloor surveying for oceanographic, commercial, and national security purposes to be performed with assured autonomy. As the desired applications and operating environments of UVs become more complex, there is a need for increasingly capable and robust TTC.

The inclusion of a dynamical vehicle model in TTC algorithms has been shown to improve performance, but model accuracy is crucial [14]. While the form (i.e model structure) of UV plant models can be determined analytically through first principles [4], [19], [24], parameters such as mass, vehicle hydrodynamic added mass and drag parameters, vehicle displacement and center of buoyancy (CB) parameters, control-actuator and control-surface parameters (e.g. propeller thrust and torque and fin lift and drag coefficients) must be determined empirically, and moreover are subject to change over time due to vehicle and payload reconfiguration, environmental conditions, or unexpected faults. Although a substantial body of adaptive control research has addressed plant parameter uncertainty, these methods generally assume that actuator model parameters are known *a priori*.

We report a novel 6-degree-of-freedom (DOF) nullspace adaptive trajectory-tracking control (NS-ATTC) algorithm, which does not require exact knowledge of plant or actuator parameters *a priori*. This differs from all previously reported adaptive trajectory-tracking control (ATTC) approaches for second-order mechanical systems, such as those for robot arms [3], [22], [26] and UVs [15], [27], which adaptively identify plant parameters only. The NS-ATTC algorithm referred herein is capable of online operation, is valid for general fully-actuated 6-DOF UVs, and requires access to position, orientation, and velocity signals but does not require instrumentation of acceleration — a signal which is often difficult to measure. By maintaining an online estimate of the plant and actuator parameters that determine vehicle input-output behavior, this control approach is inherently fault-aware and fault-tolerant. NS-ATTC parameter estimation also may enable other model-based autonomy subsystems such as motion planning, state estimation, and fault detection.

To the best of our knowledge, the NS-ATTC approach reported herein is the first indirect ATTC method to estimate a full thruster allocation matrix simultaneously with plant parameters and the first formulation of the ATTC problem using a parameterization with nullspace structure. We also present an analytical proof of locally asymptotically stable convergence of trajectory-tracking error in the presence of unknown plant and actuator parameters and stable parameter estimation error. Finally, we report a simulation evaluation demonstrating NS-ATTC on a UV model for reference trajectories with simultaneous motion in 6-DOF.

II. Related Work

Trajectory-tracking control (TTC) for UVs is challenging due to their nonlinear and coupled dynamics. A conventional model-free approach is to decouple the control problem, apply linear proportional-integral-derivative (PID) control approaches to subsystems or individual DOFs, e.g. as a heading or depth autopilot, and treat unmodeled dynamics as disturbances [4]. Comparative experimental evaluations have shown that fixed (i.e. non-adaptive) model-based TTC methods can provide improved performance over model-free methods [13], [23], however these methods depend on *a priori* knowledge of model parameters.

Adaptive control addresses the problem of parameter uncertainty either directly through a control tuning algorithm or indirectly through parameter estimation. There is a rich body of work in ATTC methods for plants such as robot arms [3], [22], [26]. Work on ATTC for UVs includes parameter bound estimation methods [30], adaptive sliding mode control [28], [27], and adaptive linearizing control [16], [15]. ATTC for UVs has been shown to outperform PID [31] and proportional derivative control (PDC) [16] in the presence of plant model parameter error and disturbances.

A limitation of these ATTC methods, however, is that with few exceptions they address uncertainty in the plant pa-

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rameters only. Indeed, most model-based control approaches assume that a desired force/moment vector may be achieved, e.g. [4], [13], [23], thus actuator model parameters must be known *a priori*. Although a class of model-reference adaptive controllers (MRACs) for linear plants considers uncertainty in the input mapping [18], these results do not apply to TTC for nonlinear plants. [15], [16] showed that unmodeled thruster dynamics can destabilize parameter adaptation and reported a two-step algorithm which estimates subsets of parameters successively to increase robustness of the parameter adaptation, but does not estimate the actuator model parameters. [5] addresses uncertainty in the input mapping for a 3-DOF UV, but only as a scalar multiplier. In contrast, the NS-ATTC algorithm reported herein estimates plant and actuator parameters simultaneously, exploiting the nullspace structure of plants and actuator models which are linear in the parameters. The authors reported a similar approach to the related but distinct problem of nullspace adaptive identification (NS-AID) [6].

An alternative category of model-free TTC approaches uses neural networks (NNs) to learn complex control mappings from input-output data alone [25], [29]. Some hybrid approaches combine the advantages of physics-based modeling with the approximation capabilities of NNs by supplementing a known dynamics model with learned unknown dynamics [2], [11]. In general, NN approaches may be limited by the data and offline training time required for generalizable performance and may lack online adaptation. Moreover, the ability of NS-ATTC to estimate a physics model of both plant and actuator behavior is useful not only for the TTC objective, but in enabling more accurate online model-based motion planning, state estimation [7], fault detection [12], and other autonomy subsystems.

Another challenge of vehicle control with respect to a world inertial frame is the choice of orientation representation. Approaches using quaternions [1] or exponential coordinates of rotation [16] avoid Euler-angle singularities. Although the NS-ATTC algorithm utilizes Euler angles and is thus subject to a bounded pitch assumption, this choice of representation is not fundamental to the nullspace parameter adaptation approach and is a potential area of future work.

III. Mathematical Conventions

For a vector $x \in \mathbb{R}^n$, we denote the Euclidean (L^2) norm by $||x||_2$ and the maximum (L^∞) norm by $||x||_\infty$. For a matrix $A \in \mathbb{R}^{n\times n}$, we denote the spectral norm by $||A||_2$ and Frobenius norm by $||A||_F$ and make use of the fact that $||A||_2 \leq ||A||_F$. When not specified, the $||\cdot||_2$ norm is implied for both vectors and matrices. For a positive-definite symmetric (PDS) matrix $P \in \mathbb{R}^{n\times n}$, we denote minimum and maximum eigenvalues as $\lambda_{min}(P), \lambda_{max}(P)$.

We also make use of the diagonal matrix operator $\text{diag}(\cdot) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, Kronecker product $\otimes$, skew-symmetric operator $(\cdot)^\wedge : \mathbb{R}^3 \to so(3)$, stacking operator $\text{vec}(\cdot) : \mathbb{R}^{n\times n} \to \mathbb{R}^{(n*n)\times 1}$ [20], and adjoint operator $ad_{se(3)}(\cdot) : \mathbb{R}^6 \to \mathbb{R}^{6\times 6}$, which is defined $\forall v = [\nu^T \ \omega^T]^T \in \mathbb{R}^6$ as

$$ad_{se(3)}(v) = \begin{bmatrix} \omega^\wedge & 0_{3\times 3} \\ \nu^\wedge & \omega^\wedge \end{bmatrix}. \tag{1}$$

IV. 6-DOF Vehicle Plant and Actuator Model

The commonly accepted second-order finite-dimensional model of a submerged fully-actuated 6-DOF UV subject to quadratic drag and gravitational forces, [4], [19], is

$$\dot{\eta} = J(\varphi)v \tag{2}$$

$$M\dot{v} + C(v)v + D(v)v - \mathcal{G}(\varphi) = \tau(\xi), \tag{3}$$

where $\eta(t) = [x^\top \ \varphi^\top] \in \mathbb{R}^6$ represents the position $x \in \mathbb{R}^3$ and orientation $\varphi \in \mathbb{R}^3$ of a body-fixed frame with respect to an inertial frame of reference; $v(t) = [\nu^\top \ \omega^\top] \in \mathbb{R}^6$ represents the body-frame linear velocity $\nu \in \mathbb{R}^3$ and angular velocity $\omega \in \mathbb{R}^3$; $\dot{v}(t) \in \mathbb{R}^6$ is the time derivative of the body velocity (termed the "plant acceleration"); $\tau(\xi) \in \mathbb{R}^6$ is the vector of control-actuator forces and moments; and $\xi \in \mathbb{R}^6$ is the vector of control inputs. We represent the vehicle orientation φ in Euler angles and assume that the pitch φ_2 is bounded to avoid the kinematic singularities arising therein. The kinematic Jacobian $J(\varphi) \in \mathbb{R}^{6\times 6}$ described in Section 2.2 of [4], composed of the body-to-world rotation matrix $R(\varphi) \in SO(3)$ and the angular velocity transformation matrix $T(\varphi) \in \mathbb{R}^{3\times 3}$, is

$$J(\varphi) = \begin{bmatrix} R(\varphi) & 0_{3\times 3} \\ 0_{3\times 3} & T(\varphi) \end{bmatrix}. \tag{4}$$

$M \in \mathbb{R}^{6\times 6}$ is the positive definite symmetric (PDS) mass matrix, $C(v) \in \mathbb{R}^{6\times 6}$ is the Coriolis matrix, $D(v) \in \mathbb{R}^{6\times 6}$ is the positive semidefinite (PSD) drag matrix, and $\mathcal{G}(\eta) \in \mathbb{R}^6$ is the force and moment vector due to gravity and buoyancy.

The model vehicle studied in this work is the Johns Hopkins University (JHU) remotely operated vehicle (ROV), a fully-actuated ROV actuated via 6 current-controlled direct-drive brushless electric thrusters [10]. We use a simplified steady-state thruster model in which thrust is proportional to motor current input — a reasonable assumption when vehicle advance velocity is small relative to thruster jet velocity — representing the current inputs as $\xi \in \mathbb{R}^6$ and the directionality, location relative to the center of gravity (CG) in the vehicle body frame, and thrust coefficients of the thrusters with a thruster allocation matrix $A \in \mathbb{R}^{6\times 6}$. Then we can write the overall body-frame force/moment vector as

$$\tau(\xi) = A\xi. \tag{5}$$

A. Parameterization and Regressor Form

For this preliminary study, we assume that the body frame is coincident with the vehicle CG and that $M, D(v)$ are diagonal. Since the parameters in (3) enter linearly, we can factor a parameter vector from each of the terms in (3) and factor the nonlinear terms as regressor matrix-valued functions, described for each term in (3) as follows.

Under the diagonal mass matrix assumption, we define

$$\theta_m \triangleq [m_1 \ m_2 \ m_3 \ m_4 \ m_5 \ m_6]^\top \in \mathbb{R}^6 \tag{6}$$

$$W_m(\dot{v}) \triangleq \text{diag}(\dot{v}), \tag{7}$$

which results in

$$M\dot{v} = \text{diag}(\theta_m)\text{diag}(\dot{v}) \tag{8}$$

$$= W_m(\dot{v})\theta_m, \tag{9}$$

where θ_m also parameterizes the Coriolis matrix

$$C(v_a)v_b = \begin{bmatrix} 0_{3\times3} & -(M_{11}\nu_a)^\wedge \\ -(M_{11}\nu_a)^\wedge & -(M_{22}\omega_a)^\wedge \end{bmatrix} \begin{bmatrix} \nu_b \\ \omega_b \end{bmatrix} \quad (10)$$

$$= ad_{se(3)}(v_b)Mv_a \quad (11)$$

$$= ad_{se(3)}(v_b)W_m(v_a)\theta_m. \quad (12)$$

We distinguish between $v_a \in \mathbb{R}^6$, the argument to the Coriolis matrix $C(v_a)$, and $v_b \in \mathbb{R}^6$, the outside argument. Similarly for the diagonal drag matrix, we define

$$\theta_d \triangleq [d_1\ d_2\ d_3\ d_4\ d_5\ d_6]^\top \in \mathbb{R}^6 \quad (13)$$

$$W_d(v_a, v_b) \triangleq \text{diag}(|v_a|)\text{diag}(v_b), \quad (14)$$

which results in

$$D(v_a)v_b = \text{diag}(|v_a|)\text{diag}(\theta_d)v_b \quad (15)$$

$$= W_d(v_a, v_b)\theta_d. \quad (16)$$

To factor the gravity/buoyancy term, we define

$$\theta_{\mathcal{G}} \triangleq [g\ b^\top]^\top \in \mathbb{R}^4 \quad (17)$$

$$W_{\mathcal{G}}(\varphi) \triangleq \begin{bmatrix} R(\varphi)^\top e_3 & 0_{3\times3} \\ 0_{3\times1} & (R(\varphi)^\top e_3)^\wedge \end{bmatrix}, \quad (18)$$

where $e_3 = [0\ 0\ 1]^\top$; $g = g_c(m - \rho\nabla) \in \mathbb{R}$ is the net effective buoyant force with gravitational acceleration constant g_c, dry mass m, fluid density ρ, and displacement volume ∇; and $b = g_c\rho\nabla r_{cb} \in \mathbb{R}^3$ is the net righting torque with vector $r_{cb} \in \mathbb{R}^3$ from the CG to the CB. This results in

$$\mathcal{G}(\varphi) = W_{\mathcal{G}}(\varphi)\theta_{\mathcal{G}}. \quad (19)$$

Combining (9,12,16,19), we define the plant parameter vector $\theta_p \in \mathbb{R}^{16}$ and plant regressor matrix-valued function $W_p(\dot{v}, v_a, v_b, \varphi) \in \mathbb{R}^{6\times16}$ as

$$\theta_p \triangleq [\theta_m^\top\ \theta_d^\top\ \theta_{\mathcal{G}}^\top]^\top \quad (20)$$

$$W_p(\dot{v}, v_a, v_b, \varphi) \triangleq \begin{bmatrix} \Big(W_m(\dot{v}) + ad_{se(3)}(v_b)W_m(v_a)\Big)^\top \\ W_d(v_a, v_b)^\top \\ -W_{\mathcal{G}}(\varphi)^\top \end{bmatrix}^\top, \quad (21)$$

resulting in the following expression for the LHS of (3)

$$M\dot{v} + C(v)v + D(v)v - \mathcal{G}(\eta) = W_p(\dot{v}, v, v, \varphi)\theta_p. \quad (22)$$

Stacking the rows of the thruster allocation matrix A, we define the actuator parameter vector and actuator regressor matrix-valued function

$$\theta_a \triangleq \text{vec}(A^\top) \in \mathbb{R}^{36} \quad (23)$$

$$W_a(\xi) \triangleq I_{6\times6} \otimes \xi^\top \in \mathbb{R}^{6\times36}, \quad (24)$$

so that the RHS of (3) may be written as

$$\tau(\xi) = W_a(\xi)\theta_a. \quad (25)$$

Finally, we construct the combined plant and actuator parameter vector from (20,23)

$$\theta \triangleq [\theta_p^\top\ \theta_a^\top]^\top \in \mathbb{R}^{52}. \quad (26)$$

and combined regressor matrix-valued function from (21,24)

$$\mathbb{W}(\dot{v}, v, v, \varphi, \xi) \triangleq \begin{bmatrix} W_p(\dot{v}, v, v, \varphi) & -W_a(\xi) \end{bmatrix} \quad (27)$$

so that, using (22,25), we can write the LHS and RHS of the dynamic equation of motion (3) in regressor form

$$W_p(\dot{v}, v, v, \varphi)\theta_p = W_a(\xi)\theta_a \quad (28)$$

and using (26,27,28) obtain a nullspace relationship

$$\mathbb{W}(\dot{v}, v, v, \varphi, \xi)\theta = 0. \quad (29)$$

Thus the true parameters θ are not only a unique point in parameter space, but members of a true parameter set consisting of all vectors θ^* that also belong to the persistent nullspace of $\mathbb{W}(\dot{v}, v, v, \varphi, \xi)$. This set $P(\theta)$ may be defined

$$P(\theta) = \{\theta^* : \theta^* \neq 0, \text{and } \mathbb{W}(\dot{v}, v, v, \varphi, \xi)\theta = 0 \iff \mathbb{W}(\dot{v}, v, v, \varphi, \xi)\theta^* = 0\}. \quad (30)$$

For example, any scalar multiple of θ equivalently satisfies (29). We note that, while we adopt a particular UV model and parameterization for this preliminary study, the control approach reported herein is applicable to any fully-actuated second-order model that can be written in the form (3) or, equivalently (29) — a broad class of systems including aerial and marine vehicles, spacecraft, and robot arms.

V. Nullspace Adaptive Model-Based Trajectory-Tracking Control

A. Problem Statement

Given a smooth bounded reference trajectory in the world inertial reference frame $\eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t)$ and unknown true plant and actuator parameters θ (26), our task is to design control inputs $\xi(t)$ and a parameter estimate $\hat{\theta}(t)$ with parameter estimate update law $\dot{\hat{\theta}}(t)$ to achieve asymptotically stable trajectory tracking. We define the tracking error coordinates

$$\Delta\eta(t) \triangleq \eta(t) - \eta_d(t) \quad (31)$$

$$\Delta\dot{\eta}(t) = \dot{\eta}(t) - \dot{\eta}_d(t) \quad (32)$$

$$\Delta\ddot{\eta}(t) = \ddot{\eta}(t) - \ddot{\eta}_d(t), \quad (33)$$

as well as the parameter estimate error coordinates

$$\Delta\theta(t) \triangleq \hat{\theta}(t) - \theta. \quad (34)$$

We will also refer to the plant part $\hat{\theta}_p \in \mathbb{R}^{16}$ and actuator part $\hat{\theta}_a \in \mathbb{R}^{36}$ of the parameter estimate $\hat{\theta}$, with

$$\hat{A} \triangleq \text{vec}^{-1}(\hat{\theta}_a)^\top \quad (35)$$

$$\Delta A \triangleq \hat{A} - A \quad (36)$$

$$\Delta\theta_a \triangleq \hat{\theta}_a - \theta_a \quad (37)$$

$$\Delta\theta_p \triangleq \hat{\theta}_p - \theta_p. \quad (38)$$

We make the following assumptions:

- $\eta_d(t), \dot{\eta}_d(t), \ddot{\eta}_d(t)$ are smooth and bounded;
- $\eta(t), \eta_d(t)$ are bounded in pitch $\varphi_2 \in [-\frac{\pi}{4}, \frac{\pi}{4}]$ to avoid the singularities arising from $J(\varphi)$ at $p = \pm\frac{\pi}{2}$;
- $\omega(t)$ is bounded by $\omega_{\max} \geq ||\omega(t)||_\infty \forall t \geq t_0$;
- $v(t)$ overall is bounded by $v_{\max} \geq ||v(t)||_\infty \forall t \geq t_0$;
- θ is constant;

and denote $m_{\max} \triangleq \max_i m_i$, $d_{\min} \triangleq \min_i d_i$, $d_{\max} \triangleq \max_i d_i$, $k_{p,\min} \triangleq \min_i k_{pi}$, $k_{d,\min} \triangleq \min_i k_{di}$, $k_{d,\max} \triangleq \max_i k_{di}$, $\gamma_{\max} \triangleq \max_i \gamma_i$.

B. Adaptive Control Law and Parameter Update Law

We define a reference velocity signal and error coordinates

$$v_d(t) \triangleq J(\varphi)^{-1}\dot{\eta}_d \tag{39}$$

$$\Delta v(t) \triangleq v(t) - v_d(t). \tag{40}$$

We note that v_d is *not* the projection of $\dot{\eta}_d$ onto the body frame at η_d, but a reference signal related to $\dot{\eta}_d, \ddot{\eta}_d$ by

$$\dot{\eta}_d = J(\varphi)v_d \tag{41}$$

$$\dot{v}_d = J(\varphi)^{-1}\ddot{\eta}_d + \tfrac{d}{dt}(J(\varphi)^{-1})\dot{\eta}_d. \tag{42}$$

Using (21,39,42), we define the desired force/moment vector

$$\tau^* \triangleq W_p(\dot{v}_d, v, v_d, \varphi)\hat{\theta}_p - J(\varphi)^\top K_p\Delta\eta - K_d\Delta v \tag{43}$$

with positive definite diagonal (PDD) gain matrices $K_p \triangleq \text{diag}([k_{p1}, ..., k_{p6}]^\top)$, $K_d \triangleq \text{diag}([k_{d1}, ..., k_{d6}]^\top)$. If the true actuator parameters θ_a are known, then the control inputs ξ^* to achieve the desired force/moment vector τ^* may be computed as

$$\xi^* = A^{-1}\tau^*. \tag{44}$$

However, since only an estimate $\hat{\theta}_a$ is available, we choose the NS-ATTC control law ξ computed using the estimate $\hat{\theta}_a$

$$\xi = \hat{A}^{-1}\tau^*. \tag{45}$$

With ξ (45) applied to the actuators (5), we have

$$\tau(\xi) = A\hat{A}^{-1}\tau^*. \tag{46}$$

The NS-ATTC parameter update law is given by

$$\dot{\hat{\theta}} = -\Gamma\mathbb{W}(\dot{v}_d, v, v_d, \varphi, \xi)^\top(\Delta v + \epsilon J(\varphi)^{-1}\Delta\eta), \tag{47}$$

where $\epsilon \in \mathbb{R}_{>0}$ and $\Gamma \triangleq \text{diag}([\gamma_1, ..., \gamma_{52}]^\top)$is a PDD matrix of adaptation gains with principal submatrices $\Gamma_m, \Gamma_d, \Gamma_{\mathcal{G}}, \Gamma_a$ associated with $\theta_m, \theta_d, \theta_{\mathcal{G}}, \theta_a$, respectively.

C. Error Dynamics

Equating (46) to (25), substituting this term into the RHS of the plant dynamics (28) and using the identity $A\hat{A}^{-1} = I - \Delta A\hat{A}^{-1}$, the controlled plant takes the form

$$W_p(\dot{v}, v, v, \varphi)\theta_p = A\hat{A}^{-1}\tau^* \tag{48}$$

$$W_p(\dot{v}, v, v, \varphi)\theta_p = (I - \Delta A\hat{A}^{-1})\tau^* \tag{49}$$

$$\begin{aligned} W_p(\dot{v}, v, v, \varphi)\theta_p = {} & W_p(\dot{v}_d, v, v_d, \varphi)\hat{\theta}_p \\ & - J(\varphi)^\top K_p\Delta\eta - K_d\Delta v - \Delta A\hat{A}^{-1}\tau^*. \end{aligned} \tag{50}$$

From the control law (45), the fact that $\Delta A = \text{vec}^{-1}(\Delta\theta_a)^\top$ (36,37), and the regressor form of the actuator force/moment vector (24,25), we have that

$$\Delta A\hat{A}^{-1}\tau^* = \Delta A\xi \tag{51}$$

$$= W_a(\xi)\Delta\theta_a. \tag{52}$$

Substituting (52) and $\hat{\theta}_p = \theta_p + \Delta\theta_p$ (34) into (50), we can express the controlled plant as

$$\begin{aligned} W_p(\dot{v}, v, v, \varphi)\theta_p = {} & W_p(\dot{v}_d, v, v_d, \varphi)(\theta_p + \Delta\theta_p) \\ & - W_a(\xi)\Delta\theta_a \\ & - J(\varphi)^\top K_p\Delta\eta - K_d\Delta v. \end{aligned} \tag{53}$$

Combining terms using (21,22) yields

$$\begin{aligned} W_p(\Delta\dot{v}, v, \Delta v, 0)\theta_p = {} & -J(\varphi)^\top K_p\Delta\eta - K_d\Delta v \\ & + W_p(\dot{v}_d, v, v_d, \varphi)\Delta\theta_p - W_a(\xi)\Delta\theta_a, \end{aligned} \tag{54}$$

and we can use (27,34) to obtain

$$\begin{aligned} W_p(\Delta\dot{v}, v, \Delta v, 0)\theta_p = {} & -J(\varphi)^\top K_p\Delta\eta - K_d\Delta v \\ & + \mathbb{W}(\dot{v}_d, v, v_d, \varphi, \xi)\Delta\theta \end{aligned} \tag{55}$$

$$\begin{aligned} M\Delta\dot{v} + [C(v) + D(v)]\Delta v = {} & -J(\varphi)^\top K_p\Delta\eta - K_d\Delta v \\ & + \mathbb{W}(\dot{v}_d, v, v_d, \varphi, \xi)\Delta\theta. \end{aligned} \tag{56}$$

Thus the velocity error dynamics $\Delta\dot{v}$ may be written as

$$\begin{aligned} M\Delta\dot{v} = {} & -[C(v) + D(v) + K_d]\Delta v - J(\varphi)^\top K_p\Delta\eta \\ & + \mathbb{W}(\dot{v}_d, v, v_d, \varphi, \xi)\Delta\theta. \end{aligned} \tag{57}$$

Using (2,32,41), we can express the tracking error dynamics

$$\Delta\dot{\eta} = J(\varphi)\dot{v}_d - J(\varphi)\dot{v} \tag{58}$$

$$= J(\varphi)\Delta v, \tag{59}$$

and, since the true parameters are constant (i.e. $\dot{\theta} = 0$), the parameter error dynamics are simply $\Delta\dot{\theta} = \dot{\hat{\theta}}$ (34, 47), thus

$$\Delta\dot{\theta} = -\Gamma\mathbb{W}(\dot{v}_d, v, v_d, \varphi, \xi)^\top(\Delta v + \epsilon\Delta\eta). \tag{60}$$

D. Stability and Boundedness Analysis

This section reports proofs of stability about the origin of the full error system $z \triangleq [\Delta\eta^\top\ \Delta v^\top\ \Delta\theta^\top]^\top$, asymptotic convergence of $\Delta\eta, \Delta v$ to 0, and boundedness of all signals.

We consider the Lyapunov function candidate

$$V(z) = \frac{1}{2}z^\top \begin{bmatrix} K_p & \epsilon J(\varphi)^{-\top}M & 0 \\ \epsilon M J(\varphi)^{-1} & M & 0 \\ 0 & 0 & \Gamma^{-1} \end{bmatrix} z, \tag{61}$$

which is bounded below by

$$\begin{aligned} V(z) \geq {} & \frac{1}{2}\big(\lambda_{min}(K_p)||\Delta\eta||^2 + \lambda_{min}(M)||\Delta v||^2 \\ & + \frac{1}{\lambda_{max}(\Gamma)}||\Delta\theta||^2\big) - \epsilon|\Delta\eta^\top J(\varphi)^{-\top}M\Delta v|, \end{aligned} \tag{62}$$

where, using properties of the spectral and Frobenius norms,

$$\begin{aligned} |\Delta\eta^\top J(\varphi)^{-\top}M\Delta v| & \leq ||\Delta\eta||\ ||\Delta v||\ ||J(\varphi)^{-\top}||\ ||M|| \\ & \leq ||\Delta\eta||\ ||\Delta v||\ ||J(\varphi)^{-\top}||_F\lambda_{\max}(M) \end{aligned} \tag{63}$$

$$\leq ||\Delta\eta||\ ||\Delta v|| \text{tr}(J(\varphi)^{-1}J(\varphi)^{-\top})\lambda_{\max}(M). \tag{64}$$

By direct computation, we have that $\text{tr}(J(\varphi)^{-1}J(\varphi)^{-\top}) = 6$. Thus for diagonal K_p, M, Γ, combining (62,64) and defining $\bar{z} \triangleq [||\Delta\eta||\ ||\Delta v||\ ||\Delta\theta||]^\top$ yields

$$V(z) \geq \frac{1}{2}\bar{z}^\top \begin{bmatrix} k_{p,\min} & -6\epsilon m_{\max} & 0 \\ -6\epsilon m_{\max} & m_{\min} & 0 \\ 0 & 0 & \frac{1}{\gamma_{\max}} \end{bmatrix} \bar{z}, \tag{65}$$

and by choosing $\epsilon < \frac{1}{6m_{\max}}\sqrt{\frac{k_{p,\min}}{m_{\min}}}$, we ensure that $V(z)$ is positive-definite in z. Furthermore, $V \in \mathcal{C}^1$, is equal to zero if and only if $z = 0$, and is radially unbounded, satisfying the requirements for a Lyapunov function. Taking the error dynamics (59,57,60) and the time derivative of (61), we have

$$\begin{aligned}\dot{V}(z) =& \Delta\eta^\top K_p \Delta\dot{\eta} + \Delta v^\top M \Delta\dot{v} + \Delta\theta^\top \Gamma^{-1} \Delta\dot{\theta} \\ &+ \epsilon\big[\Delta\eta^\top J(\varphi)^{-\top} M \Delta\dot{v} + \Delta\dot{\eta}^\top J(\varphi)^{-\top} M \Delta v \\ &+ \Delta\eta^\top \tfrac{d}{dt}(J(\varphi)^{-\top}) M \Delta v\big] \qquad (66)\\ =& -\epsilon\Delta\eta K_p \Delta\eta - \Delta v^\top [D(v) + K_d - \epsilon M]\Delta v \\ &- \epsilon\Delta\eta^\top \big[J(\varphi)^{-\top}[C(v) + D(v) + K_d] \\ &- \tfrac{d}{dt}(J(\varphi)^{-\top})M\big]\Delta v, \qquad (67)\end{aligned}$$

which, defining $\bar{D}(v) \triangleq D(v) + Kd$, is bounded above by

$$\begin{aligned}\dot{V}(z) \leq & -\epsilon\lambda_{\min}(K_p)||\Delta\eta||^2 \\ &- \big[\lambda_{\min}(\bar{D}(v)) - \epsilon\lambda_{\max}(M)\big]||\Delta v||^2 \\ &+ \epsilon|\Delta\eta^\top \big[J(\varphi)^{-\top}[C(v) + \bar{D}(v)] \\ &- \tfrac{d}{dt}(J(\varphi)^{-\top})M\big]\Delta v|, \qquad (68)\end{aligned}$$

where the last term is further bounded above by

$$\begin{aligned}&|\Delta\eta^\top \big[J(\varphi)^{-\top}[C(v) + \bar{D}(v)] - \tfrac{d}{dt}(J(\varphi)^{-\top})M\big]\Delta v| \\ &\leq \big(||J(\varphi)^{-\top}||\ ||C(v) + \bar{D}(v)|| \\ &\quad + ||\tfrac{d}{dt}J(\varphi)^{-\top}||\ ||M||\big)\ ||\Delta\eta||\ ||\Delta v|| \qquad (69)\\ &\leq \big(||J(\varphi)^{-\top}||_F\ ||C(v) + \bar{D}(v)|| \\ &\quad + ||\tfrac{d}{dt}J(\varphi)^{-\top}||_F\ ||M||\big)\ ||\Delta\eta||\ ||\Delta v||. \qquad (70)\end{aligned}$$

Term-by-term from (70), we have

- by direct computation,
$$||J(\varphi)^{-\top}||_F = \mathrm{tr}(J(\varphi)^{-1}J(\varphi)^{-\top}) = 6; \qquad (71)$$
- from Proposition 5.11 in [21] and the skew-symmetricity of $C(v)$,
$$\begin{aligned}||C(v) + \bar{D}(v)||_2 &\leq 2 \sup_{||y||=1} |y^*(C(v) + \bar{D}(v))y| \qquad (72)\\ &\leq 2\lambda_{\max}\bar{D}(v); \qquad (73)\end{aligned}$$
- by direct computation for bounded pitch $\varphi_2 \in [-\frac{\pi}{4}, \frac{\pi}{4}]$,
$$\begin{aligned}||\tfrac{d}{dt}J(\varphi)^{-\top}||_F &= \mathrm{tr}(\tfrac{d}{dt}(J(\varphi)^{-1})\tfrac{d}{dt}(J(\varphi)^{-\top})) \qquad (74)\\ &= \dot{\varphi}^\top B(\varphi)\dot{\varphi} \qquad (75)\\ &\leq ||\omega||^2 ||T(\varphi)^\top B(\varphi) T(\varphi)||_F \qquad (76)\\ &\leq 38\omega_{\max}^2, \qquad (77)\end{aligned}$$
  where
$$B(\varphi) \triangleq \begin{bmatrix} 3 + c^2\varphi_2 & 0 & -2s\varphi_2 \\ 0 & 3 & 0 \\ -2s\varphi_2 & 0 & 2 \end{bmatrix}; \qquad (78)$$
- and for the PDS matrix M, $||M||_2 = \lambda_{\max}(M)$.

Thus the last term of (68) is bounded above by

$$\begin{aligned}&|\Delta\eta^\top \big[J(\varphi)^{-\top}[C(v) + \bar{D}(v)] - \tfrac{d}{dt}(J(\varphi)^{-\top})M\big]\Delta v| \\ &\leq \big(12\lambda_{\max}(\bar{D}(v)) + 38\lambda_{\max}(M)\omega_{\max}^2\big)||\Delta\eta||\ ||\Delta v||. \qquad (79)\end{aligned}$$

Since $M, D(v), K_p, K_d$ are assumed to be diagonal, their minimum and maximum eigenvalues are their minimum and maximum diagonal elements. We note that

$$\lambda_{\min}(\bar{D}(v)) = k_{d,\min} \qquad (80)$$
$$\lambda_{\max}(\bar{D}(v)) = d_{\max}v_{\max} + k_{d,\max} \qquad (81)$$

and abbreviate

$$\bar{d}_{\max} \triangleq 6(d_{\max}v_{\max} + k_{d,\max}) \qquad (82)$$
$$\bar{m}_{\max} \triangleq 19(m_{\max}\omega_{\max}^2). \qquad (83)$$

Then the upper bound on $\dot{V}(z)$ in (68) can be written as

$$\dot{V}(z) \leq -\bar{z}^\top \begin{bmatrix} G & 0_{12\times 52} \\ 0_{52\times 12} & 0_{52\times 52} \end{bmatrix} \bar{z} \qquad (84)$$

$$G \triangleq \begin{bmatrix} \epsilon k_{p,\min} & -\epsilon(\bar{d}_{\max} + \bar{m}_{\max}) \\ -\epsilon(\bar{d}_{\max} + \bar{m}_{\max}) & k_{d,\min} - \epsilon m_{\max} \end{bmatrix}. \qquad (85)$$

If the gains satisfy

$$m_{\max}k_{p,\min} + (\bar{d}_{\max} + \bar{m}_{\max})^2 > 0 \qquad (86)$$
$$\epsilon < \frac{k_{p,\min}k_{d,\min}}{m_{\max}k_{p,\min} + (\bar{d}_{\max} + \bar{m}_{\max})^2}, \qquad (87)$$

then $G > 0$ and $\dot{V}(z)$ is negative-definite in $\Delta\eta, \Delta v$ and negative semi-definite overall. Thus by choosing

$$0 < \epsilon < \min\left\{\frac{1}{6m_{\max}}\sqrt{\frac{k_{p,\min}}{m_{\min}}}, \frac{k_{p,\min}k_{d,\min}}{m_{\max}k_{p,\min} + (\bar{d}_{\max} + \bar{m}_{\max})^2}\right\}, \qquad (88)$$

then $V(z), \dot{V}(z)$ satisfy the requirements of a Lyapunov function to show that the error system (57,59,60) is uniformly stable about the origin and that $\Delta\eta, \Delta v, \Delta\theta$ are bounded.

From the boundedness of $\Delta\theta$ (34) and constant θ, $\hat{\theta}$ is bounded. Since $\Delta\eta$ (31) is bounded and η_d is assumed bounded, η is bounded, as well as $J(\varphi)$ under the bounded pitch assumption. Then bounded Δv implies bounded $\Delta\dot{\eta}$ (59), and together with the boundedness assumption on $\dot{\eta}_d$, this implies that $\dot{\eta}$ is bounded. Furthermore $v_d, \dot{v}_d$ (39,42) are bounded from the boundedness of $\eta, \dot{\eta}_d, \ddot{\eta}_d, J(\varphi)^{-1}, \dot{J}(\varphi)^{-1}$. We have already assumed that v is bounded.

To show that $\dot{v}$ is bounded, we must first verify the boundedness of $\tau(\xi)$ (46) and thus that $\hat{A}^{-1}$ exists and is bounded. As a consequence of Corollary 5.6.16 in [8], $\hat{A}^{-1} = (A + \Delta A)^{-1}$ is invertible if

$$||\Delta A||_F < ||A^{-1}||_F^{-1}. \qquad (89)$$

We can verify by direct computation that $||\Delta A||_F = ||\Delta\theta_a||_2$, which is bounded above from the definition of $V(z)$ (61) and the positivity of its error terms,

$$V(z) \geq \frac{1}{2}\lambda_{\min}(\Gamma_a^{-1})||\Delta\theta_a||^2. \qquad (90)$$

Furthermore, since $\dot{V} \leq 0$ (84), $V(z(t))$ is bounded above by its initial value $V(z(t_0))$, and we have

$$||\Delta\theta_a||^2 \leq 2\lambda_{\min}(\Gamma_a^{-1})^{-1} V(z(t)) \quad (91)$$
$$\leq 2\lambda_{\max}(\Gamma_a) V(z(t_0)). \quad (92)$$

Thus from (89), $\hat{A}(t)$ remains invertible $\forall t \geq t_0$ if

$$\sqrt{2\lambda_{\max}(\Gamma_a)V(z(t_0))} < ||A^{-1}||_F^{-1}. \quad (93)$$

From (43,46), we see that all other signals in $\tau(\xi)$ have been shown to be bounded, so that $\dot{v}$ and consequently $\Delta\dot{v} = \dot{v} - \dot{v}_d$ are bounded from the boundedness of (3).

Regarding convergence of the trajectory-tracking error, we can verify from (61,84) that $[\Delta\eta^\top \Delta v^\top]^\top \in \mathcal{L}_2 \cap \mathcal{L}_\infty$. By a corollary of Barbalat's Lemma [9], this implies that

$$\lim_{t\to\infty} \Delta\eta(t) = 0 \quad (94)$$
$$\lim_{t\to\infty} \Delta v(t) = 0. \quad (95)$$

thus achieving the NS-ATTC goal. We can further conclude from (94,95) and the boundedness of all signals in (47) that

$$\lim_{t\to\infty} \dot{\hat{\theta}} = 0. \quad (96)$$

It is an open problem, however, to guarantee convergence of $\hat{\theta}$ to $P(\theta)$ (30). In [6], the authors showed that a persistence of excitation (PE) condition is sufficient for such convergence in the context of nullspace adaptive identification, a distinct task sharing the same nullspace parameterization. Potential application of such PE conditions to parameter convergence in NS-ATTC is the subject of future work.

VI. Simulation Evaluation

This Section reports a preliminary simulation evaluation of the NS-ATTC algorithm applied to a 6-DOF model of the JHU ROV [10]. The world-frame reference trajectory consisted of a sinusoid for each DOF, whose amplitude and period are each given in Table I. Table II gives the NS-ATTC algorithm gains. The simulated true parameter values were adapted from those reported in [17]. The true parameter values and initial estimate values are given in Table III.

TABLE I
NS-ATTC Reference Trajectory

DOF	Ampl.	Period
x_1 (North)	1 m	300 sec
x_2 (East)	-1 m	240 sec
x_3 (Down)	0.5 m	120 sec
φ_1 (Roll)	1.5°	60 sec
φ_2 (Pitch)	3°	60 sec
φ_3 (Heading)	180°	300 sec

TABLE II
NS-ATTC Gains

Gain	Value
Γ_m, Γ_d	1e4 $I_{6\times6}$
$\Gamma_{\mathcal{G}}$	diag([1;1;1;2e3])
Γ_a	5 $I_{36\times36}$
K_p	60 $I_{6\times6}$
K_d	1000 $I_{6\times6}$
ϵ	0.05

Figure 1 shows the reference trajectories, actual trajectories and corresponding tracking error versus time for 900 seconds of numerically simulated closed loop motion under the NS-ATTC control law (45) and parameter update law (47). Consistent with the analytical stability and convergence results (94,95) derived in Section V, the world position and orientation tracking error $\Delta\eta(t)$ and body velocity tracking error $\Delta v(t)$ converge quickly to zero. Consistent with (96), Figure 2 shows that the parameter estimate error magnitude $||\Delta\theta(t)||$ converges to an approximately constant value.

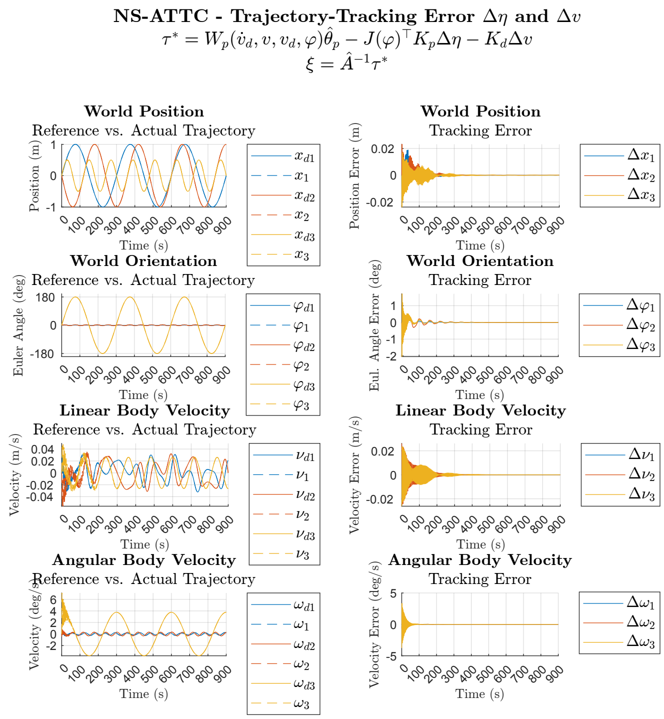

Fig. 1. The left column shows the reference and actual world position and orientation $x_d, x, \varphi_d, \varphi$ and body linear and angular velocities $\nu_d, \nu, \omega_d, \omega$ versus time. The right column shows error in world position and orientation $\Delta\eta = [\Delta x; \Delta\varphi]$ and body velocity $\Delta v = [\Delta\nu; \Delta\omega]$ versus time.

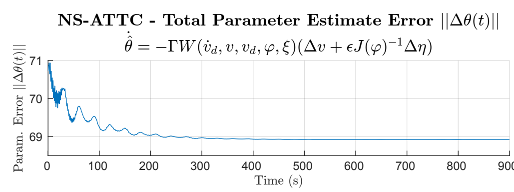

Fig. 2. The parameter estimate error magnitude $||\Delta\theta(t)||$ versus time

VII. Conclusion

NS-ATTC is a novel approach for TTC of fully-actuated UVs in the presence of unknown plant and actuator parameters. We report the NS-ATTC algorithm derivation, an analytical proof of asymptotically stable trajectory-tracking error convergence and stable parameter estimation, and a preliminary simulation evaluation using a 6DOF UV model. Future work includes experimental evaluation with comparison to other TTC methods and study of control feasibility, generalization to non-UV platforms, and investigation of parameter convergence and the case of underactuated vehicles.

TABLE III
NS-ATTC True Parameter and Initial Estimate Values

Group	True Parameters θ	Init. $\hat{\theta}(t_0)$
θ_m	[996.9; 1275; 1378; 308.7; 322.3; 467.4]	θ_m*0.97
θ_d	[347.8; 433.9; 497.6; 244.2; 55.34; 157.9]	θ_d*0.98
$\theta_{\mathcal{G}}$	[21.77; 5.966; -0.9802; 342.8]	$\theta_{\mathcal{G}}$*1.05
$a_{1,i}$	[-15; 15; 0; 0; 0; 0]	$a_{1,i}$*0.95
$a_{2,i}$	[0; 0; -15; 15; 0; 0]	$a_{2,i}$*1.03
$a_{3,i}$	[0; 0; 0; 0; -15; -15]	$a_{3,i}$*1.05
$a_{4,i}$	[0; 0; -3.371; -4.44; 0; 0]	$a_{4,i}$*0.94
$a_{5,i}$	[1.794; -1.794; 0; 0; 12.37; -11.08]	$a_{5,i}$*0.9
$a_{6,i}$	[-8.565; -8.565; 0; 1.665; 0; 0]	$a_{6,i}$*1.1

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