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Efficient Composite Learning Robot Control Under Partial Interval Excitation

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Abstract— **Parameter convergence in adaptive control is crucial for improving the stability and robustness of robotic systems. Nevertheless, a stringent condition named persistent excitation (PE) needs to be satisfied to ensure parameter convergence in the conventional adaptive robot control. Composite learning robot control (CLRC) is an innovative methodology that guarantees parameter convergence under a condition of interval excitation (IE) that is strictly weaker than PE. This paper puts forward a time-division multi-channel (TDMC) CLRC strategy such that parameter convergence is achieved even without the IE condition. In the TDMC mechanism, a filtered regressor is integrated with multiple time intervals to generate a generalized prediction error for parameter update, such that excitation information of regressor channels at different instants is exploited more effectively and efficiently to achieve fast and accurate parameter estimation. Global exponential stability with parameter convergence of the closed-loop system is achieved under a partial IE condition that is much weaker than IE. Experiments on a collaborative robot with 7 degrees of freedom have demonstrated the superiority of the proposed approach in both parameter estimation and trajectory tracking compared to start-of-the-art approaches.**

I. INTRODUCTION

Parameter convergence has significant advantages in enhancing stability and robustness for adaptive control of robotic systems [1]. In general, adaptive robot control can be divided into two schemes, including a direct scheme where parameter adaptation is driven by tracking errors regarding joint motions and an indirect scheme where prediction errors on control torques are employed to update parameter estimates to be used in the certainty-equivalence control law [2]. Composite adaptive robot control (CARC) that combines direct and indirect adaptive control can utilize both prediction and tracking errors to update parameter estimates [3]. The distinctive feature of CARC is that multi-source information, rather than tracking errors exclusively from joint motion, is employed to enhance both estimation and control. CARC has attracted great attention in the past decades, and many results can be found in existing literature [4]. However, its parameter convergence depends on persistent excitation (PE), which implies that robot trajectories include considerably rich spectral information all the time, which is generally infeasible in practice [5].

The PE requirement for exponential stability with parameter convergence in CARC results from using only instantaneous data for parameter adaptation. A natural idea to relax excitation conditions is to exploit online historical data (OHD), which is beneficial for improving parameter convergence. Inspired by this idea, some parameter learning schemes have been proposed for adaptive robot control, typically including concurrent learning and composite learning. Concurrent learning resorts to online selecting and forgetting data to establish a dynamic data stack, where data update is judged by maximizing the minimum singular value (MSV) of the data stack [6]. If sufficiently rich data are stored, exponential parameter convergence can be unconstrained from PE. Concurrent learning has been applied to deal with several robot control problems with real-world applications [7]. Nevertheless, concurrent learning does not show the connection to composite adaptation with the concept of prediction errors and does not solve the problem of requiring joint accelerations. Composite learning utilizes interval data-driven memory regressor extension to define a generalized prediction error and applies this prediction error together with the tracking error to update parameter estimates, such that exponential stability with parameter convergence is achieved under a condition termed interval excitation (IE) that is much weaker than PE, in which parameter estimation does not depend on acceleration information [8]–[10]. Composite learning has been widely applied to real-world robot control problems, including trajectory tracking, physical human-robot interaction, and visual servoing [7], [11]–[13].

A key feature of the above-mentioned learning schemes for adaptive control is that the storage and forgetting of OHD are determined by the exciting strength of an excitation matrix, which requires that all regressor channels are activated in a certain instant simultaneously. We propose a time-division multi-channel (TDMC) mechanism that utilizes multiple regressor channels at different instants for CLRC such that exponential parameter convergence is ensured under partial IE, removing the strict condition that all regressor channels must be activated simultaneously. In the TDMC scheme, the strength of each regressor channel is determined online to construct some new excitation matrices comprised of only active channels. The storage and forgetting of OHD are decided by maximizing the MSV of the summation of these excitation matrices, which are not affected by inactive channels. Using multiple asynchronous regressor channels enhances the memory on OHD for CLRC. Experiments on a collaborative robot with 7 degrees of freedom (DoFs) are carried out to verify the proposed approach.

Notations: $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the spaces of real numbers, positive real numbers, real n-vectors, and real $m \times n$-

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matrices, respectively, I is an identity matrix, L_∞ is the space of bounded signals, $\max\{\cdot\}$ and $\min\{\cdot\}$ are the maximum and minimum operators, respectively, $\|\boldsymbol{x}\|$ is the Euclidean norm of $\boldsymbol{x}$, $\sigma_{\min}(A)$ is the MSV of A, $\arg\max_{x\in S} f(x) := \{x \in S | f(y) \le f(x), \forall y \in S\}$ with $f : \mathbb{R} \mapsto \mathbb{R}$ and $S \subset \mathbb{R}$, and $\mathrm{diag}(x_1, x_2, ..., x_n)$ is a diagonal matrix with diagonal elements $x_1, x_2, ..., x_n$, where $A \in \mathbb{R}^{n\times n}$, $\boldsymbol{x} = [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$, and $x_i \in \mathbb{R}$ is the *i*th element of $\boldsymbol{x}$.

II. PROBLEM FORMULATION

Consider a class of n-DoF robotic systems as follows [14]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + G(\boldsymbol{q}) + F(\dot{\boldsymbol{q}}) = \boldsymbol{\tau}, \tag{1}$$

in which $\boldsymbol{q}(t) := [q_1(t),\ q_2(t),\ \cdots,\ q_n(t)]^T \in \mathbb{R}^n$ is a joint position, $\boldsymbol{\tau} \in \mathbb{R}^n$ is a control input, $M(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is an inertia matrix, $C(\boldsymbol{q}, \dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ is a centripetal-Coriolis matrix, $G(\boldsymbol{q}) \in \mathbb{R}^n$ denotes a gravitational torque, and $F(\dot{\boldsymbol{q}}) \in \mathbb{R}^n$ denotes a friction torque. Assume that $F(\dot{\boldsymbol{q}}) = \mathrm{diag}(f_\mathrm{v})\dot{\boldsymbol{q}} + \mathrm{diag}(f_\mathrm{c})\tanh(20\dot{\boldsymbol{q}}) + f_\mathrm{o}$, where f_v, f_c and $f_\mathrm{o} \in \mathbb{R}^n$ are the coefficient vectors of viscous friction, Coulomb friction, and its offset, respectively [14]. Some properties of (1) from [14] and definitions from [10] are presented as follows.

Property 1: $M(\boldsymbol{q})$ is a symmetric and positive-definite matrix that meets $m_0\|\boldsymbol{x}\|^2 \le \boldsymbol{x}^T M(\boldsymbol{q})\boldsymbol{x} \le \bar{m}\|\boldsymbol{x}\|^2, \forall \boldsymbol{x} \in \mathbb{R}^n$, where $m_0, \bar{m} \in \mathbb{R}^+$ are unknown constants.

Property 2: $\dot{M}(\boldsymbol{q}) - 2C(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is skew-symmetric such that $\boldsymbol{x}^T(\dot{M}(\boldsymbol{q}) - 2C(\boldsymbol{q}, \dot{\boldsymbol{q}}))\boldsymbol{x} = 0, \forall \boldsymbol{x} \in \mathbb{R}^n$.

Property 3: The left-hand side expression of (1) can be linearly parameterized as follows:

$$M(\boldsymbol{q})\dot{\boldsymbol{v}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{v} + G(\boldsymbol{q}) + F(\dot{\boldsymbol{q}}) = \Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{v}, \dot{\boldsymbol{v}})W \tag{2}$$

where $\boldsymbol{v} \in \mathbb{R}^n$ is an auxiliary variable, $\Phi : \mathbb{R}^{4n} \mapsto \mathbb{R}^{N\times n}$ is a $\mathcal{C}^1$ regressor, $W \in \mathbb{R}^N$ is an unknown base parameter vector, and N is the number of base parameters.

Definition 1: A bounded signal $\Phi(t) \in \mathbb{R}^{N\times n}$ is of PE if $\exists$ $\sigma, \varsigma_\mathrm{d} \in \mathbb{R}^+$ such that $\int_{t-\varsigma_\mathrm{d}}^{t} \Phi(\varsigma)\Phi^T(\varsigma)d\varsigma \ge \sigma I, \forall t \ge 0$.

Definition 2: A bounded signal $\Phi(t) \in \mathbb{R}^{N\times n}$ is of IE if $\exists$ $T_\mathrm{e}, \varsigma_\mathrm{d}, \sigma \in \mathbb{R}^+$ such that $\int_{T_\mathrm{e}-\varsigma_\mathrm{d}}^{T_\mathrm{e}} \Phi(\varsigma)\Phi^T(\varsigma)d\varsigma \ge \sigma I$.

Definition 3: A bounded signal $\Phi(t) \in \mathbb{R}^{N\times n}$ is of partial IE, if $\exists$ $T_\mathrm{e}, \varsigma_\mathrm{d}, \sigma \in \mathbb{R}^+$ such that $\int_{T_\mathrm{e}-\varsigma_\mathrm{d}}^{T_\mathrm{e}} \Phi_\mathrm{s}(\varsigma)\Phi_\mathrm{s}^T(\varsigma)d\varsigma \ge \sigma I$, where $\Phi_\mathrm{s} \in \mathbb{R}^{m\times n}$ is a sub-regressor constituted by some row vectors of Φ with $1 \le m < N$.

Assume that Φ is known, but W is unknown in (2). For convenience, a column vector $\boldsymbol{\phi}_i \in \mathbb{R}^n$ $(i = 1, 2, ..., N)$ of the regressor $\Phi^T(t) \in \mathbb{R}^{n\times N}$ is named as a channel. Thus, one has $\Phi(t) = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, ..., \boldsymbol{\phi}_N]^T$. A channel $\boldsymbol{\phi}_i$ $(i = 1, 2, ..., m)$ of $\Phi(t)$ is named an *active channel* if $\boldsymbol{\phi}_i(t) \ne \mathbf{0}$, conversely termed an *inactive channel*. Let $\boldsymbol{q}_\mathrm{d}(t) := [q_{\mathrm{d}1}(t), q_{\mathrm{d}2}(t), \cdots, q_{\mathrm{d}n}(t)]^T \in \mathbb{R}^n$ denotes the desired joint trajectory that satisfies $\boldsymbol{q}_\mathrm{d}$, $\dot{\boldsymbol{q}}_\mathrm{d}$ and $\ddot{\boldsymbol{q}}_\mathrm{d} \in L_\infty$. Define tracking errors $\boldsymbol{e}_1(t) := \boldsymbol{q}_\mathrm{d}(t) - \boldsymbol{q}(t)$ and $\boldsymbol{e}_2(t) := \dot{\boldsymbol{e}}_1(t) + \Lambda\boldsymbol{e}_1(t)$, in which $\Lambda \in \mathbb{R}^{n\times n}$ is a positive-definite diagonal matrix of control gains. The objective here is to design an effective control strategy for the system (1) to guarantee closed-loop stability and parameter convergence under the weak partial IE condition.

III. COMPOSITE LEARNING CONTROL DESIGN

A. Composite Learning Robot Control

Following (1), the definition of $\boldsymbol{e}_2$ and Property 3, one gets the open-loop error dynamics

$$M(\boldsymbol{q})\dot{\boldsymbol{e}}_2 = \Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{r}, \ddot{\boldsymbol{q}}_\mathrm{r})W - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{e}_2 - \boldsymbol{\tau} \tag{3}$$

where $\dot{\boldsymbol{q}}_\mathrm{r}(t) := \dot{\boldsymbol{q}}_\mathrm{d}(t) + \Lambda\boldsymbol{e}_1(t)$ is a joint "reference velocity". The robot control law is given by

$$\boldsymbol{\tau} = K_\mathrm{c}\boldsymbol{e}_2 + \Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{r}, \ddot{\boldsymbol{q}}_\mathrm{r})\hat{W}(t) \tag{4}$$

in which $\hat{W}(t) \in \mathbb{R}^N$ is an estimate of W, and $K_\mathrm{c} \in \mathbb{R}^{n\times n}$ is a positive-definite diagonal matrix of control gains. Applying (4) to (3) and letting $\tilde{W} := W - \hat{W}$ result in

$$M(\boldsymbol{q})\dot{\boldsymbol{e}}_2 = \Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{r}, \ddot{\boldsymbol{q}}_\mathrm{r})\tilde{W} - K_\mathrm{c}\boldsymbol{e}_2 - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{e}_2 \tag{5}$$

which is a closed-loop tracking error system.

Using Property 3, (1) is linearly parameterized to be

$$\boldsymbol{\tau}(t) = \Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})W \tag{6}$$

which involves $\ddot{\boldsymbol{q}}$ that is hardly accessible from measurement. To avoid using $\ddot{\boldsymbol{q}}$ in parameter estimation, a stable linear filter $L(s) := \frac{\alpha}{s+\alpha}$ with a filtering constant $\alpha \in \mathbb{R}^+$ is applied to each side of (6) resulting in [3]

$$\boldsymbol{\tau}_\mathrm{f} = \Phi_\mathrm{f}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})W \tag{7}$$

where $\boldsymbol{\tau}_\mathrm{f} = L(s)[\boldsymbol{\tau}]$ and $\Phi_\mathrm{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}) := L(s)[\Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})]$ are filtered values of $\boldsymbol{\tau}$ and Φ, respectively, and s is the complex Laplace operator. A torque prediction model is given by

$$\hat{\boldsymbol{\tau}}_\mathrm{f} = \Phi_\mathrm{f}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})\hat{W} \tag{8}$$

with $\hat{\boldsymbol{\tau}}_\mathrm{f} \in \mathbb{R}^n$ being a predicted value of $\boldsymbol{\tau}_\mathrm{f}$. Let

$$\Theta(t) := \int_{t-\varsigma_\mathrm{d}}^{t} \Phi_\mathrm{f}(\varsigma)\Phi_\mathrm{f}^T(\varsigma)d\varsigma, \tag{9}$$

$$\boldsymbol{\psi}(t) := \int_{t-\varsigma_\mathrm{d}}^{t} \Phi_\mathrm{f}(\varsigma)\boldsymbol{\tau}_\mathrm{f}(\varsigma)d\varsigma = \Theta(t)W \tag{10}$$

with $\Phi_\mathrm{f}(\varsigma) := \Phi_\mathrm{f}(\boldsymbol{q}(\varsigma), \dot{\boldsymbol{q}}(\varsigma))$. Based on (9), the IE condition in Definition 2 is equivalent to $\Theta(T_e) \ge \sigma I$, in which σ is regarded as an exciting strength determined by the MSV of Θ. Define a general torque prediction error

$$\boldsymbol{\epsilon}(t) := \boldsymbol{\tau}_\mathrm{f}(t) - \Phi_\mathrm{f}(t)\hat{W} \tag{11}$$

and a generalized torque prediction error

$$\boldsymbol{\xi}(t) := \begin{cases} \boldsymbol{\psi}(t) - \Theta(t)\hat{W}(t), & t < T_\mathrm{e} \\ \boldsymbol{\psi}(t_\mathrm{e}) - \Theta(t_\mathrm{e})\hat{W}(t), & t \ge T_\mathrm{e} \end{cases} \tag{12}$$

with $t_\mathrm{e} := \arg\max_{\zeta\in[T_\mathrm{e},t]} \sigma_{\min}(\Theta(\zeta))$, where it is assumed $\Theta(T_\mathrm{e}) \ge \sigma I$. A composite learning law is given by

$$\dot{\hat{W}} = \Gamma(\Phi(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_\mathrm{r}, \ddot{\boldsymbol{q}}_\mathrm{r})\boldsymbol{e}_2 + \kappa\boldsymbol{\xi}) \tag{13}$$

where $\Gamma \in \mathbb{R}^{N\times N}$ is a positive-definite matrix of learning rates, and $\kappa \in \mathbb{R}^+$ is a weighting factor. It is shown in [10] that the robotic system (1) driven by the CLRC law (4) with (12) and (13) achieves global exponential stability in the sense that $\boldsymbol{e}_2$, $\tilde{W} \to \mathbf{0}$ exponentially under IE. IE implies that all regressor channels $\boldsymbol{\phi}_i(t)$ $(1 \leq i \leq N)$ must be activated simultaneously in a certain instant t. If there is a channel $\boldsymbol{\phi}_i(t) \equiv \mathbf{0}$, the exciting strength σ of the excitation matrix Θ is always 0, and the generalized prediction error $\boldsymbol{\xi}$ is always calculated by the upper case of (12). In this case, OHD corresponding to active channels can not be stored to promote the convergence of $\hat{W}$. This phenomenon will also be shown in Sec. IV.

B. Time-Division Multi-Channel Learning

This section aims to relax the IE condition for parameter convergence in CLRC. For convenience, let $\boldsymbol{\phi}_{\mathrm{f}i}$ denote the ith column vector of the filter regressor Φ_{f}^T. To avoid storing and forgetting OHD are affected by some inactive channels, the exciting strength of each channel, defined by $\|\boldsymbol{\phi}_{\mathrm{f}i}(t)\|$, needs to be determined in advance, and only active channels are applied to construct a novel excitation matrix

$$\Theta_{\mathrm{s}} := \int_{t-\varsigma}^{t} \Phi_{\mathrm{s}}(\varsigma)\Phi_{\mathrm{s}}^T(\varsigma)d\varsigma \tag{14}$$

where Φ_{s} is a sub-regressor of Φ_{f} as in Definition 3. Then, storing and forgetting OHD can be decided by maximizing the exciting strength σ of Θ_{s}, which are not influenced by inactive channels. Moreover, as different channels may be activated at different instants, it is essential to collect these useful data points from different instants. Let $\mathcal{Z}(t)$ and $\mathcal{Y}(t)$ be data sets that record Θ and $\boldsymbol{\psi}$ given by (9) and (10) at different instants as data stacks at the time t, respectively. Based on the above argument, define a novel generalized prediction error

$$\boldsymbol{\xi}(t) := \frac{1}{N}\sum_{k=1}^{N}(\boldsymbol{\psi}_k - \Theta_k\hat{W}) \tag{15}$$

where $\Theta_k \in \mathbb{R}^{N\times N}$ and $\boldsymbol{\psi}_k \in \mathbb{R}^N$ are the kth elements of $\mathcal{Z}$ and $\mathcal{Y}$, respectively, and $\mathcal{Z}$ and $\mathcal{Y}$ are updated by Algorithm 1. Note that the data set Z_{s} and excitation matrix Θ_{s} are used only to calculate the exciting strength of the excitation matrix $\sum_{j=1}^{N}\mathcal{Q}_j$ constructed from all active channels [see Lines 8-16], and the threshold $\delta \in \mathbb{R}^+$ is set as a small value to avoid the impact of noise and disturbances [see Lines 7-8].

For the composite learning law in (12)-(13), OHD are stored and updated only if IE holds. In the proposed TDMC-CLRC, all active channels $\boldsymbol{\phi}_{\mathrm{f}i}$ are collected according to the channel strength $\|\boldsymbol{\phi}_{\mathrm{f}i}\|$ to construct an excitation matrix set Z_{s}, such that the exciting strength σ of the excitation matrix Θ_{s} can be reflected [see Lines 7-11 in Algorithm 1]. If the current exciting strengths of some active channels $\boldsymbol{\phi}_{\mathrm{f}i}$ are greater than their maximum exciting strengths in the past, the corresponding indexes are collected to denote an index set Υ [see Lines 12 in Algorithm 1]. A certain element in $\mathcal{Z}$ whose index belongs to Υ will be replaced by the current excitation matrix $\Theta(t)$. Hence, the data sets $\mathcal{Z}$ and $\mathcal{Y}$ can be updated when the exciting strength of any channel $\boldsymbol{\phi}_{\mathrm{f}i}$ increases. The generalized prediction error $\boldsymbol{\xi}$ in (15) is obtained from multiple time-interval integrals of the filtered regressor Φ_{f} at different instants such that the exciting strength of $\sum_{k=1}^{N}\Theta_k$ is monotonously nondecreasing [see Lines 13-21 in Algorithm 1]. The following theorem shows closed-loop properties regarding stability and parameter convergence under the proposed method.

Algorithm 1 Time-division multi-channel learning

```
 1: **Initialize**: $k \leftarrow 0$
 2: **if** $k < N$ **then**
 3:     $k \leftarrow k + 1$
 4:     $\Theta_k \leftarrow \Theta(t)$, $\boldsymbol{\psi}_k \leftarrow \boldsymbol{\psi}(t)$
 5:     $\sigma_{\mathrm{old}} \leftarrow \sigma_{\min}(\Sigma_{j=1}^{k}\mathcal{Z}_j(t))$
 6: **else**
 7:     **if** $\exists i$ such that $\|\boldsymbol{\phi}_{\mathrm{f}i}(t)\| \leq \delta$ **then**
 8:         Ignore the column and row of $\Theta(t)$ and matrices in $\mathcal{Z}(t)$ that correspond to $\|\boldsymbol{\phi}_{\mathrm{f}i}(t)\| \leq \delta$, and the resulting matrix and set are denoted as $\Theta_{\mathrm{s}}$ and $\mathrm{Z}_{\mathrm{s}}$, respectively.
 9:     **else**
10:         $\mathrm{Z}_{\mathrm{s}} \leftarrow \mathcal{Z}$, $\Theta_{\mathrm{s}}(t) \leftarrow \Theta(t)$
11:     **end if**
12:     Find $i$ satisfying $\|\boldsymbol{\phi}_{\mathrm{f}i}(t)\| \geq \max_{\zeta\in[0,t)}\{\|\boldsymbol{\phi}_{\mathrm{f}i}(\zeta)\|\}$ to constitute an index set $\Upsilon$
13:     **for** $i = 1$ to length($\Upsilon$) **do**
14:         $\mathcal{Q} \leftarrow \mathrm{Z}_{\mathrm{s}}$
15:         $\mathcal{Q}_{\Upsilon_i} \leftarrow \Theta_{\mathrm{s}}$
16:         $E_i \leftarrow \sigma_{\min}(\Sigma_{j=1}^{N}\mathcal{Q}_j)$
17:     **end for**
18:     Find $\max\{E\}$ and let $l$ denote the corresponding index
19:     **if** $\max\{E\} > \sigma_{\mathrm{old}}$ **then**
20:         $\Theta_{\Upsilon_l}(t) \leftarrow \Theta(t)$, $\boldsymbol{\psi}_{\Upsilon_l}(t) \leftarrow \boldsymbol{\psi}(t)$
21:         $\sigma_{\mathrm{old}} \leftarrow \max\{E\}$
22:     **end if**
23: **end if**
```

Theorem 1. Consider the robotic system with Properties 1-3 driven by the TDMC-CLRC law in (4) and (13), in which the general prediction error $\boldsymbol{\xi}$ is updated by (15) and Algorithm 1. If only partial IE holds and there exists a time $T_\sigma > 0$ such that all channels are activated at least once on $t \in [0, T_\sigma]$, the closed-loop system is exponential stability in the sense that the tracking error $\boldsymbol{e}_2$ and the parameter estimation error $\tilde{W}$ exponentially converge to $\mathbf{0}$ on $t \in [T_\sigma, \infty)$.

Proof: If only partial IE holds and there exists a time $T_\sigma > 0$ such that all channels are activated at least once on $t \in [0, T_\sigma]$, multiple values of the excitation matrix Θ at different instants can be recorded in the data set $\mathcal{Z}$. The data points in memory are linearly independent, which ensures that there exists a constant $\sigma > 0$ such that $\sum_{k=1}^{N}\Theta_k \geq \sigma I$. Therefore, a Lyapunov function candidate for the closed-loop system composed of (5) and (13) is chosen as follows:

$$V(\boldsymbol{e}_2, \tilde{W}) = \frac{1}{2}\boldsymbol{e}_2^T M(\boldsymbol{q})\boldsymbol{e}_2 + \frac{1}{2}\tilde{W}^T\Gamma^{-1}\tilde{W}. \tag{16}$$

The time derivative of V in (16) is given by

$$\dot{V} = \boldsymbol{e}_2^T M(\boldsymbol{q})\dot{\boldsymbol{e}}_2 + \frac{1}{2}\boldsymbol{e}_2^T\dot{M}(\boldsymbol{q})\boldsymbol{e}_2 - \tilde{W}^T\Gamma^{-1}\dot{\hat{W}}.$$

Noting Property 2 and applying (5) and (13), one gets

$$\begin{aligned}\dot{V} = &-\boldsymbol{e}_2^T K_{\mathrm{c}}\boldsymbol{e}_2 + \boldsymbol{e}_2^T\Phi^T(\boldsymbol{q}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_{\mathrm{r}}, \ddot{\boldsymbol{q}}_{\mathrm{r}})\tilde{W}\\ &- \tilde{W}^T\Gamma^{-1}\dot{\hat{W}} + \boldsymbol{e}_2^T(\dot{M}(\boldsymbol{q}) - 2C(\boldsymbol{q}, \dot{\boldsymbol{q}}))/2\end{aligned}$$

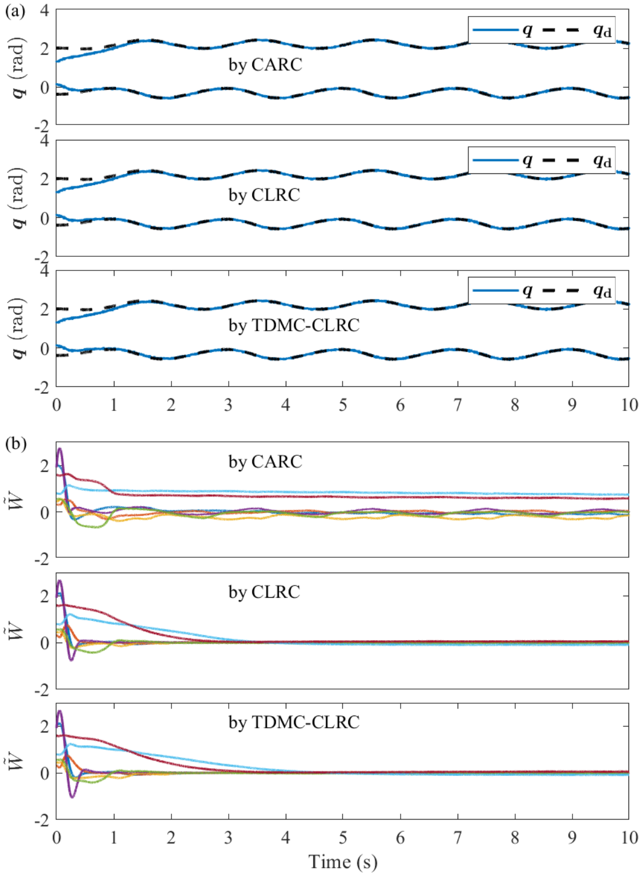

Fig. 1. Performance comparisons of three controllers for the tracking in simulations. (a) The trajectories of $\boldsymbol{q}_{\mathrm{d}}$ and $\boldsymbol{q}$. (b) The estimation errors $\tilde{W}$.

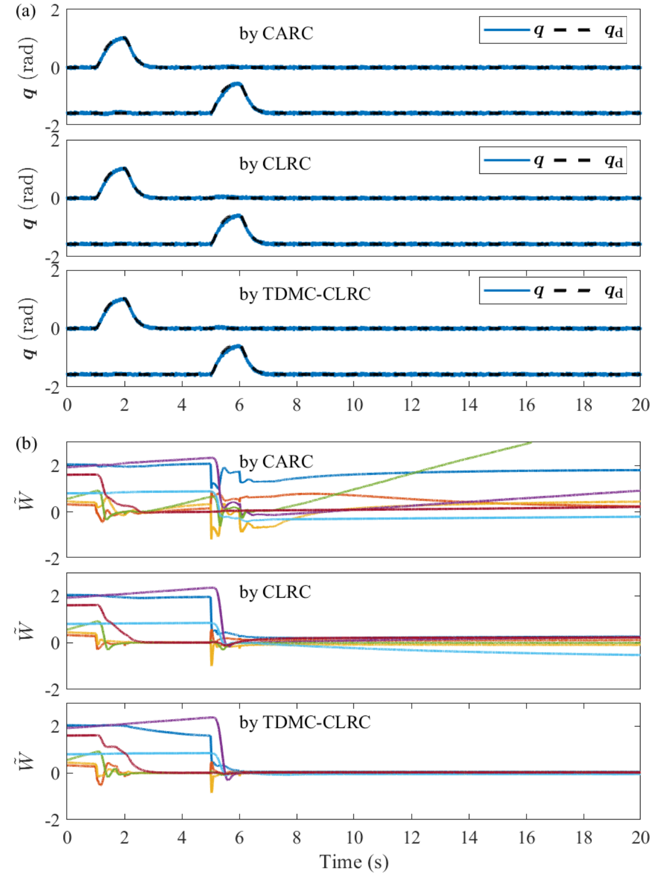

Fig. 2. Performance comparisons of three controllers for the regulation in simulations. (a) The trajectories of $\boldsymbol{q}_{\mathrm{d}}$ and $\boldsymbol{q}$. (b) The estimation errors $\tilde{W}$.

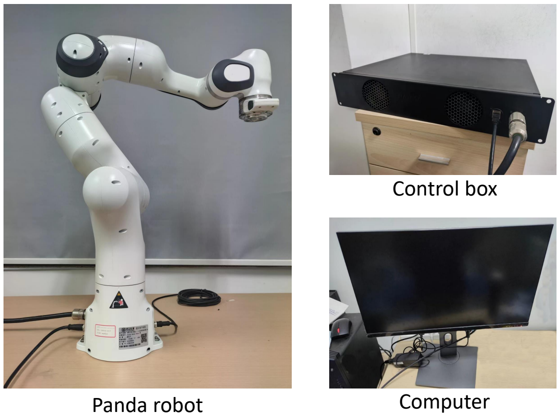
Panda robot
Control box
Computer

Fig. 3. A 7-DoF robot named Franka Emika Panda for experiments.

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\begin{aligned}
&= -\boldsymbol{e}_2^T K_{\mathrm{c}} \boldsymbol{e}_2 - \frac{\kappa}{N}\tilde{W}^T \sum_{k=1}^{N} \Theta_k \tilde{W} \\
&\le -\boldsymbol{e}_2^T K_{\mathrm{c}} \boldsymbol{e}_2 - \frac{\kappa\sigma}{N}\tilde{W}^T \tilde{W}.
\end{aligned}
$$

It follows from Property 1 and the above inequality that

$$\dot{V} \le -k_{\mathrm{c}} \boldsymbol{e}_2^T M(\boldsymbol{q}) \boldsymbol{e}_2 / \bar{m} - \kappa\sigma\lambda_{\min}(\Gamma)\tilde{W}^T \Gamma^{-1} \tilde{W}/N$$

with $k_{\mathrm{c}} := \lambda_{\min}(K_{\mathrm{c}}) \in \mathbb{R}^+$. Thus, one obtains

$$\dot{V}(t) \le k_{\mathrm{m}} V(t), \forall t \ge T_\sigma,$$

with $k_{\mathrm{m}} := \min\{k_{\mathrm{c}}/\bar{m}, \kappa\sigma\lambda_{\min}(\Gamma)/N\} \in \mathbb{R}^+$, which shows that the closed-loop system has global exponential stability in the sense of $\lim_{t\to\infty} \|\boldsymbol{e}_2(t)\| = 0$ and $\lim_{t\to\infty} \|\tilde{W}(t)\| = 0$ with exponentially convergent rates on $t \in [T_\sigma, \infty)$. ■

Remark 1: In the proposed method, the generalized prediction error $\boldsymbol{\xi}$ in (15) is constructed by some excitation matrices Θ_k at different instants that ignore all inactive channels, and storing and forgetting OHD follows the exciting strengths of the active channels $\boldsymbol{\phi}_{\mathrm{f}i}$ and the excitation matrix $\sum_{j=1}^{N} \mathcal{Q}_j$. If there always exists at least one inactive channel, only partial IE exists for each excitation matrix Θ_k. However, the excitation matrix $\sum_{j=1}^{N} \mathcal{Q}_j$ can store data under different partial IE stages simultaneously, such that $\sum_{j=1}^{N} \mathcal{Q}_j$ is positive definite if all channels are activated at least once. In this manner, exponential stability with parameter convergence can be achieved under partial IE that is much weaker than PE and IE, which imposes robustness against various perturbations, such as measurement noise, external disturbances, and unmodeled dynamics [2]. The TDMC algorithm is natural to resolve a slowly time-varying or suddenly changing parameter W if storing and forgetting OHD is decided by the current prediction error $\boldsymbol{\epsilon}$ in (11).

Remark 2: The proposed TDMC algorithm possesses several

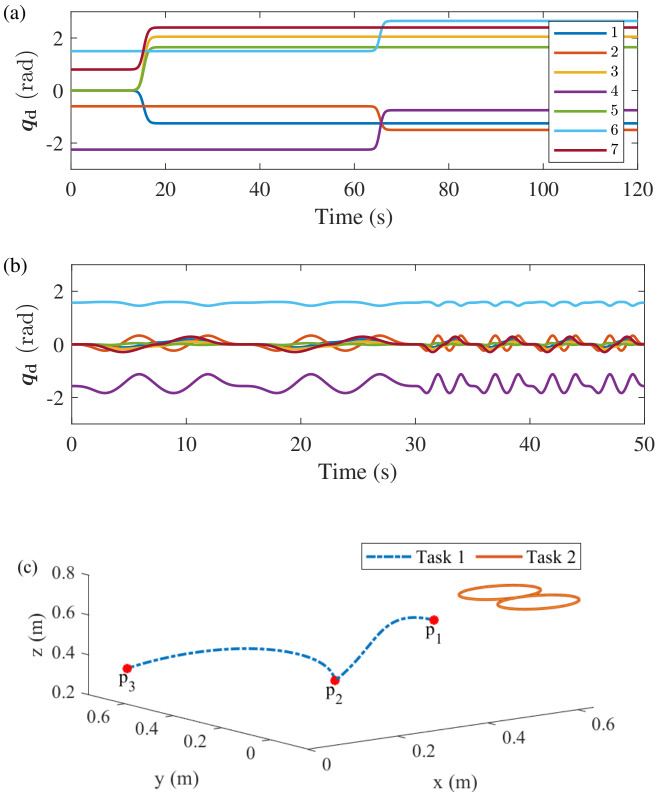

Fig. 4. The desired output $\boldsymbol{q}_{\rm d}$ composed of two tasks for experiments. (a) The trajectories of $\boldsymbol{q}_{\rm d}$ in Task 1. (b) The trajectories of $\boldsymbol{q}_{\rm d}$ in Task 2. (c) The corresponding Cartesian trajectories of $\boldsymbol{q}_{\rm d}$. Noted that Task 1 is with the order $\mathrm{p}_1 \to \mathrm{p}_2 \to \mathrm{p}_3$ at $t \in [0, 120]$ s.

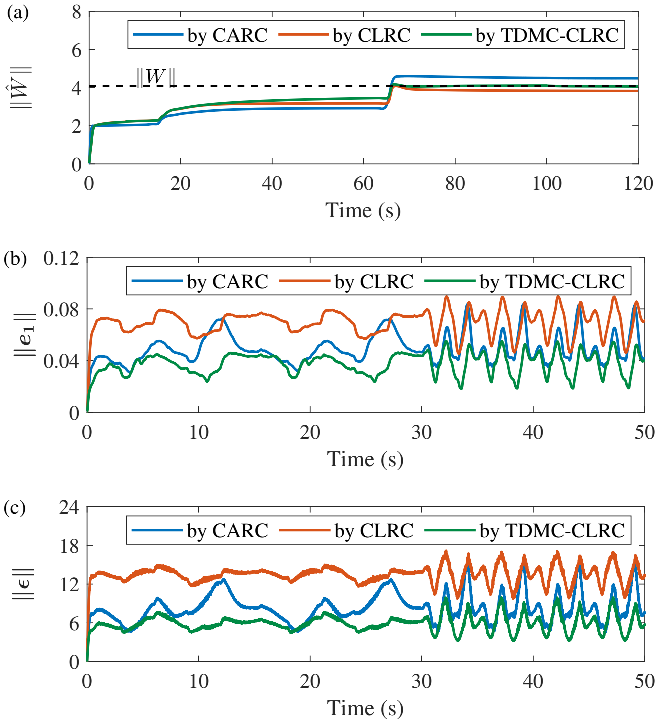

Fig. 5. Performance comparisons of three controllers in experiments. (a) The parameter estimate norms $\|\hat{W}\|$ in Task 1. (b) The tracking error norm $\|\boldsymbol{e}_1\|$ in Task 2. (c) The prediction error norms $\|\boldsymbol{\epsilon}\|$ in Task 2.

differences from the MSV maximizing algorithm in concurrent learning [6]: 1) It stores the time-interval integrals of $\Phi_{\rm f}\Phi_{\rm f}^T$ as data points, which can average the effect of measurement noise, whereas the algorithm of [6] lacks the averaging effect; 2) it can eliminate the influence that recording and forgetting OHD is dominated by inactive channels [see Lines 7-11 in Algorithm 1], whereas recording and forgetting OHD in the algorithm of [6] is determined by maximizing the MSV that is affected by the weakest channel; 3) it can still record and forget some data points corresponding to active channels if an inactive channel is present throughout, whereas recording and forgetting OHD in the algorithm of [6] may be invalid since the exciting strength σ is always 0.

Remark 3: The excitation matrices Θ_k are selectively storing and forgetting OHD, which possibly results in a discontinuous adaptive law in (13). However, (13) is still locally Lipschitz in state variables and piecewise continuous with respect to time t, which implies the existence of a unique Filippov solution to the differential equation (13) that represents a general class of adaptive systems with switching elements [15].

IV. SIMULATION RESULTS

This section considers a planar robot arm with two revolute joints whose model information can be referred to [10, Sec. 4]. Set the control parameters $K_{\rm c} = 10I$, $\kappa = 1$, $\varsigma_{\rm d} = 5$, $\Lambda = 2I$, $\Gamma = 3I$, $\alpha = 5$, and $\delta = 10^{-4}$, and the initial value of the parameter estimate $\hat{W}(0) = \boldsymbol{0} \in \mathbb{R}^7$. The CLRC in [10] and the CARC with a constant learning rate in [3] are selected as baselines, where their shared parameters are set to be the same values for fair comparisons. Simulations are carried out in MATLAB with fixed-step ode3 solver and a sample frequency of 1000Hz. Gaussian white noise with mean $=0$, standard deviation (SD) $=0.03$, and bound $=3\times$SD is added to the feedback of $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ as noisy measurement.

Consider the tracking problem in Case 2 of [10], where the purpose is to verify that the performance of the TDMC-CLRC is comparable to the CLRC when all channels are activated. The desired output $\boldsymbol{q}_{\rm d}$ and performance comparisons of all controllers are depicted in Fig. 1, and the PE condition is met in this case. It is observed that the estimation error $\tilde{W}$ by the CARC does not converge to $\boldsymbol{0}$ even PE exists [see Fig. 1(b)], because it is affected by the exciting strength and measurement noise. For the CLRC and TDMC-CLRC, each element of $\tilde{W}$ converges to 0 after running 4 s [see Fig. 1(b)]. Besides, all controllers exhibit high tracking accuracy after the transient process of learning [see Fig. 1(a)].

Consider a regulation problem where the following reference model is introduced to generate the desired output $\boldsymbol{q}_{\rm d}$:

$$\dot{\boldsymbol{q}}_{\rm d} = \begin{bmatrix} 0 & 1 \\ -36 & -12 \end{bmatrix} \boldsymbol{q}_{\rm d} + \begin{bmatrix} 0 \\ 36 \end{bmatrix} q_{ci}$$

with $i = 1, 2$, $\boldsymbol{q}_{\rm d}(0) = [-\pi/2, 0]^T$, $\boldsymbol{q}(0) = [-\pi/2, 0]^T$, and $\dot{\boldsymbol{q}}(0) = \boldsymbol{0}$, in which $(q_{\rm c1}, q_{\rm c2}) = (-\pi/2, -\pi/6)$, $(\pi/3, 0)$ and $(-\pi/2, 0)$ for $t \in [1, 2)$ s, $t \in [5, 6)$ s and the rest of the time t, respectively. In this case, all regressor channels are impossible to activate simultaneously, implying the possible existence of partial IE rather than IE. Performance comparisons are depicted in Fig. 2. It is observed that the CARC does not show parameter convergence and even encounters parameter drift due to the absence of PE and the existence of measurement noise, and the CLRC only shows partial convergence of $\tilde{W}$ to $\mathbf{0}$ due to the lack of IE. In contrast, the proposed TDMC-CLRC exhibits full convergence of $\tilde{W}$ to $\mathbf{0}$ rapidly after 5 s, which implies that: 1) The exponential stability of the robotic system (1) is guaranteed under partial IE that significantly relaxes IE; 2) the summation of multiple time-interval integrals for $\Phi_{\mathrm{f}}\Phi_{\mathrm{f}}^T$ is beneficial for parameter convergence; 3) the convergence of the parameter estimate $\hat{W}$ in (13) with (15) to its true value is still guaranteed when regressor channels are activated asynchronously.

V. EXPERIMENTAL RESULTS

An experimental platform of a 7-DoF robot named Franka Emika Panda shown in Fig. 3 is applied to verify the proposed method, where control algorithms run on a personal computer with a sampling time of 1 ms. Choose the control parameters $K_{\mathrm{c}} = \mathrm{diag}\{20, 20, 20, 20, 8, 8, 5\}$, $\Lambda = 10I$, $\Gamma = 0.2I$, $\kappa = 1$, $\varsigma_{\mathrm{d}} = 8$ s, and $\alpha = 10$, and the initial value of the parameter estimate $\hat{W}(0) = \mathbf{0} \in \mathbb{R}^{64}$. Note that the proposed control law can achieve global stability of closed-loop system, and thus parameter convergence is irrelevant to the choice of the initial value $\hat{W}(0)$. The desired output $\boldsymbol{q}_{\mathrm{d}}$ with two tasks for experiments are depicted in Fig. 4. Task 1 is composed of two regulation sub-tasks for achieving parameter convergence [see Fig. 4(a)], and Task 2 is a time-varying trajectory with low and high speeds for verifying the accuracy of parameter estimation [see Fig. 4(b)]. Note that in each regulation point of Task 1, only a part of the robot joints delivers movements.

Performance comparisons of all controllers in experiments are exhibited in Fig. 5, where the black dashed line in Fig. 5(a) is the norm of the true parameter W obtained from [16] for reference. It is observed that the parameter estimate norms $\|\hat{W}\|$ by all controllers converge to certain constants after the 2nd regulation sub-task [see Fig. 5(a)]. The proposed TDMC-CLRC achieves the best estimation accuracy compared to the CARC and CLRC, implying that the true parameter W can be estimated in the absence of IE. In regard to the tracking performance, the TDMC-CLRC significantly outperforms the other two methods all the time [see Fig. 5(b)]. For the torque prediction, the TDMC-CLRC also performs better than the other two methods [see Fig. 5(c)], which indirectly verifies the successful estimation of W under partial IE.

VI. CONCLUSIONS

This paper has presented a feasible adaptive control strategy named TDMC-CLRC for robotic systems to achieve parameter convergence under the weakened partial IE condition. Simulations on a 2-DoF planar robot have verified directly the theoretical result on parameter convergence for the proposed approach. Experiments on a 7-DoF robot have also verified the theoretical result indirectly, where the proposed approach significantly outperforms state-of-the-art controllers with respect to both parameter estimation and trajectory tracking. Further work would focus on investigating the stability and robustness of the proposed approach rigorously.

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