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Decentralized multi-phase formation control for cattle herding

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Abstract— **Herding is performed by people or trained animals to control the movement of livestock under the desired direction of an operator. This paper presents a novel decentralized control strategy for a group of robots to herd animals which consists of two phases, a surrounding phase and a driving phase. In the surrounding phase, a custom artificial potential field is employed to simultaneously guide the robots to encircle the herd by tracking the outmost animals and maintaining a safe distance from other neighboring robots. Once the encirclement is complete, the robots transition to drive the animals toward a designated goal by simply maintaining their initial formation and traversing to it. Unlike existing works on herding using flocking control, local observations of the nearest animals and communication with other robots within the sensing range are the only requirements for the robots to surround and herd the animals effectively. Moreover, the animal-robot behavior model resembles the interaction of livestock in the presence of an external predatory threat, where robots act as predators. An analytical proof and empirical results collected from different simulators demonstrate that the proposed control enables the robots to converge around the boundary of the animals and guide them toward the designated goal.**

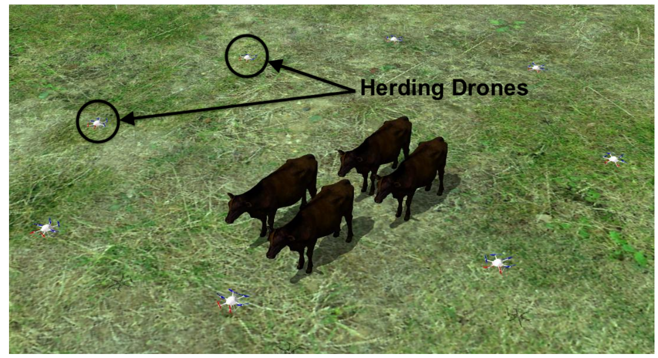

Fig. 1. The surrounding phase of our decentralized control strategy for the group of robots (drones) to herd animals (cows).

I. INTRODUCTION

Existing research in multi-robot surrounding and herding often needs knowledge of a global center of mass (GCM). Robots track the positions of all target animals and determine the GCM of the entire group. The herding process is then conducted by gathering animals that are far away from the herd's GCM. Once all animals are gathered, the robots proceed with driving the herd towards a target indirectly, by positioning themselves at strategic locations that can trigger a reaction from the animals. This herding strategy resembles how animals are herded in real life, which has the advantage of requiring significantly fewer robots than animals. However, this approach comes with a few drawbacks. First, calculating a GCM of the animals usually requires all the robots in the system to have access to a centralized tracking mechanism, which mandates a centralized coordinating architecture on the multi-robot system. Secondly, strategies that utilize a GCM are inefficient as the robots must regularly alternate between herding and collecting to ensure the animals stay together as a cohesive group [1].

This paper presents a decentralized control strategy to herd a group of animals, which aims to address the aforementioned issues with the traditional centralized GCM-based method. The strategy consists of two main phases, an encircling phase and a driving phase. In the encircling phase, an artificial potential field function is utilized as a force that propels each individual robot along the perimeter group of the animals, resulting in encircling behavior that envelops the entire group. From this encirclement, it is possible to induce the GCM of the animal group in a decentralized manner. This surrounding phase is similar to the work introduced in [2],[3], where one group of agents approaches and surrounds another group of different agents. Much like the scenario presented in Pierson's work [4], this situation can be characterized as a non-cooperative multi-robot problem. In this context, the robots' primary goal is to guide the animals, albeit without encountering any active resistance or inclination from the animals themselves. An analytical proof and supporting empirical results are also provided to demonstrate the convergence property of the system under this control rule.

The contribution is in decentralized multi-phase animal herding, introducing a novel approach based on local observations and robot communication without the reliance on centralized coordination. Decentralized approaches have been widely adopted in recent years for their redundancy and robustness to adversaries, compared to centralized approaches [5]. Finally, unlike alternative herding strategies, the animals in this work exhibit a behavioral model that mimics how animals behave in real life.

II. LITERATURE REVIEW

The idea of herding and steering a herd of animals using a group of robot herders was first introduced by [6]. The authors demonstrated that employing multiple shepherds can improve control over the herd's motion, providing a general strategy for the herding task. Subsequently, researchers have extensively studied the natural behaviors between predators and prey for the purpose of herding animals, [7], [1], [8]. The predator-prey approach is intuitive and straightforward to implement as the interaction can be modeled as attraction

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and repulsion forces. However, a major drawback is that it does not guarantee convergence in finite time as the flock may disperse heavily under the presence of the robots.

To address the issue of convergence, researchers have reformulated the original shepherding problem to incorporate aspects of a multi-robot system formation control problem. The general idea is similar to [6], where a group of robots forms a particular formation that completely surrounds the herd. Based on that idea, a herding strategy was proposed that positions the herders in a circular formation around the herds, to guide them using a reduced unicycle dynamic model [4]. A different strategy, proposed in [9], [10], [11], [12], [3], enforces particular formations around the herds with non-traversable virtual linkages between agents to force the herds to follow the motion of the herders. Although the formation control problem solves the missing convergence property of the prey/predator interaction method, it generally requires a significantly larger number of robots to perform the task. Another drawback of this approach is that the robots require the state of the herd's GCM, which can only be obtained through complex robot coordination or a centralized provider.

Furthermore, it is necessary to consider the collective behavior of the herded animals. Within the research community, there is a general consensus that animals exhibit flocking behavior, which arises from three fundamental rules: separation, alignment, and cohesion [13]. Variations of the original model have been proposed, utilizing potential fields and consensus theory to model the aforementioned rules [14], [15]. Notably, [16] extends the original concept proposed by Reynolds, presenting a theoretical and mathematical analysis that includes a proof of stability and convergence properties for the original model.

III. PROBLEM FORMULATION

Consider a 2-D empty space free of obstacles, where there are M robots and N animals. Each robot/animal is a holonomic agent with a limited sensing range and restricted inter-agent communication within a specified radius. The robots are also capable of detecting and measuring the position and velocity of all the animals, as well as broadcasting information to any robot within their sensing range. The animals are considered able to follow predefined interaction rules and react to the influence of other robots. It is presumed that the robot's maximum speed and sensing range are at least equal to those of the animals. The multi-robot herding problem can be defined as follows: Determine the control strategy to guide a group of M robots to track, surround, and drive a group of N animals from their initial location toward a specified location.

IV. GENERAL MODEL OF A HERD

Livestock are social animals that evolved to gather as a cohesive group (flocking) for protection against predatory threats. As such, it is reasonable to assume that animals follow this behavior model, within the scope of this research. The three flocking rules of separation, alignment, and cohesion can be mathematically modeled and are summarized in the following equation [16]:

$$u_i^{\alpha} = \sum_{j \in N_i} \phi_\alpha \left(\|x_j - x_i\|_\sigma \right) n_{ij} + \sum_{j \in N_i} a_{ij}(x)(\dot{x_i} - \dot{x_j}) \quad (1)$$

where ϕ_α describes a pairwise potential function between two agents, i and j, based on their respective position, x. This function is designed to have a global minimum of 0 when the distance between the pair reaches the value d. n_{ij} represents the unit vector along the line connecting agent i and j. The first term of (1) enforces the separation rule mathematically, where animals must respect each other's living space. The second term, $a_{ij}(x)$, is a function defining the spatial relationship between pairs of agents. The effect of this term is associated with alignment or velocity matching.

The rule of cohesion encompasses livestock behavior associated with forming groups as a protective mechanism from predatory threats. To exhibit such behavior, a variant of the local crowded horizon approach is employed. A mathematical model of the selfish herd theory is implemented. In this approach, an animal has the tendency to move toward the nearest animal group within its sensing range whilst avoiding predators. Even though this rule appears to be simple, it captures the overall notion of the selfish herd theory and creates the most realistic representations of an animal herd under external threats [17]. The equation for the gamma term can be described as follows:

$$\begin{aligned} u_i^{\gamma} = & \sum_{j \in N_i} \frac{1}{1 + k\|x_i - x_j\|} \frac{x_i - x_j}{\|x_i - x_j\|} \\ & + \sum_{r \in M_i} \frac{1}{\|x_i - x_r\|^2} \frac{x_i - x_r}{\|x_i - x_r\|} \end{aligned} \quad (2)$$

where x_i and x_r are the positions of animal, i, and robot, r, respectively. The set of neighboring herd members and robots within the sensing range of animal, i, are N_i and M_i.

V. MULTI-PHASES DECENTRALIZED HERDING STRATEGY

A. Surrounding Behavior

A novel potential function of a robot is proposed to describe the interaction force between itself and another agent, whether it be an animal or a neighboring robot within its sensing range.

$$\begin{aligned} f(x_i) &= f_{attract}(x_i, x_j) - f_{repulse}(x_i, x_j) \\ &= \frac{a(1 - \exp(-\frac{\|x_i - x_j\| - d}{c}))}{1 + \|x_i - x_j\|}(x_i - x_j) \end{aligned} \quad (3)$$

where x_i and x_j represent the positions of robot, i, and agent, j, respectively, which can either be a neighboring robot or an animal. d is the desired distance between i and j, where the upper bound of d is set to be no more than the sensing range of the robots, while the lower bound value changes depending on whether agent j is an animal or another robot. For robot-animal interaction, the animals possess a specific radius r, that causes them to react and move under the influence of the robots. As such, it is necessary that $d < r$ as the control strategy exploits this

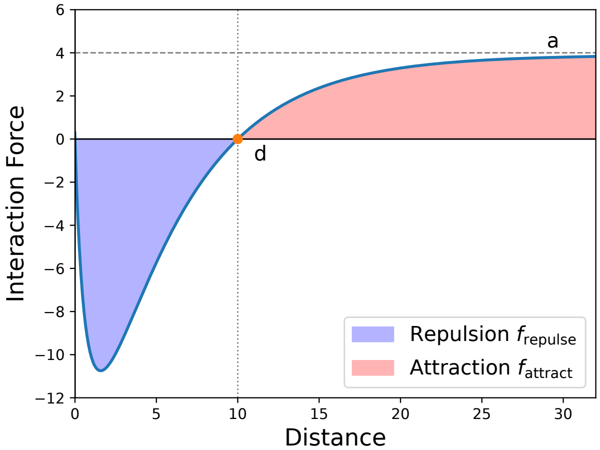


Fig. 2. The 2D graph of equation 3

constraint to enable the prey-predator behavior between the animals and robots. a and c are parameters that describe the magnitude of the attraction and repulsion forces, respectively. The potential function possesses a unique property of being an odd function, meaning that $f(-y) = -f(y)$. This property allows 3 to converge and remain stable around $x = d$ [18], as demonstrated as follows.

Theorem 1: Consider a multi-agent interaction model described in the equation 3 with the attraction and repulsion function $f(.)$. As $t \to \infty$, $x \to \Omega$.

$$\Omega = \{x_i : \|x_i - x_j\| = d\} \tag{4}$$

where d is a predefined distance from robot i to agent j.

Proof: Let $e_{ij} = x_i - x_j$ and the following Lyapunov function:

$$V(x) = \frac{1}{2}(e_{ij})^T(e_{ij}) \tag{5}$$

Taking the time derivative of $V(x)$ and with further manipulation, the upper bound of $\dot{V}(x)$ can be obtained:

$$\dot{V}(x) \leq -\sum_{j=1}^{M} \frac{1}{1 + \|e_{ij}\|} a(1 - exp(-\frac{\|e_{ij}\| - d}{c}))\|e_{ij}\|^2 \tag{6}$$

At $t = 0$, $\|e_{ij}\| \in (d, +\infty)$, also within this interval, $exp(-\frac{\|e_{ij}\|-d}{c})$ is a decreasing function with a unique minimum at $\|e_{ij}\| = d$. Thus, the Lyapunov function becomes:

$$\dot{V}(x) \leq 0 \tag{7}$$

From LaSalle's invariance theorem, as $t \to \infty$, the state x_i converges to the set defined in 4. This means that given enough time, the robot, i, should converge to a position where the relative distance between itself and agent, j, is equal to d. ■

Theorem 2: Consider a multi-agent interaction model described in the equation above with the attraction and repulsion function $f(.)$. As $t \to \infty$, $x \to \Omega$.

$$\Omega = \{x_i : \dot{x_i} = 0\} \tag{8}$$

Proof: Let $e_{ij} = x_i - x_j$ and the following Lyapunov function:

$$J(x) = \frac{1}{2} \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} a \Bigg[-\ln(\|e_ij\| + 1) + \frac{ce^{\frac{d-\|e_ij\|}{c}} \|x\| + \mathrm{E}_2\left(\frac{\|e_ij\|+1}{c}\right) ce^{\frac{d+1}{c}}}{\|e_ij\| + 1} + \|e_ij\| \Bigg] \tag{9}$$

where E_2 is the exponential integral operation. The gradient of $J(x)$ with respect to each x^i is $\nabla_{x_i} J(x) = -\dot{x_i}$. From that, taking the time derivative of $V(x)$, leads to the following phase:

$$\dot{J}(x) = [\nabla_x J(x)]^T \dot{x_i} = -\sum_{i=1}^{M} \|\dot{x_i}\|^2 \leq 0 \tag{10}$$

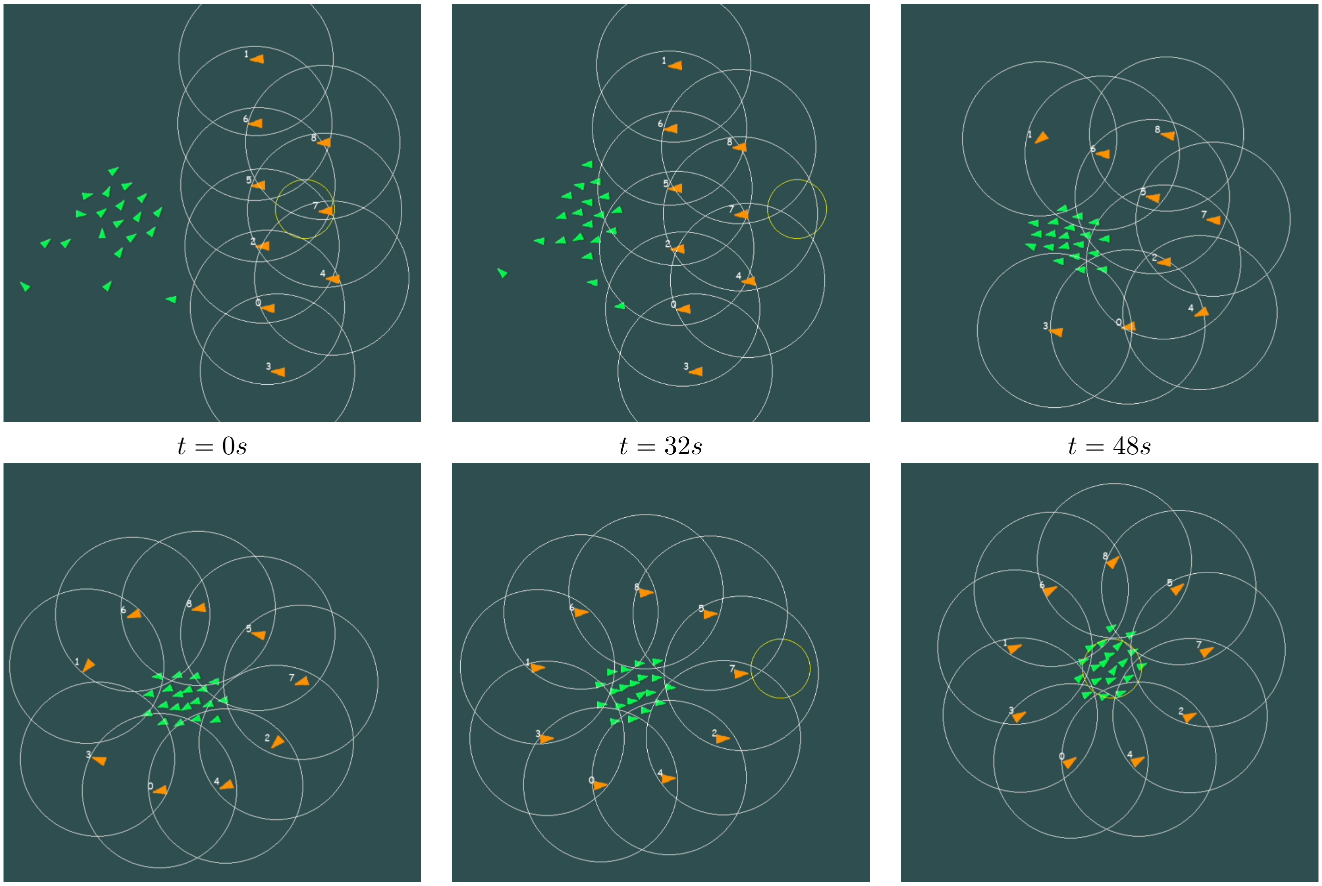
From LaSalle's invariance theorem, as $t \to \infty$, the state x_i converges to the set defined in 8. Thus, as a robot, i approaches a position where its relative distance to the neighboring agent, j, is equal to d, the robot will stop moving and remain stable. ■

From the two analyses shown above, it is reasonable to conclude that given enough time, under the effect of the proposed attraction/repulsion function, 3, an individual robot converges to a location where the relative distance between itself and the nearest neighboring agent is d. After convergence, the robots remain stable at that location and do not oscillate. From these characteristics, it is possible to have a group of robots track and surround a herd of animals by utilizing two distinct interactions based on 3. These interactions describe the robot-to-animal interaction and the robot-to-robot interaction, respectively, which are given as follows:

$$\begin{aligned} u_i &= -\left[C_a \sum_{j=1}^{j \in N_i} f(x_i, x_j) + C_r \sum_{k=1}^{k \in M_i} f(x_i, x_r) \right] \\ &= C_a f_a(x_i) + C_r f_r(x_i) \end{aligned} \tag{11}$$

where C_a and C_r are weighting parameters for animal-robot and robot-robot interaction, respectively. f_a describes the animal-robot potential field with distance d_s, while f_r describes the robot-robot potential field with distance d_r. u_i stands for the target velocity vector of the robot i. The encircling behavior is a product of the combination of two distinct potential fields 3. Furthermore, it is imperative for $C_a \leq kC_r$ for the surrounding behavior to occur. This relationship implies that the robots must pay more attention to the animals than their peer robots to enable them to approach and interact with the herd. If $C_a \leq kC_r$, then the attraction forces from the animals are not strong enough to trigger the robot's surrounding behavior.

Equation 11 does not take into consideration the actions of animals as they are approached by robots, which is described in 2. Consequently, the robots may not be able to successfully encircle the animals as they would flee away if the robots



$t = 0s$ $t = 32s$ $t = 48s$

$t = 56s$ $t = 62s$ $t = 105s$

Fig. 3. Herding with 9 robots and 20 animals at various time steps. Animals are represented as green triangles, while robots are depicted as orange triangles. The sensing range for each robot is plotted as a white circle. The yellow circle illustrates the position and radius of the target location where the robots have to drive the animals. At $t = 0s$, the robots start to approach the herd. As they come closer, members of the herd start moving away to maintain a certain distance from the incoming threat, as observed at $t = 32s$. The robots then quickly converge around the edge of the herd, as shown during $t = 48s$, before completely surrounding the herd at $t = 56s$. The robots then guide the animals toward the target location, as shown at $t = 62s$. The herding behavior stops at $t = 105s$ when the animals are all positioned within the target location specified by the yellow circle.

come too close. Therefore, a velocity-tracking controller must be incorporated for the robots to track the movement of the animals.

$$\begin{aligned} u_i &= C_a f_a(x_i) + C_r f_r(x_i) + C_v \frac{1}{N_i} \sum_{j=1}^{j \in N_i} v_j \\ &= C_a f_a(x_i) + C_r f_r(x_i) + C_v f_t \end{aligned} \tag{12}$$

where C_v represents the velocity gain of the average velocity vector of all animals within the sensing range of robot i. The average velocity is sufficient to drive the tracking controller of the robots as it is assumed that the animals will try to follow the velocity of the members around them, as described in 1. This additional tracking term is derived from the idea of the velocity consensus behavior of animals.

B. Metrics for analyzing the surrounding behavior

To analytically and quantitatively evaluate the surrounding behavior of a robot under the proposed control strategy, four distinct metrics of distribution evenness, distribution range, the distance between the two centroids, and isolation rate are utilized. First, the distribution evenness describes the variances of the distance between the robot and the nearest animal and two other neighboring robots. This metric is used to identify whether the robot has converged to a stable location with respect to both the animals and the neighboring robots. Next, the value of the distribution range is defined as the distance between the group's centroid and the outermost member of that group. This metric is used to measure the overall spread of one specific group. Finally, the distance to the herd's centroid represents the distance between the robots' centroid and the herd's centroid. This is an important metric as it shows that under the effect of 12, the centroid of the robots should approach and overlap the animal's centroid, indicating successful surrounding behavior.

C. Herding Behavior

The robots begin with the surrounding phase, in which they are attracted toward the animals under the effect of 3. As the robots approach and gradually surround the animals, they determine their local distribution evenness and publicly share this information with the neighboring robots within their vicinity. Once a consensus of distribution evenness is reached, the robots transition from the surrounding phase to

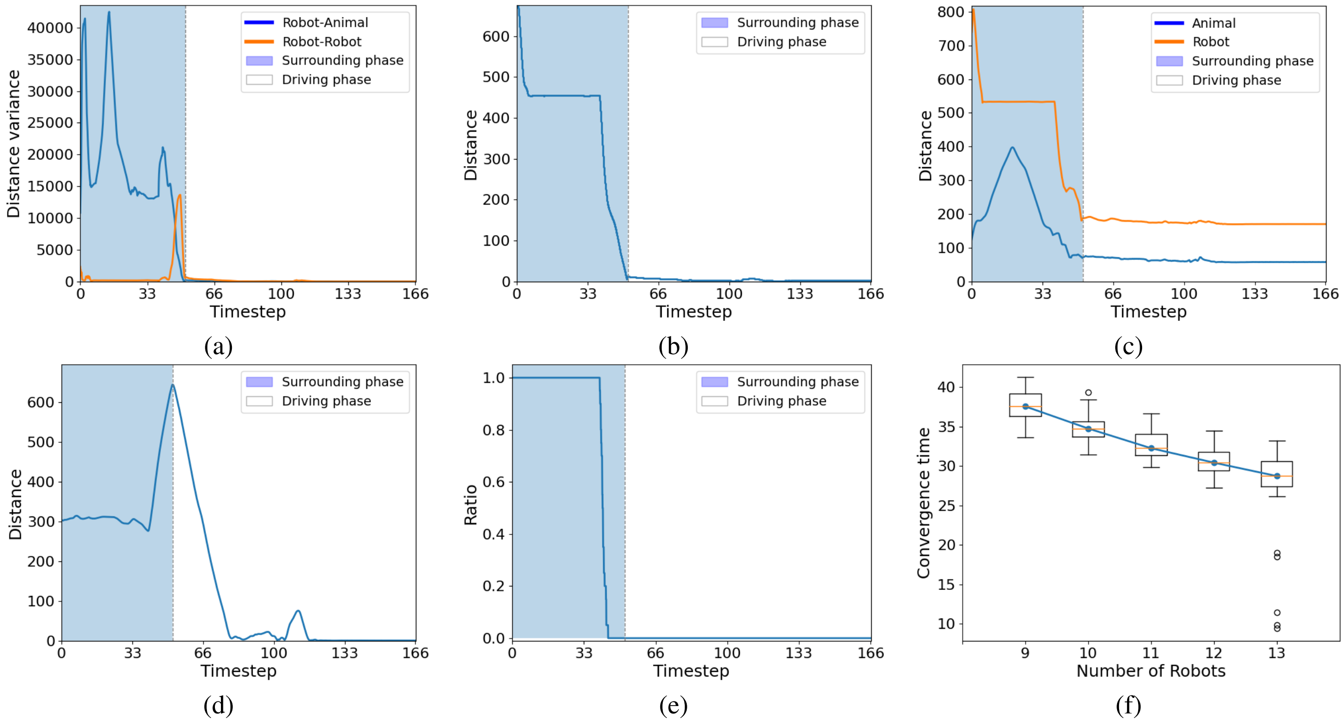


Fig. 4. Results of collecting a herd of 20 members using a group of 8 robots (a to e) and M robots (f); (a) The distribution evenness of the robots; (b) The distance between the herd's centroid and the robot's centroid; (c) The distribution range of both the herd and the robot; (d) The distance between the herd's centroid to the target location; (e) The isolation rate of the animals; (f) The relationship between convergence time and number of robots

the driving phase. During the driving phase, an additional proportional controller between the centroid of all robots and the target location is added, as shown in equation 12, enabling the robots to guide the animals toward the target location.

$$u_i = C_a f_a(x_i) + C_r f_r(x_i) + C_v f_t + C_d\,(x_c - x_t) \quad (13)$$

where x_c and x_t represent the centroid of all robots and the target location, respectively. It is sufficient to use the centroid of the robots as the reference for the added proportional controller. After the surrounding phase, the centroid of the robots should overlap the centroid of the animals.

VI. EXPERIMENT AND RESULTS

Simulations for the proposed work have been conducted extensively in a 2D simulator, shown in Fig. 3, and Gazebo depicted in Fig. 5. The setup for both cases is in an open space scenario where the animals are located around a randomized initial area. The robots, on the other hand, are randomly positioned at a different location some distance away from the animals. The robots then approach, surround and drive the animals from their initial locations to another location.

All metrics for analyzing the surrounding behaviors are collected alongside the distance between the centroid of the animals and the target. The simulations are conducted to provide empirical support for the analytical proof of the convergence and stability properties of the artificial potential field function, as well as to find the relationship between the convergence time and the robot population. Convergence time is defined as when the distribution evenness of all robots reaches consensus. The animals are initialized to have the same sensing range of 17 m, a maximum speed of 5 m/s, as well as a personal space radius of 1 m. Conversely, the robots start the simulation with a sensing range of 20 m and a maximum speed of 7.5 m/s. The parameters a and c in 3 are set to be 10 and 8, for both robot-animal and robot-robot interaction, while d_s and d_r are 4 m and 3 m, respectively.

Figure 4(a) shows the distribution evenness of robot to robot and robot to animal. It can be observed that the evenness between robot and animal fluctuates and remains high as the robots approach the animal during the surrounding phase due to the differences between the two initial locations. As the surrounding phase happens and the robots gradually encircle the animal herd, this value decreases until it reaches a minimum value of 0. On the other hand, the distribution evenness between a pair of robots remains stable during this process. It only increases slightly as the robots spread out to encircle the animals completely. The surrounding phase ends and transitions to the driving phase when both distribution evenness reaches their respective minimum values and stabilizes, indicating a successful surrounding action. This transition is shown in 4(b), where the centroid of the robots gradually converges and then overlaps with the centroid of the animals. Furthermore, during the entire process, the distribution range of the robots is always larger than that of the animals, as indicated in Fig. 4(c). Based on the collected data, the empirical results support the aforementioned theorems on the convergence and stability properties of the proposed potential function. Figure 4(d)

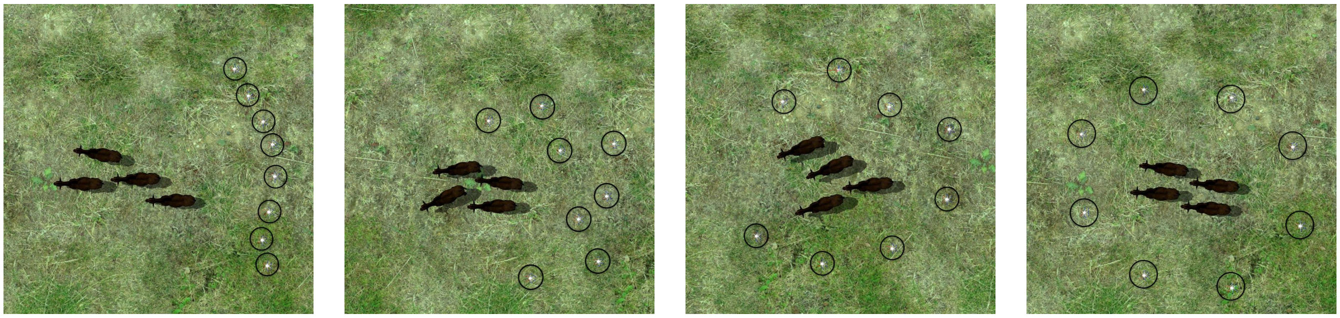


Fig. 5. Gazebo simulation of collecting 4 animals using 8 drones. Similar to the 2D simulation, the drones start at the same location, away from the animals. As they approach, the animals move away to avoid the drones while staying together as a cohesive group. The drones then chase the animals while simultaneously spreading out to surround them.

illustrates the distance between the herd's centroid to the target over time. The figure shows that the distance value oscillates during the surrounding phase as the robots chase and surround the animals. After that, it reduces gradually toward 0, indicating that the robots have successfully driven the animals toward the target location. Figure 4(e) shows the isolation rate of the animals over the course of the herding process. It can be observed from the graph that the isolation rate reaches 0 before the system transitions from the surrounding phase to the herding phase. Finally, the results from Fig. 4(f) illustrate the relationship between the number of robots and the convergence time of the system given a fixed animal population (20 animals). It can be observed that there is a proportionally inverse relationship between the number of robots and the convergence time of the system, indicating that more robots lead to more time-efficient execution.

VII. CONCLUSIONS

This paper introduces a novel decentralized control strategy for a group of robots to herd a group of animals, which is a combination of a surrounding phase and a driving phase. In the surrounding phase, a custom artificial potential field is employed to ensure the robots encircle the animals while maintaining a safe distance from other neighboring robots. Unlike most previous works, the control strategy does not require any global measurement of either the robots or the animals. It relies solely on what the robots sense within their sensing range. Mathematical proofs for the convergence and stability properties of the control strategies are provided, which are supported by the empirical results collected from various simulations. It is also evidently clear that more robots lead to the efficient and timely execution of the herding task. Future work includes the validation of the performance of the approach in real robotic experiments with real cows and sheep on farms. It would be interesting to see how real cattle animals react and move in the presence of the drones.

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