

# Stable, Safe, and Passive Teleoperation of Multi-Robot Systems

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**Abstract**—In this paper, we present a unified framework to ensure the stability, safety, and passivity of a multi-robot teleoperation system in a holistic fashion. The proposed approach consists of encoding these three properties as constraints in an optimization-based controller using control Lyapunov and (integral) control barrier functions. The result is a stability-safety-passivity (SSP) filter implemented as a convex optimization control policy, which can be efficiently evaluated in an online fashion. The developed filter minimally modifies the teleoperation input in order to ensure that the robotic system remains stable, safe, and passive. The effectiveness of the developed approach is showcased using a team of mobile robots in a human-multi-robot teleoperation scenario.

## I. INTRODUCTION

In the teleoperation of multi-robot systems, a human operator and a team of robots can be modeled as interconnected systems that interact by exchanging energy through power ports [1]. The dynamics of the human operator, as well as those of the robots and the communication channel between them can lead to unpredictable behaviors of the interconnected system. In these settings, it is convenient to analyze the human-multi-robot teleoperation systems from an energetic point of view, which entails keeping track of the energy that the interconnected systems exchange between each other. Along this line of inquiry, passivity-based approaches to deal with the control of interconnected systems have been developed and successfully implemented [2], which demonstrated to be suitable in many application domains, ranging from telemanipulation [3] to the teleoperation of multi-robot systems [4].

Rich behaviors of multi-robot systems can be encoded by means of *stability* conditions. In [5], the authors show how different coordinated controllers for teams of robots can be achieved by means of an appropriately defined energy function. Moreover, asymptotic stability of a system to be teleoperated can be used to ensure boundedness of its state in response to teleoperation inputs [6]. For these reasons, stability is paramount in human-multi-robot teleoperation.

Besides stability, *passivity* is another amenable property when dealing with interconnected systems, as it ensures that the energy generated by the system does not exceed the one injected through its input. Energy injected through external inputs can make a system become non-passive, as illustrated in [7], and passivity theory allows us to analyze dynamical systems from an energetic point of view, so it is a suitable design tool for interconnected systems and, therefore, for robot teleoperation systems. Passivity is even more important

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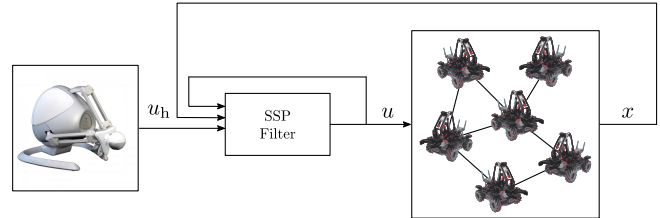


Fig. 1. Stability, Safety, and Passivity (SSP) filter. A coordinated team of robots interacts with a human teleoperator by means of the human input  $u_h$  supplied via a haptic device. Its stability, safety, and passivity properties are ensured thanks to a convex optimization control policy that effectively filters the human input before passing it to the multi-robot system.

when the presence of communication delays between the human teleoperator and the multi-robot system is not negligible [8]. In fact, in the worst case, the effect of delays can be a cause of instability of the system [9]. Passivity-based and other energy-based methods have been employed in a variety of applications for the control of robotic systems [7], [10], [11], [12], [13]. In particular, in [14], the authors introduce the concept of energy tanks, which are then extended in [15], [16], [17]. These works consider additional dissipative forces on a teleoperation system that has to kept passive in order to prevent energy tanks from depleting, so as to keep a positive passivity margin, representing the energy dissipated by the system over time.

Finally, apart from stability and passivity, in many teleoperated applications, it is also desirable to ensure the *safety* of the system, intended as the forward invariance of a subset of the system state space. This is of particular interest when robotic systems interact or collaborate with humans in order to perform a task. The safety of the human operator can be framed as a forward invariance property of a subset of the robot state space, and this property can be efficiently enforced by leveraging control barrier functions (CBFs) [18], a control-theoretic tool developed to ensure safety of dynamical systems.

In this paper, we present a holistic approach to enforce stability, safety, and passivity of human-multi-robot teleoperation systems. We do so by defining a stability, safety and passivity (SSP) filter (see Fig. 1), whose goal is that of minimally modifying the human input to the robotic system in order to render the latter stable, safe, and passive. As will be explained in the following sections, the net effect of the filter is that of effectively modifying the structure of the system itself in order to ensure that these three fundamental properties hold. The proposed approach is flexible, insofar as it is agnostic to the teleoperation input, and it introduces

only a minimal computational overhead, as it entails solving a convex optimization program, making the approach amenable for the real-time implementation in many robotic applications.

To summarize, the main contributions of this paper with respect to the state-of-the-art are the following:

- (i) Development of a framework that allows to enforce the passivity of robotic systems pointwise in time, rather than at an integral level as in [13], [27], in a minimally-invasive fashion, i.e. by altering the desired robot behavior as little as possible. This framework, implemented as the SSP filter, is designed leveraging the recently developed integral control barrier functions;
- (ii) Introduction of stability-like conditions in the SSP filter to allow for the execution of rich behaviors of robotic systems;
- (iii) Provable guarantees on the behavior of the robotic system executing a controller filtered using the SSP filter.

## II. MATHEMATICAL BACKGROUND

Throughout this paper, we consider a multi-robot system comprised of  $N$  robots, each of which is modeled by the following nonlinear control system:

$$\begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = g_i(x_i) \end{cases}$$

where  $i = 1, \dots, N$ ,  $x_i \in \mathbb{R}^{n_i}$ ,  $u_i \in \mathbb{R}^{m_i}$  and  $y_i \in \mathbb{R}^{m_i}$  are the state, input and output, respectively, of robot  $i$ ,  $f_i: \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{n_i}$  is a Lipschitz continuous vector fields and  $g_i: \mathbb{R}^{n_i} \rightarrow \mathbb{R}^{m_i}$ . Notice how this modeling choice allows us to account for the case of heterogeneous multi-robot system. For simplicity of notation, we introduce the vectors  $x = [x_1^T, \dots, x_N^T]^T$ ,  $u = [u_1^T, \dots, u_N^T]^T$ , and  $y = [y_1^T, \dots, y_N^T]^T$  to denote the compound state, input, and output of the multi-robot system. Similarly  $f$  and  $g$  are defined as follows:

$$f(x, u) = \begin{bmatrix} f_1(x_1, u_1) \\ \vdots \\ f_N(x_N, u_N) \end{bmatrix}, \quad g(x) = \begin{bmatrix} g_1(x_1) \\ \vdots \\ g_N(x_N) \end{bmatrix},$$

so that

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x) \end{cases} \quad (1)$$

To account for the teleoperation input, we consider the input  $u$  to be split as follows:

$$u = u_c + u_h, \quad (2)$$

where  $u_c$  denotes an input signal used to achieve a desired coordinated behavior (e.g. via a state feedback control design), and  $u_h$  represents the teleoperation input given by the human.  $u_h$  can be used to realize different human-multi-robot interaction strategies, including individual control of all the robots or control of a leader robot.

As we are interested in provably guaranteeing stability, passivity and safety properties of the multi-robot system, in

the following we briefly introduce these three concepts. First, we recall that the origin of the system (1) is *exponentially stable* if and only if the linearization of  $f$  at the origin is Hurwitz [6]. The following theorem provides a computationally amenable way of ensuring the stability of a system by means of an appropriately defined Lyapunov function  $V$ .

**Theorem 1** (Exponential stability [6]). *Let  $f$  in (1) be locally Lipschitz over a domain  $\mathcal{D} \subset \mathbb{R}^n$ ,  $0 \in \mathcal{D}$ . Let  $V$  be a continuously differentiable function such that*

$$\begin{aligned} k_1 \|x\|^a \leq V(x) \leq k_2 \|x\|^a \\ \dot{V}(x, u) \leq -k_3 \|x\|^a \end{aligned} \quad (3)$$

for all  $x \in \mathcal{D}$ , where  $k_1, k_2, k_3, a$  are positive constants. Then, the origin is an exponentially stable equilibrium point.

As argued in the previous section, besides stability, we would like the teleoperation system to be passive. In this paper, we will adopt the following, pointwise, definition of passivity, compared to [13] which employs its integral definition in order to design robot controllers.

**Definition 2** (Passivity [6]). *The system (1) is passive if there exists a continuously differentiable positive definite storage function  $S: \mathbb{R}^n \rightarrow \mathbb{R}$  such that, for all  $x$  and  $u$ ,*

$$\dot{S} = \frac{\partial S}{\partial x} f(x, u) \leq u^T y.$$

While the definition of passivity is a statement about the system—which holds for all possible values of the input  $u$  and the output  $y$ —safety is tied to the definition of a *safe set*, i.e. a subset of the state space of the system we want to render forward invariant. Control barrier functions (CBFs) are a technique which demonstrated to be effective in ensuring safety in a variety of robotic systems and scenarios [19], [20], [21], [22], [23]. In the following, we introduce the definition of CBFs and the main result used in this paper to ensure controlled forward invariance.

**Definition 3** (Control Barrier Functions (CBFs) [18]). *Let  $\mathcal{C} \subset \mathcal{D} \subset \mathbb{R}^n$  be the zero superlevel set of a continuously differentiable function  $h: \mathcal{D} \rightarrow \mathbb{R}$ . Then  $h$  is a control barrier function if there exists an extended class  $\mathcal{K}_\infty$  function  $\gamma$  such that, for the system (1),*

$$\sup_{u \in \mathbb{R}^m} \{L_f h(x, u) + \gamma(h(x))\} \geq 0. \quad (4)$$

for all  $x \in \mathcal{D}$ .

The notation  $L_f h(x)$  denotes the Lie derivative of  $h$  along the vector field  $f$ . Given this definition of CBFs, the following theorem highlights how they can be used to ensure set forward invariance (safety).

**Theorem 4** (Safety [18]). *Let  $\mathcal{C} \subset \mathbb{R}^n$  be a set defined as the zero superlevel set of a continuously differentiable function  $h: \mathcal{D} \subset \mathbb{R}^n \rightarrow \mathbb{R}$ . If  $h$  is a CBF on  $\mathcal{D}$  with  $0$  a regular value, then any Lipschitz continuous controller  $u(x) \in \{u \in \mathbb{R}^m: L_f h(x, u) + \gamma(h(x)) \geq 0\}$  for the system (1) renders the set  $\mathcal{C}$  forward invariant (safe).*

The condition of passivity recalled in Definition 2 involves the control input  $u$ . Recently introduced integral CBFs (I-CBFs) [24], which generalize control dependent CBFs [25], can be leveraged to enforce passivity conditions by means of a constraint on  $u$ . In order to take advantage of I-CBFs, the system (1) needs to be dynamically extended as follows:

$$\begin{cases} \dot{x} = f(x, u) \\ \dot{u} = \phi(x, u, t) + v \\ y = g(x) \end{cases} \quad (5)$$

where  $v \in \mathbb{R}^m$  is the new control input and  $\phi: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^m$  will be designed to ensure that  $u = u_c + u_h$ , as in (2), as desired. Here,  $n = \sum_{i=1}^N n_i$  and  $m = \sum_{i=1}^N m_i$ . We now have the necessary constructions to introduce I-CBFs.

**Definition 5** (Integral Control Barrier Functions [24]). *For the system (1), with corresponding safe set  $\mathcal{S} = \{(x, u) \in \mathbb{R}^n \times \mathbb{R}^m : h(x, u) \geq 0\} \subset \mathbb{R}^n \times \mathbb{R}^m$  defined as the zero superlevel set of a function  $h: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$  with 0 a regular value. Then,  $h$  is an integral control barrier function if for any  $(x, u) \in \mathbb{R}^n \times \mathbb{R}^m$  and  $t \geq 0$ :*

$$\frac{\partial h}{\partial u}(x, u) = 0 \implies \frac{\partial h}{\partial x}(x, u)f(x, u) + \gamma(h(x, u)) \geq 0. \quad (6)$$

The implication in (6) guarantees that, by means of an I-CBF  $h$ , the inequality  $\dot{h}(x, u) + \gamma(h(x, u)) = \frac{\partial h}{\partial x}(x, u)f(x, u) + \frac{\partial h}{\partial u}(x, u)\phi(x, u, t) + \gamma(h(x, u)) \geq 0$ —which, by the comparison lemma [6], guarantees the forward invariance of the set  $\mathcal{S}$ —can always be satisfied by a proper choice of  $\phi(x, u, t)$ . With the definition of I-CBFs, we concluded the introduction of the control-theoretic tools employed in the next section to design an input filter to render a dynamical system stable, safe, and passive.

### III. STABILITY-SAFETY-PASSIVITY FILTER

In this section, we develop the SSP filter, whose structure is broken down into four components:

- 1) Stability-preserving component (Section III-A)
- 2) Safety-preserving component (Section III-B)
- 3) Passivity-preserving component (Section III-C)
- 4) Tracking controller (Section III-D)

These four components will be then combined in Section III-E where Proposition 10 is stated, which ensures that the designed SSP filter achieves all the desired properties.

#### A. Ensuring Stability

Stability conditions can be employed to encode consensus-like [5] or more complex coordinated behaviors [26] of multi-robot systems. In order to ensure exponential stability of the multi-robot system, we need to enforce the condition  $\dot{V}(x, u) \leq -k_3 \|x\|^a$  given in (3),  $k_3$  and  $a$  being positive constants. However, since this inequality does not contain the input of the dynamically extended system,  $v$ , it cannot be enforced by an appropriate choice of  $v$ . This is equivalent to a relative-degree-2 condition on the control Lyapunov function  $V$ . To circumvent this issue, we can proceed as in [23] and define the auxiliary control barrier function

$h_V(x, u) := -\dot{V}(x, u) - k_3 \|x\|^a$ . Then, in order to ensure the stability of the system, we may select any control input  $v$  in the following set:

$$K_V(x, u) = \{v \in \mathbb{R}^m : L_f h_V(x, u, v) + \gamma_V(h_V(x, u)) \geq 0\}. \quad (7)$$

#### B. Ensuring Safety

Assume we would like the state of the system to remain within the safe set  $\mathcal{S}_x$  defined as the zero superlevel set of a continuously differentiable function  $h_x$ , i.e.

$$\mathcal{S}_x = \{x \in \mathbb{R}^n : h_x(x) \geq 0\}. \quad (8)$$

The condition that the state  $x$  of the system (1) remains confined in  $\mathcal{S}_x$  for all times  $t$ —corresponding to safety—can be enforced using the result in Theorem 4.

However, if  $h_x$  has relative degree 1 with respect to the input  $u$ —i.e. the time derivative  $\dot{h}_x$  explicitly depends on  $u$ —then it has relative degree 2 with respect to the input  $v$ , based on the dynamic extension introduced in (5). In order for Theorem 4 to be applicable to the system (5), we need  $L_f h_x(x, u)$  to depend on the input  $v$ , a condition that does not hold if the relative degree of  $h_x$  with respect to  $v$  is greater than 1.

Proceeding as in the previous section, following the strategy adopted in [24], let  $h'_x(x, u) := \dot{h}_x(x, u) + \gamma(h_x(x))$ . Since  $h'_x$  depends on  $u$ ,  $L_f h'_x$  depends on  $v$ . Then, in order to ensure the safety of  $\mathcal{S}_x$ , we may choose any control input  $v$  satisfying the inequality  $L_f h'_x(x, u, v) + \gamma_x(h'_x(x, u)) \geq 0$ . This way, by Theorem 4,  $h'_x(x, u) \geq 0$  for all times, which is equivalent to  $\dot{h}_x(x, u) + \gamma(h_x(x)) \geq 0$  for all times. The repeated application of Theorem 4 shows that  $h_x(x) \geq 0$ , i.e.  $\mathcal{S}_x$  is safe. In order to ensure the safety of the system with respect to the set  $\mathcal{S}_x$ , we may select any control input in the following set:

$$K_x(x, u) = \{v \in \mathbb{R}^m : L_f h'_x(x, u, v) + \gamma_x(h'_x(x, u)) \geq 0\}. \quad (9)$$

#### C. Ensuring Passivity

As pointed out before, as passivity is a condition on the control input  $u$  rather than the state  $x$ , in this paper, we employ the integral version of CBFs to enforce it. The following result, analogous to Theorem 4 for I-CBFs, will be leveraged.

**Theorem 6** ([24]). *Consider the control system (1) and suppose there is a corresponding dynamically defined controller  $\dot{u} = \phi(x, u, t)$ . If the safe set  $\mathcal{S} \subset \mathbb{R}^n \times \mathbb{R}^m$  is defined by an integral control barrier function  $h: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$ , then modifying the dynamically defined controller to be*

$$\dot{u} = \phi(x, u, t) + v^*, \quad (10)$$

where  $v^*$  solves the following quadratic program (QP):

$$\begin{aligned} & \underset{v \in \mathbb{R}^m}{\text{minimize}} && \|v\|^2 \\ & \text{subject to} && \frac{\partial h}{\partial u}(x, u)v + \frac{\partial h}{\partial x}(x, u)f(x, u) \\ & && + \frac{\partial h}{\partial u}(x, u)\phi(x, u, t) + \gamma(h(x, u)) \geq 0 \end{aligned} \quad (11)$$

results in safety, i.e.  $\mathcal{S}$  is forward invariant for the control system (5) with the dynamically defined controller (10), or, in other words, if  $(x(0), u(0)) \in \mathcal{S}$  then  $(x(t), u(t)) \in \mathcal{S}$  for all  $t \geq 0$ .

We now define an I-CBF to ensure the passivity of the system (1). Let  $S: \mathbb{R}^n \rightarrow \mathbb{R}$  be a continuously differentiable positive definite function, and define the following I-CBFs:

$$h_u(x, u) := g(x)^T u - L_f S(x, u). \quad (12)$$

The corresponding safe set  $\mathcal{S}_u$  is defined as

$$\mathcal{S}_u = \{(x, u) \in \mathbb{R}^n \times \mathbb{R}^m : g(x)^T u - L_f S(x, u) \geq 0\}. \quad (13)$$

With this construction, the following theorem is leveraged to ensure the passivity of a dynamical system as a safety condition [27].

**Theorem 7** (Passivity as safety). *If the set  $\mathcal{S}_u$  is forward invariant for the system (5), then the system (1) is passive.*

*Proof.* Assume  $\mathcal{S}_u$  is safe, i.e.  $(x, u) \in \mathcal{S}_u$  for all  $t$ . From (13), one has that  $g(x)^T u - L_f S(x, u) = y^T u - \frac{\partial S}{\partial x} f(x, u) = y^T u - \dot{S} \geq 0$  for all  $t$ . Thus,  $y^T u \geq \dot{S}$  and, by Definition 2, the system (1) is passive with storage function  $S$ , with respect to input  $u$  and output  $y$ .  $\square$

Similarly to what has been done before, the result in Theorem 6 suggests the definition of the following set of control inputs:

$$K_u(x, u) = \left\{ v \in \mathbb{R}^m : \begin{aligned} & \frac{\partial h_u}{\partial u}(x, u)v + \frac{\partial h_u}{\partial x}(x, u)f(x, u) \\ & + \frac{\partial h_u}{\partial u}(x, u)\phi(x, u, t) \\ & + \gamma(h_u(x, u)) \geq 0 \end{aligned} \right\}. \quad (14)$$

By Theorem 6, the safety of  $\mathcal{S}_u$ —and, by Theorem 7, the passivity of (1)—is enforced using the I-CBF  $h_u$  by picking a controller in  $K_u(x, u)$ .

With this result in place, we are now ready to combine stability, safety, and passivity. Before presenting the SSP filter, in the following section we show how to ensure that the dynamically extended system (5) asymptotically behaves as the original system (1).

#### D. Tracking Desired Control Inputs

The dynamic extension of the system (1) proposed in (5) is required in order to enforce constraints on the input  $u$ —the passivity constraints—through a proper choice of  $v$ . On the other hand, due to this extension, we are not able to control the original system (1) using  $u$  anymore, but rather we have to design a suitable function  $\phi$  in (5) in order to track the desired  $u$  using  $v$ . The objective of this section is that of presenting a controller that serves this purpose.

Assume we want  $u = u_c + u_h$  as in (2). As  $\dot{u} = \phi(x, u) + v$ , one could set

$$\dot{u} = \dot{u}_c + \dot{u}_h + v^* = \underbrace{L_f u_c(x, u) + \dot{u}_h + v^*}_{=: \phi(x, u, t)}, \quad (15)$$

where  $v^*$  is the solution of (11). This choice, however, may cause  $u(t)$  to diverge over time more and more from its desired value  $u_c(t) + u_h(t)$ , due to the fact that  $v^*$  that solves (11) is the minimizer of the difference between the time derivatives of the input functions. In fact, from (10),  $\|v^*\| = \|\dot{u} - \phi(x, u, t)\|$ , is the norm of the difference between the derivative of  $u$ —rather than the input function itself—and  $\phi$ . The following theorem presents a dynamically defined control law which results in  $u(t)$  converging to the desired value  $u_c(t) + u_h(t)$  as  $t \rightarrow \infty$  whenever safety is not violated.

**Proposition 8.** *Consider the system (1) and a desired nominal input signal (2). Consider an I-CBF  $h: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}$  defined to ensure the safety of the set  $\mathcal{S} \subset \mathbb{R}^n \times \mathbb{R}^m$  defined as its zero superlevel set. Then, the dynamically defined controller*

$$\dot{u} = \underbrace{L_f u_c(x, u) + \dot{u}_h + \frac{\alpha}{2}(u_c + u_h - u)}_{=: \phi(x, u, t)} + v^*, \quad (16)$$

where  $\alpha > 0$  and  $v^*$  is the solution of (11), will ensure the safety of the set  $\mathcal{S}$ , as well as the tracking of the nominal control signal (2) whenever the controller  $\phi(x, u, t)$  is safe.

*Proof.* First of all, by  $\phi(x, u, t)$  being safe we mean that the constraints in (11) are inactive and, consequently,  $v^*(x, u, t) = 0$ . Then, by Theorem 6, the controller (16) results in the forward invariance, i.e. safety, of the set  $\mathcal{S}$ . Therefore, we only need to confirm that, if the controller (16) is safe, then  $u$  will track the nominal controller (2). To this end, let us consider the following Lyapunov function candidate for the system in (5) with  $\phi(x, u, t)$  given in (15):

$$W(u, x, u_h) = \frac{1}{2} \|u - u_c - u_h\|^2.$$

Its time derivative evaluates to:  $\dot{W} = \frac{\partial W}{\partial u} \dot{u} + \frac{\partial W}{\partial x} \dot{x} + \frac{\partial W}{\partial u_h} \dot{u}_h = (u - u_c - u_h)^T (\dot{u} - L_f u_c(x, u) - \dot{u}_h)$ . Substituting the proposed controller (16), we obtain  $\dot{W} = (u - u_c - u_h)^T (L_f u_c(x, u) + \dot{u}_h + \frac{\alpha}{2}(u_c + u_h - u) - L_f u_c(x, u) - \dot{u}_h) = \frac{\alpha}{2}(u - u_c - u_h)^T (u_c + u_h - u) = -\alpha W(u, x, u_h)$ . Thus,  $W(t) \rightarrow 0$ , or equivalently  $u(t) \rightarrow u_c(t) + u_h(t)$ , as  $t \rightarrow \infty$ , i.e. the input  $u$  will track the desired control signal (2).  $\square$

**Remark 9.** *The variable  $\dot{u}$  only appears in the software implementation of the passive and safe controller for the system (1). The actual input given to the system is its integral  $u(t)$ . Therefore, the value  $\alpha$  in the expression of the dynamically defined controller (16) can be chosen arbitrarily large, being aware of not introducing numerical errors while solving the QP (11). As can be noticed in the proof of Proposition 8, the larger the value of  $\alpha$  is, the faster the convergence of  $u$  to the desired controller (2) when  $v^* = 0$  (i.e. when no safety-related modifications of  $\dot{u}$  are required).*

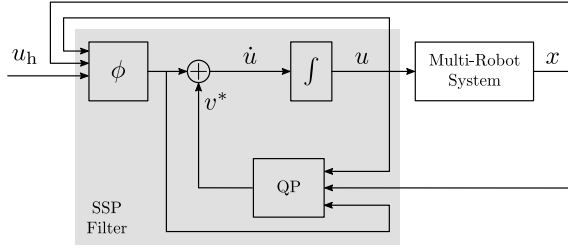


Fig. 2. The structure of the SSP filter depicted in Fig. 1 is detailed using the results of this section of the paper. The SSP filter includes the computation of the function  $\phi$ , whose expression is given in (16), the solution of the convex quadratic program (QP) (17), and an integrator, in order to compute the control input  $u$  supplied to the multi-robot system.

### E. Stability-safety-passivity filter design

In this section, we combine the results of the previous four subsections to design a safety and passivity input filter.

**Proposition 10 (Main result).** *Consider a dynamical system (1), a continuously differentiable control Lyapunov function  $V$  used to ensure its exponential stability, a set  $S_x$  where we want the state  $x$  of the system to remain confined for all times, and a continuously differentiable positive definite function  $S$  with respect to which we want the system to be passive. Let  $K_{Vxu}(x, u) := K_V(x, u) \cap K_x(x, u) \cap K_u(x, u) \subset \mathbb{R}^m$ , and  $K_V(x, u)$ ,  $K_x(x, u)$ , and  $K_u(x, u)$  be given in (7), (9), and (14), respectively. If the controller*

$$v^* = \underset{v \in K_{Vxu}(x, u)}{\operatorname{argmin}} \|v\|^2, \quad (17)$$

*exists for all times  $t$ , then the system (1) is stable, safe, and passive.*

*Proof.* The proof of this proposition is based on the combination of the results of Theorems 1, 4, 6, and 7. If the QP (17) has a solution for all  $t$ , then  $v^* \in K_{Vxu}(x, u)$  for all  $t$ . Then, by Theorem 1, as  $v^* \in K_V(x, u)$ , the origin is exponentially stable. Moreover, as  $v^* \in K_x(x, u)$ , by Theorem 4,  $S_x$  defined in (8) using  $h_x$  is forward invariant, i.e. safe. Furthermore, as  $v^* \in K_u(x, u)$  for all  $t$ , Theorem 6 ensures that  $S_u$  defined in (13) using  $h_u$  is safe. Thus, by Theorem 7, the system (1) is passive.  $\square$

**Remark 11.** *Solving the QP (17) can be interpreted as filtering the desired control input given in (2)—comprised of a state feedback component,  $u_c$ , and a human input,  $u_h$ —to obtain  $v^*$ . The filtered controller is then integrated in software to obtain the actual control input  $u$  supplied to the multi-robot system (1) to ensure its stability, safety, and passivity (see Fig. 2). The described filtering to synthesize stable, safe, and passive controllers is implemented as a convex optimization control policy. As such, it can be efficiently computed in online settings, even under real-time constraints, in a large number of robotic applications.*

In the following section, we illustrate the effectiveness of the SSP filter developed in this section in a human-multi-robot teleoperation application.

## IV. EXPERIMENTAL RESULTS

The results of the implementation of the SSP filter developed in the previous section are showcased in this section in a human-multi-robot teleoperation scenario. The experimental setup is as in Fig. 1. The robotic system is comprised of 6 DJI RoboMaster EP mobile platforms deployed in the Robohub at the University of Waterloo. Each robot is acceleration-controlled and modeled using the following damped double-integrator dynamics:

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = -\sigma x_{i2} + u_i \\ y_i = x_{i2}, \end{cases} \quad (18)$$

where  $x_{i1}, x_{i2}, u_i, y_i \in \mathbb{R}^2$ , are position, velocity, acceleration input and velocity output, respectively, of robot  $i$ , and  $\sigma > 0$ . Its dynamic extension (5) is:

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = -\sigma x_{i2} + u_i \\ \dot{u}_i = \phi_i(x_i, u_i, t) + v_i \\ y_i = x_{i2}, \end{cases}$$

where  $v_i \in \mathbb{R}^2$ . The human, through the input  $u_h$  supplied via a Falcon Haptic Device, directly controls robot 1 in order to drive it to a known desired goal point located at  $x_G = [1.2, -0.5]^T$  m, i.e.  $u_1 = u_h$ . Moreover,  $u_c = 0$ . The state of the robots is estimated using a Kalman filter in which robot position measurements are supplied by a Vicon-based tracking system.

The stability of the multi-robot system is leveraged to assemble a geometric shape consisting of a regular hexagon whose side length is 1m. This is achieved using the control Lyapunov function  $V(x) = \sum_{i=1}^6 \sum_{j \in \mathcal{N}_i} (\|x_{1i} - x_{1j}\|^2 - W_{ij}^2)^2$ , where  $\mathcal{N}_i$  is the neighborhood of robot  $i$ , i.e. the set of robots with which robot  $i$  shares an edge in a rigid graph and  $W_{ij}$  is the distance to be maintained between robots  $i$  and  $j$  (see [5], [29]). The exponential stability condition given in (3) is affine in  $v$ , i.e. it can be expressed as  $A_V v \leq b_V$ . To ensure the passivity of the system, the storage function  $S: \mathbb{R}^n \rightarrow \mathbb{R}: x \mapsto \|x\|^2$  has been employed to define the I-CBF  $h_u$  as in (12). For the system (18), the passivity condition also takes an affine form,  $A_u v \leq b_u$ . Finally, safety has been defined as the condition that robots 4, 5, and 6 shall not enter the circular region of radius 1m centered at  $x_O = [0, -1]^T$  m. To this end, the CBFs  $h_{x_i}(x) = \|x_i - x_O\|^2 - 1$ ,  $i = 4, 5, 6$  have been defined. As  $h_x$  has relative degree 2 with respect to  $u$ , the same recursive approach utilized for  $V$  has been employed for  $h_x$  in order to write the safety condition (4) in the form  $A_x v \leq b_x$ .

Stability, safety, passivity conditions are then combined in the following single QP:

$$\begin{aligned} v^* = \underset{v \in \mathbb{R}^{2N}, \delta \in \mathbb{R}}{\operatorname{argmin}} \quad & \|v\|^2 + \kappa \delta^2 \\ \text{subject to} \quad & A_V(x)v \leq b_V(x, u, t) + \delta \\ & A_x(x)v \leq b_x(x, u, t) \\ & A_u(x)v \leq b_u(x, u, t), \end{aligned} \quad (19)$$

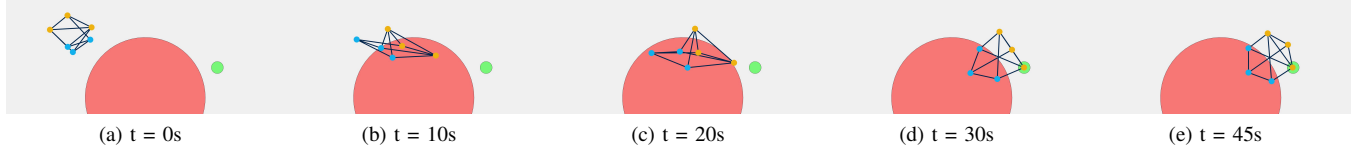


Fig. 3. Experiment conducted without employing the SSP filter. A multi-robot system comprised of 6 robots—depicted as orange and blue dots—is tasked with assembling a hexagonal formation, while keeping the blue robots away from the red-shaded circular region. One of the robots is controlled by a human input that drives it to the prescribed green location on the right of each frame. Without employing the desired SSP filter, the safety of the system is violated: the blue robots enter the red region. Moreover, the teleoperation system is not passive. The video of the experiment is available as a multimedia attachment to the paper.

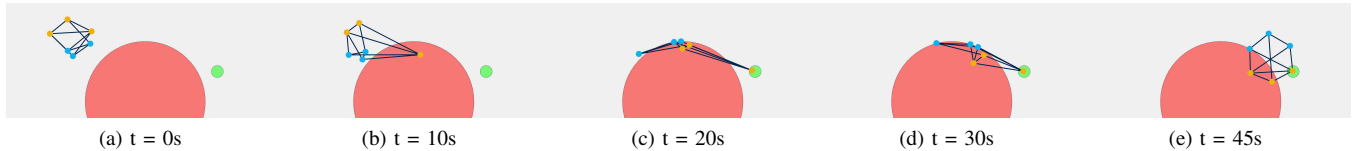


Fig. 4. Experiment conducted by employing the SSP filter to achieve stability, safety, and passivity of the human-multi-robot teleoperation system. The 6 robots—orange and blue dots—are driven to the green goal location by a human input sent to one of the robots. Moreover, a hexagonal formation is assembled by the aid of a control Lyapunov function. At the same time, the safety specification according to which the blue robots shall not enter the red-shaded region is achieved. Furthermore, the SSP filter is also able to ensure that the whole teleoperation system is passive (see also Fig. 5). The video of the experiment is available as a multimedia attachment to the paper.

where  $\kappa = 10^3$ . This QP is equivalent to (17) up to the relaxation variable  $\delta$ , introduced to prioritize safety and passivity over stability. As a result, the multi-robot system will sacrifice the formation control to assemble the desired hexagonal shape whenever safety and/or passivity are endangered. The result of the implementation of the solution of (19) to control the system of 6 mobile robots are reported in the following.

Figure 3 shows a the motion of the robots during the course of an experiment conducted without employing the SSP filter. Instead, only the stability was enforced, to drive the robots to a desired formation. The 6 robots are shown as orange and blue dots. The latter depict robots 4, 5, and 6, which, according to the safety specification shall not enter the red-shaded circle at the bottom center of each frame. Additionally, the goal point for robot 1, to which the latter is driven by a human input, is plotted as a small green circle at the bottom right of each frame. As can be seen, the hexagonal formation of robots is successfully driven by the human input towards the green goal point, and at time  $t = 45s$  the formation is achieved at the desired location. Nevertheless, the robots depicted as blue dots navigate through the unsafe red region.

In order to ensure safety and passivity, alongside stability, the SSP filter has been applied to minimally modify the human input sent to the robots in such a way that the teleoperation system is stable, safe, and passive. Figure 4 shows snapshots recorded during the course of the experiment where now the SSP filter has been activated. First of all, it can be clearly seen how stability is achieved: the hexagonal formation is assembled in the last frame ( $t = 45s$ ). Moreover, the multi-robot system reaches the desired formation while preventing the blue robots from entering the unsafe red region. Furthermore, as can be seen from Fig. 5, the passivity requirement is satisfied too. In fact, the value of  $h_u$  remains positive for the entire duration of the experiment. Finally, it

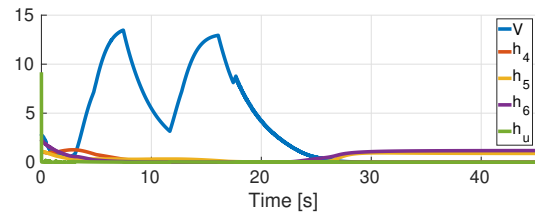


Fig. 5. Trajectories of the control Lyapunov and control barrier functions utilized in the definition of the SSP filter employed during the course of the experiment to enforce stability, safety, and passivity properties of the human-multi-robot teleoperation system in a holistic fashion. The positivity of the functions  $h_4$ ,  $h_5$ , and  $h_6$ , and  $h_u$  signifies that the system is safe and passive, while the convergence of  $V$  to 0 corresponds to the fact that the system is stable.

is worth pointing out how safety and passivity are achieved at the expense of temporarily sacrificing stability, whose constraint is relaxed by the slack variable  $\delta$  in (19). As a result, the Lyapunov function  $V$  is allowed to significantly grow around times  $t = 3s$  and  $t = 16s$ .

## V. CONCLUSIONS

In this paper, we introduced a systematic approach to guarantee the stability, safety, and passivity of a human-multi-robot teleoperation system. Stability is used to encode a long-term behavior of the robot team, safety ensures that the robotic systems remains within a desired set of configurations, while passivity guarantees that no energy is generated by the interconnection of the human teleoperator with the multi-robot system. The developed algorithm leverages control Lyapunov and (integral) control barrier functions to define a stability, safety, and passivity (SSP) filter implemented as a convex optimization control policy, efficiently computable in an online fashion. The effectiveness of the SSP filter for human-multi-robot teleoperation systems is showcased using a team of mobile robots.

## REFERENCES

- [1] S. Stramigioli, "Energy-aware robotics," in *Mathematical Control Theory I*. Springer, 2015, pp. 37–50.
- [2] T. Hatanaka, N. Chopra, and M. W. Spong, "Passivity-based control of robots: Historical perspective and contemporary issues," in *2015 54th IEEE Conference on Decision and Control (CDC)*. IEEE, 2015, pp. 2450–2452.
- [3] D. Sieber and S. Hirche, "Human-guided multirobot cooperative manipulation," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 4, pp. 1492–1509, 2018.
- [4] N. Chopra and M. W. Spong, "Passivity-based control of multi-agent systems," in *Advances in robot control*. Springer, 2006, pp. 107–134.
- [5] J. Cortés and M. Egerstedt, "Coordinated control of multi-robot systems: A survey," *SICE Journal of Control, Measurement, and System Integration*, vol. 10, no. 6, pp. 495–503, 2017.
- [6] H. K. Khalil, *Nonlinear control*. Pearson New York, 2015.
- [7] R. J. Anderson and M. W. Spong, "Bilateral control of teleoperators with time delay," *IEEE Transactions on Automatic control*, vol. 34, no. 5, pp. 494–501, 1989.
- [8] S. Zampieri, "Trends in networked control systems," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 2886–2894, 2008.
- [9] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on automatic control*, vol. 49, no. 9, pp. 1520–1533, 2004.
- [10] G. Niemeyer and J.-J. Slotine, "Stable adaptive teleoperation," *IEEE Journal of oceanic engineering*, vol. 16, no. 1, pp. 152–162, 1991.
- [11] M. R. Wohlers, *Lumped and distributed passive networks: a generalized and advanced viewpoint*. Academic press, 2017.
- [12] J. Yamauchi, M. W. S. Atman, T. Hatanaka, N. Chopra, and M. Fujita, "Passivity-based control of human-robotic networks with inter-robot communication delays and experimental verification," in *2017 IEEE International Conference on Advanced Intelligent Mechatronics (AIM)*. IEEE, 2017, pp. 628–633.
- [13] G. Notomista, X. Cai, J. Yamauchi, and M. Egerstedt, "Passivity-based decentralized control of multi-robot systems with delays using control barrier functions," in *International Symposium on Multi-Robot and Multi-Agent Systems*. IEEE, 2019, pp. 231–237.
- [14] V. Duindam and S. Stramigioli, "Port-based asymptotic curve tracking for mechanical systems," *European Journal of Control*, vol. 10, no. 5, pp. 411–420, 2004.
- [15] C. Secchi, S. Stramigioli, and C. Fantuzzi, *Control of interactive robotic interfaces: A port-Hamiltonian approach*. Springer Science & Business Media, 2007, vol. 29.
- [16] C. Secchi, A. Franchi, H. H. Bühlhoff, and P. R. Giordano, "Bilateral teleoperation of a group of uavs with communication delays and switching topology," in *2012 IEEE International Conference on Robotics and Automation*. IEEE, 2012, pp. 4307–4314.
- [17] P. R. Giordano, A. Franchi, C. Secchi, and H. H. Bühlhoff, "A passivity-based decentralized strategy for generalized connectivity maintenance," *The International Journal of Robotics Research*, vol. 32, no. 3, pp. 299–323, 2013.
- [18] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath, and P. Tabuada, "Control barrier functions: Theory and applications," in *European Control Conference*, 6 2019, pp. 3420–3431.
- [19] A. D. Ames, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs with application to adaptive cruise control," in *Conference on Decision and Control*. IEEE, 2014, pp. 6271–6278.
- [20] G. Wu and K. Sreenath, "Safety-critical control of a planar quadrotor," in *American Control Conference*. IEEE, 2016, pp. 2252–2258.
- [21] Q. Nguyen and K. Sreenath, "Exponential control barrier functions for enforcing high relative-degree safety-critical constraints," in *American Control Conference*. IEEE, 2016, pp. 322–328.
- [22] L. Wang, A. D. Ames, and M. Egerstedt, "Safety barrier certificates for collisions-free multirobot systems," *Transactions on Robotics*, vol. 33, no. 3, pp. 661–674, 2017.
- [23] G. Notomista and M. Egerstedt, "Persistification of robotic tasks," *Transactions on Control Systems Technology*, 2020.
- [24] A. Ames, G. Notomista, Y. Wardi, and M. Egerstedt, "Integral control barrier functions for dynamically defined control laws," *Control Systems Letters*, 2020.
- [25] Y. Huang, S. Z. Yong, and Y. Chen, "Guaranteed vehicle safety control using control-dependent barrier functions," in *American Control Conference*. IEEE, 2019, pp. 983–988.
- [26] P. Ogren, A. Backlund, T. Harryson, L. Kristensson, and P. Stensson, "Autonomous uav strike missions using behavior control lyapunov functions," in *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2006, p. 6197.
- [27] G. Notomista and X. Cai, "A safety and passivity filter for robot teleoperation systems," *arXiv preprint arXiv:2102.08630*, 2021.
- [28] Y. Wardi, C. Seatzu, J. Cortes, M. Egerstedt, S. Shivam, and I. Buckley, "Tracking control by the newton-raphson method with output prediction and controller speedup," *arXiv preprint arXiv:1910.00693*, 2019.
- [29] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010, vol. 33.