

Search Strategy for Layered Peg-in-Hole Using Dual Manipulator System

Haeseong Lee¹, Yonghee Lee², Junheon Yoon³, and Jaeheung Park[†]

Abstract—This study introduces a novel assembly, *layered peg-in-hole*, and suggests a search strategy. Distinct from the traditional peg-in-hole assembly, where two workpieces (i.e., peg/hole) are present, the layered peg-in-hole assembly contains three workpieces (i.e., peg, hole, and thru-hole). To handle the additional moving part, the through-hole, without relying on task-specific devices, a dual-manipulator system is preferable to a single manipulator system. However, existing research has primarily concentrated on the traditional peg-in-hole assembly regardless of the number of manipulators. In this respect, as the main contribution, a search strategy is proposed for the layered peg-in-hole assembly, which consists of two phases. In the first phase, both manipulators actively engage in the search task. In the second phase, the compliant behavior of the manipulator grasping the through-hole is advocated to assist the counterpart. Finally, the proposed search strategy is verified through the real robot experiment that replicates an industrial environment with two 7-DOF torque-controlled manipulators.

I. INTRODUCTION

Peg-in-hole assembly is one of the prototypical robotic manipulation tasks. To complete the peg-in-hole assembly, one part (i.e., peg) is guided into its counterpart (i.e., hole) to achieve a secure connection between them. Fig. 1(a) illustrates the conventional peg-in-hole assembly where two workpieces, the peg and the hole, are involved. Since it can describe a wide range of assembly scenarios, from industrial tasks to daily activities, this type of assembly has been extensively studied in previous research [1]–[3], regardless of the number of manipulators. Frequently, the assembly requires handling three workpieces as shown in Fig. 1(b). Bolt fastening and piston assembly are representative examples of the peg-in-hole assembly with three workpieces (Fig. 1(c)–(d)).

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¹Haeseong Lee is with the Artificial Intelligence Institute, Seoul National University, Seoul 08826, Republic of Korea. lhs4138@snu.ac.kr

²Yonghee Lee is with the Graduate School of Interdisciplinary Program in Artificial Intelligence, Seoul National University, Seoul 08826, Republic of Korea. henry9804@snu.ac.kr

³Junheon Yoon is with the Department of Intelligence and Information, Graduate School of Convergence Science and Technology, Seoul National University, Seoul 08826, Republic of Korea. yoonjh98@snu.ac.kr

[†]Jaeheung Park is with the Department of Intelligence and Information, Graduate School of Convergence Science and Technology, ASRI, AIIS, Seoul National University, Seoul 08826, Republic of Korea, and the Advanced Institutes and Advanced Institutes of Convergence Technology (AICT) Suwon 16229, Republic of Korea. park73@snu.ac.kr

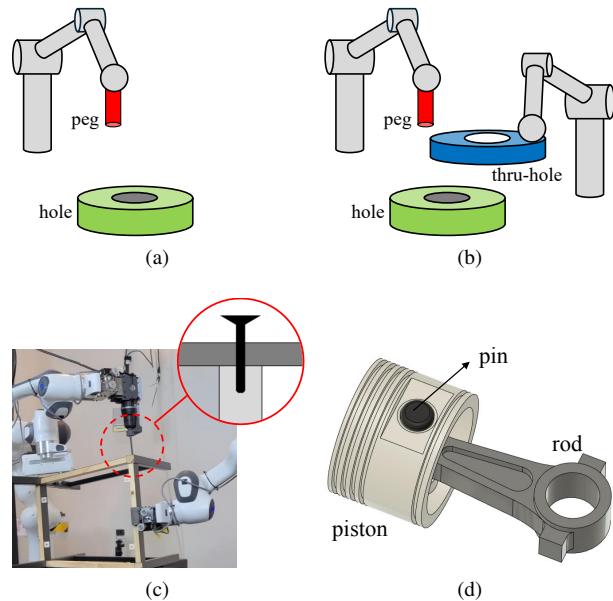


Fig. 1: Peg-in-Hole assembly: (a) conventional, (b) layered, (c) bolt fastening, and (d) piston assembly

Since the peg is connected to the other two components simultaneously, this assembly differs fundamentally from the conventional peg-in-hole assembly. To explicitly distinguish this newly defined assembly type, the authors refer to it as the *layered peg-in-hole* assembly. The term *layered* implies that the peg must penetrate a middle component to establish a complete connection. Compared to the conventional peg-in-hole assembly, there is one more moving component. Therefore, an additional task-specified device would be required for an assembly system where only a single manipulator is employed [4], [5]. To avoid such task-specified environmental settings and to address complex manipulation challenges [6], a dual manipulator system holds great potential as a versatile and effective solution.

The conventional peg-in-hole assembly can be accomplished by sequentially ordered sub-tasks: *approach*, *search*, and *insertion* [7]. In contrast, the layered peg-in-hole assembly inherently involves additional complexity, as the peg should be aligned with two distinct holes rather than one. Therefore, the conventional assembly strategy may not be directly implemented.

In that sense, as the main contributions, this paper addresses (i) a newly defined type of peg-in-hole assembly, which, to the best of the authors' knowledge, has been

underexplored in [1]–[3], and suggests (ii) the corresponding search strategy considering two moving objects: the peg and the through-hole. The proposed search strategy consists of two consecutive phases. In the first phase, both manipulators actively generate search motion to align the peg and the through-hole. In the second phase, the manipulators collaboratively coordinate their motions, guiding the peg to the hole while maintaining the connection with the through-hole. The effectiveness of the proposed method in both phases is validated through real robot experiments.

This paper extends the previous research [8], where the concept of the bi-spiral was originally introduced, by applying this search method to the newly defined layered peg-in-hole assembly. The rest of this paper is organized as follows. In Sec. II, previous studies related to the peg-in-hole assembly are introduced. The problem of layered peg-in-hole assemblies and the corresponding search strategy are explained in Sec. III and Sec. IV, respectively. The result of a real robot experiment is described in Sec. V. Finally, the conclusion of this paper is presented in Sec. VI.

II. RELATED WORKS

Recently, to address the environmental uncertainty, vision-based systems and data-driven methods have shown promising performance. With the visual information, the peg is allowed to be efficiently guided toward the hole [9]–[12]. Moreover, it enables the estimation of the current contact state between the peg and hole, combining vision data with the force/torque measurement [13], [14]. However, the vision system is susceptible to blind spots, which are areas where the view of the camera cannot reach [15]. Furthermore, in a complex environment [16], the view of the camera can be easily occluded by the grasp objects or the robot itself.

Learning from demonstration (LfD) is a representative data-driven solution for contact-rich manipulation. Based on the human demonstrations, the robots can reproduce human-like skills in various applications, including pouring [17], assembly [18], [19], and complex dual manipulation [20], [21]. However, obtaining a qualified demonstration dataset requires skilled human operators and a significant amount of time to collect sufficient training samples. Additionally, recording human demonstrations might raise safety concerns, especially when dealing with large and heavy target objects commonly used in industrial environments.

Considering the aforementioned limitations of vision-based systems and data-driven methods, blind search can be a practical and cost-effective alternative [22]–[26]. By relying on carefully designed search trajectories, it provides robust performance without the need for additional sensors or a large-scale training dataset. In particular, the spiral trajectory has been the most widely implemented.

Cooperative manipulation is a significant advantage of the dual manipulator system, particularly when handling large or heavy objects [27]. Consequently, related control strategies have been proposed, including leader/follower approach [28], manipulability ellipsoids [29], and virtual linkage model [30]. These strategies are typically developed under the

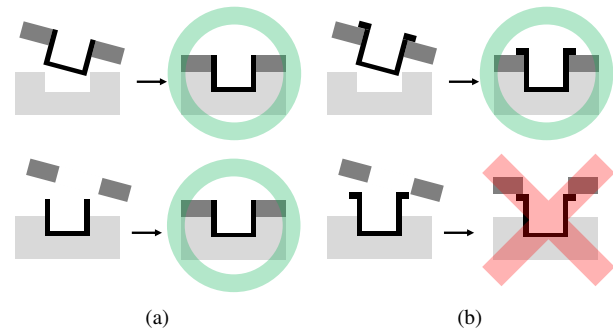


Fig. 2: Illustration for the assembly sequence of the layered peg-in-hole.

assumption that the target object is a single rigid entity. However, during the layered peg-in-hole assembly, manipulators have to handle two distinct objects (i.e., peg/through-hole). Therefore, these methods might be potentially unsuitable.

Based on the leader/follower relationship between both manipulators, experimental analysis of different compliance configurations for both manipulators during the insertion task has been conducted in [31]. In the absence of external force/torque (F/T) sensors, an active compliance method by an admittance-based control with position and velocity has been proposed to generate the motion of the follower arm [32]. However, these methods primarily address the insertion task, assuming the search task is completed. The full sequence of the traditional peg-in-hole assembly using the dual manipulators has been detailed in previous works [33]–[36]. With dexterous robot hands, an assembly strategy was applied to a keyhole-like shape peg-in-hole assembly [33], but the hole is assumed to be firmly fixed. Even though the advantage of the relative motion of both manipulators is applied during the search task [34]–[36], a certain geometric condition to guarantee the search result is not thoroughly addressed.

Ultimately, the proposed sequential search strategy is characterized by cooperative manipulation based on the spiral search trajectory.

III. PROBLEM DESCRIPTION

A. Definition of Layered Peg-in-Hole

Since the term *layered peg-in-hole* assembly is newly introduced, an explicit description is necessary for clarity. In this assembly scenario, the peg is intended to be mated with two different holes. In one possible case, the given assembly can be completed regardless of the order in which the peg is inserted, as shown in Fig. 2(a). In contrast, the other case requires a strict order: the peg and the through-hole must be aligned in advance, and then the peg is inserted into the hole while maintaining the connection with the through-hole. This order is enforced by the geometric shapes of the components, as illustrated in Fig. 2(b). An intuitive example of this case is a fixation of a certain object to a wall, either by inserting a pin or fastening it with a bolt. Although both

assembly scenarios (Fig. 2(a) and Fig. 2(b)) correspond to the layered peg-in-hole assembly, this paper focuses on the more challenging case illustrated in Fig. 2(b).

B. Challenges in Layered Peg-in-Hole

Fundamentally, the peg-in-hole assembly requires a specific method to reduce the position uncertainty to accomplish a secure connection between mating parts. In that context, a search strategy is necessary where a spiral trajectory has been widely implemented [22], [37], [38]. This paper also advocates the usage of the spiral trajectory due to its smoothness and ability to explore all directions without missing spots.

To complete the layered peg-in-hole assembly, it might be suggested to use a method in two ways: stepwise and one-shot assemblies. The stepwise assembly means that the assembly progresses step by step, as shown in Fig. 2, which is an intuitive and straightforward method. The other, one-shot assembly, represents that two moving objects are connected with the hole at the same time. Although this method is inherently efficient, designing a reliable search trajectory is a challenging problem because the position uncertainty prevents accurate measurement of the relative poses among the pegs and the corresponding holes. Therefore, splitting into two sub-assembly tasks and conquering each step would be a practical and reliable search strategy.

C. Overview of Proposed Strategy

Fig. 3 illustrates the overview of the search strategy to complete the layered peg-in-hole assembly. As mentioned in the previous subsection, the assembly splits into two phases: Preliminary and Definitive. The goal of the preliminary phase (P-phase) is to mate the peg and the through-hole. To align both objects, the manipulators are actively generating spiral search motion, which is called bi-spiral [8]. The bi-spiral is a decomposed motion from the original spiral trajectory (i.e., uni-spiral) through kinetic energy optimization. The details are provided in the following section.

In the definitive phase (D-phase), the peg is guided into the hole while maintaining its mating state with the through-hole. To satisfy this requirement, the coordinated manipulation of both manipulators is essential. Accordingly, two operation modes are considered in the D-phase: sync-mode and async-mode. In the sync-mode, the manipulators generate the target motion independently with the given exactly the same search trajectory. In contrast, in the async-mode, one manipulator leads the search task, while the other compliantly follows the leading motion.

IV. PROPOSED SOLUTION

A. Archimedean Spiral

In both phases, the search trajectory is referred to as the Archimedean spiral with a constant search speed [23].

Consequently, the spiral trajectory is defined as

$$\begin{aligned}\rho(t) &= \frac{p}{2\pi}\theta(t) \\ \theta(t) &= \sqrt{\frac{4\pi v}{p}t}\end{aligned}\quad (1)$$

where $\rho(t)$ is an increasing radius of Archimedes's spiral at time t , $\theta(t)$ is a corresponding polar angle, p is a spiral pitch, and v is a constant search speed.

B. Bi-spiral in Preliminary Phase

The basic idea of the bi-spiral is to decompose the original spiral into two components. In other words, the relative distance of these two components is geometrically identical to the radius of the original spiral. Therefore, this relationship can be formulated in polar coordinates as

$$\rho = \sqrt{\rho_p^2 + \rho_h^2 - 2\rho_p\rho_h\cos(\theta_h - \theta_p)} \quad (2)$$

where the subscripts p and h represent the peg and the hole (or through-hole), respectively.

With the angular velocity constraint, $\dot{\theta} = \dot{\theta}_p = \dot{\theta}_h$, (2) becomes simplified as

$$\rho = \sqrt{\rho_p^2 + \rho_h^2 - 2\rho_p\rho_h\cos(\Delta\theta)} \quad (3)$$

where $\Delta\theta$ is an initial polar angle offset between the two spiral trajectories.

Since the angular velocity constraint derives a proportional relationship, such as $v = kp$, (3) can be rewritten as

$$p^2 = p_p^2 + p_h^2 - 2p_p p_h \cos(\Delta\theta). \quad (4)$$

Finally, the original spiral can be decomposed into two spirals where the corresponding parameter set is $\mathbf{x} = \{p_p, p_h, v_p, v_h, \Delta\theta\}$. The details are provided in the previous study [8].

C. Optimization for Parameter Selection

Since there can exist various parameter sets of $\mathbf{x} = \{p_p, p_h, v_p, v_h, \Delta\theta\}$ to satisfy the relationship of (3), it is required to determine a feasible solution. Therefore, as one of the valid solutions, the optimal parameter set is derived by solving an optimization problem with a suggested cost function defined as below:

$$f(p_p, p_h, v_p, v_h) = \frac{1}{2}M_p v_p^2 + \frac{1}{2}M_h v_h^2 + \lambda \left(\frac{1}{p_p} + \frac{1}{p_h} \right) \quad (5)$$

where the first two terms minimize the kinetic energy of both peg and through-hole, and the last two terms ensure the search motion of both moving objects with a weight of λ .

Specifically, the mass values M_p and M_h include the average of the belted ellipsoid of the effective masses [39] at the starting configuration, $\mathbf{q}_{(*)}$ of the given assembly. In other words,

$$M_{p,h} = m_{m,p} + \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\mathbf{u}(\phi)^T \Lambda_v^{-1}(\mathbf{q}_{p_0, h_0}) \mathbf{u}(\phi)} d\phi \quad (6)$$

where $\Lambda_v \in \mathbb{R}^{3 \times 3}$ is the effective mass with respect to the end-effector frame at the initial configuration \mathbf{q}_{p_0, h_0} , and

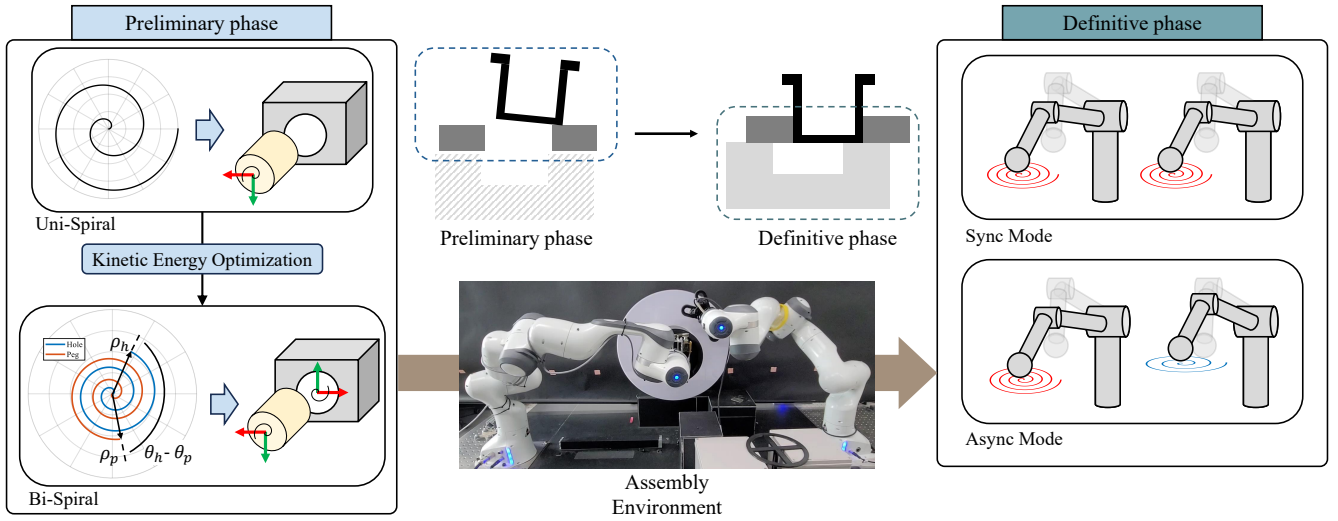


Fig. 3: Overview of the proposed assembly strategy

$\mathbf{u} = [\cos(\phi), \sin(\phi), 0.0]^T$ is a unit direction vector since the search motion is designed to the $E_x - E_y$ plane.

Finally, the standard optimization problem can be formulated as

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(p_p, p_h, v_p, v_h) \\ \text{s.t.} \quad & p_p^2 + p_h^2 - 2p_p p_h \cos(\Delta\theta) = p^2. \end{aligned} \quad (7)$$

As noted earlier, $\Delta\theta$ represents the initial angle offset between the decomposed two spirals. Therefore, this parameter is carefully addressed since it is a critical design variable in determining the relative distance between the peg and the through-hole, as shown in Fig. 3.

D. Sync/Async Mode in Definitive Phase

After the P-phase, the layered peg-in-hole assembly requires the peg to be inserted into the hole while maintaining a proper mating state between the peg and the through-hole. To satisfy, it suggests two control strategies: sync-mode and async-mode. With the given spiral parameters, p and v , the two manipulators generate geometrically equivalent spiral trajectories when the sync-mode is enabled, which is a significant distinction compared to the bi-spiral in P-phase. Whereas, in async-mode, only one manipulator executes the search motion, while the counterpart compliantly follows the motion of the leading manipulator.

V. EXPERIMENT

A. Experiment Environment

The proposed strategy is validated through layered peg-in-hole experiments using a dual-manipulator system (Fig. 4), which replicates an industrial coil packaging process. The industrial coil is made of steel or aluminum in a rolled shape. In this analogy, the industrial coil corresponds to the hole, the through-hole is designed to cover the side surface of the coil, and the peg represents a protective element for the inner edge of the coil. The hole has the following dimensions: an outer diameter of 0.5 m, an inner diameter of 0.2 m, and a depth

of 0.03 m. Thus, the peg has a diameter of 0.2 m and a depth of 0.03 m. The through-hole shares the same dimensions for both outer and inner diameters, but the thickness is 3 mm.

In this paper, the terms task arm and assist arm are introduced to clarify the role of each manipulator. The task arm grasps the peg and leads the entire assembly process, whereas the assist arm holds the through-hole and supports the task arm in completing the layered peg-in-hole assembly.

Fig. 5 briefly provides the entire assembly. From the initial configuration (Fig. 5(a)), the manipulators grasp the peg and the through-hole according to the role of the task and assist (Fig. 5(b)). Then, the manipulators are planned to prepare the layered peg-in-hole assembly as shown in Fig. 5(c). During the assembly execution in Fig. 5(d), the manipulators perform the P-phase followed by the D-phase sequentially. After completing all of the sub-tasks, the manipulators return to the initial configuration as shown in Fig. 5(e-f).

B. Robot Control

The manipulators are 7-DOF torque-controlled manipulators. Based on the motion/force hybrid control [40], all primitives, including the spiral, are defined with respect to the end-effector frame. The force direction coincides with the insertion direction (i.e., the E_z -axis), while the motion directions lie along the other two axes [41], [42].

During the spiral motion for each phase, the task arm measures an external force applied to its end-effector. Once the external force on the search plane (i.e., $E_x - E_y$ plane) exceeds a predefined threshold, the peg is considered to have detected the location of the through-hole or the hole. Subsequently, the task arm generates a force along the insertion direction for a predefined duration to insert the peg into the target [42].

Algorithm. 1 presents a pseudo-code of the proposed search strategy. The optimized bi-spiral parameters $(p_p, p_h, v_p, v_h, \Delta\theta)$ are determined within the real-time control loop based on the original spiral design parameters

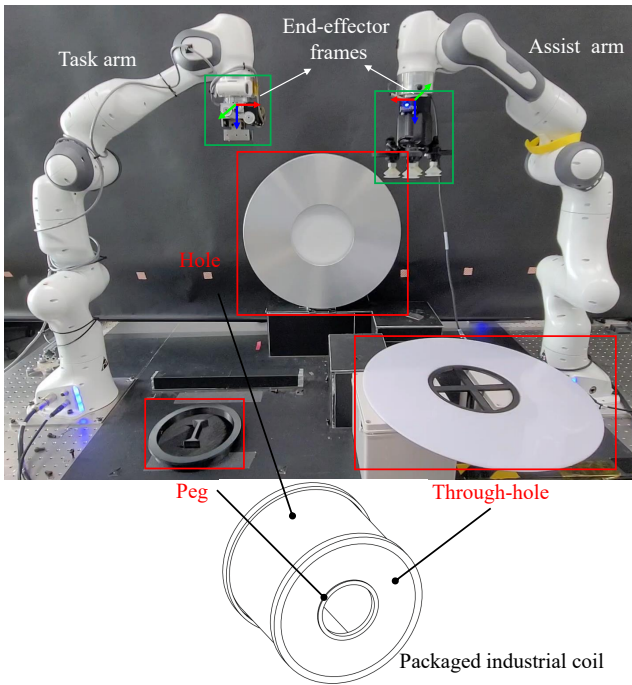


Fig. 4: Experiment environment

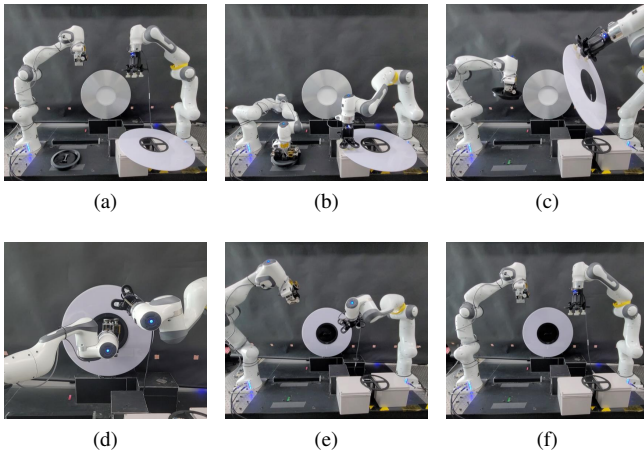


Fig. 5: Snapshots of the layered peg-in-hole assembly: (a)-(b) Grasping. (c) Planning. (d) Assembly execution with the proposed assembly strategy. (e)-(f) Returning.

(lines 1-2). During the P-phase, the manipulators execute the bi-spiral motion until the peg detects the through-hole, completing this phase with compliant insertion (lines 3-8). Similarly, in the D-phase, the assist arm operates in async-mode and both manipulators continue until the peg detects the hole, upon which the phase is completed by the same compliant insertion (lines 9-15). In both phases, once the external force measured by the task arm exceeds a predefined threshold during the search process, the contact is regarded as an indication that the hole has been found.

Since there are two different phases in the proposed assembly strategy, the verification procedure consists of two independent experiments.

Algorithm 1: Pseudo code of the proposed search strategy for the layered peg-in-hole assembly.

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1 Initialize parameters ▷ e.g.,  $p, v$ 
2 Solve (7) to determine  $p_p, p_h, v_p, v_h, \Delta\theta$ 
3 while P-phase
4   Bi-spiral with  $\{p_p, p_h, v_p, v_h, \Delta\theta\}$ 
5   if Detect the through-hole then
6     Complete P-phase
7   end
8 end
9 while D-phase
10  Async-mode
11  if Detect the hole then
12    Complete D-phase
13  end
14 end
15 Returning

```

TABLE I: Result of P-phase.

Motion	Rate (%)	Time (s)	Average Joint Power (J/s)		
			Task arm	Assist arm	Total
Uni	25/25	10.97	1.13	0.12	1.25
Bi	25/25	11.64	0.48	0.31	0.79

TABLE II: Result of D-phase.

Mode	Sync	Async
Rate	23/25	25/25

C. Experiments for P-phase

With a uniformly distributed random position error of the peg ranging from 0 mm to 5 mm, the assembly corresponds to the P-phase repeated 25 times. The main experimental result demonstrates the advantages of the bi-spiral compared to the original spiral, namely the uni-spiral. The parameters p and v of the uni-spiral are set to 1 mm and 10 mm/s, respectively. Since the optimization requires the belted ellipsoid of the effective masses, which vary in each trial due to joint redundancy, the parameters, $\{p_p, p_h, v_p, v_h, \Delta\theta\}$, are determined in real time before the execution of the bi-spiral using OSQP [43].

Table. I summarizes the experimental results of the P-phase. For both cases, 100 % success rates are achieved with the average search times of 10.97 s and 11.64 s, respectively. Due to the random position error and detection delay, this difference in search time can be considered negligible, and the two results are therefore regarded as practically identical. However, a significant advantage of the bi-spiral search motion is observed in the required joint power, which is reduced by 36.8 % compared to the uni-spiral.

The details of this reduction are provided in Fig. 6, which represents the root mean square (RMS) of torque and velocity

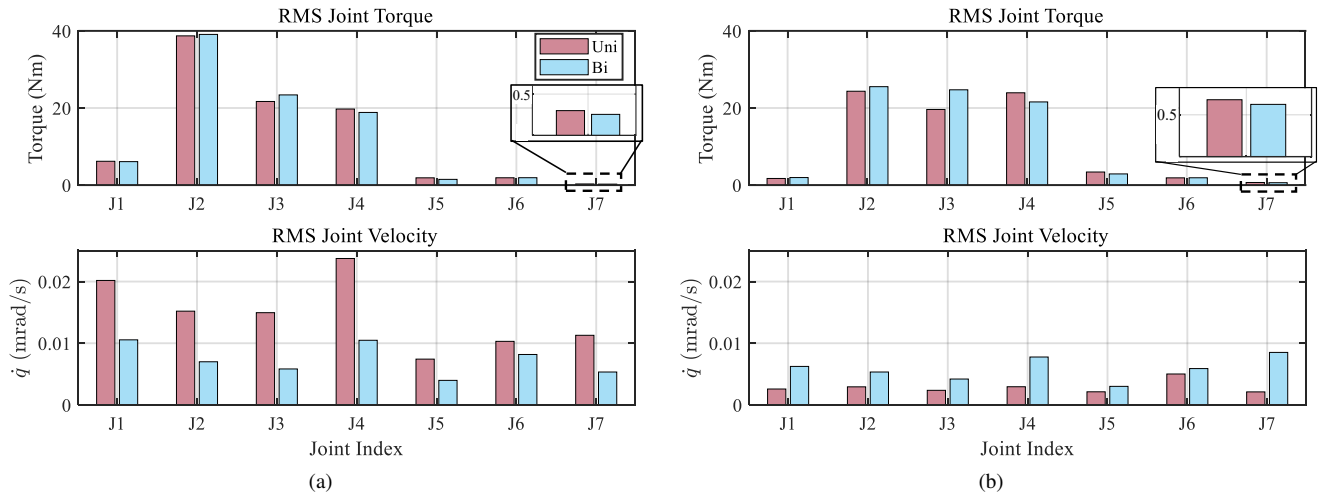


Fig. 6: RMS of the joint torque and joint velocity : (a) Task arm, and (b) Assist arm.

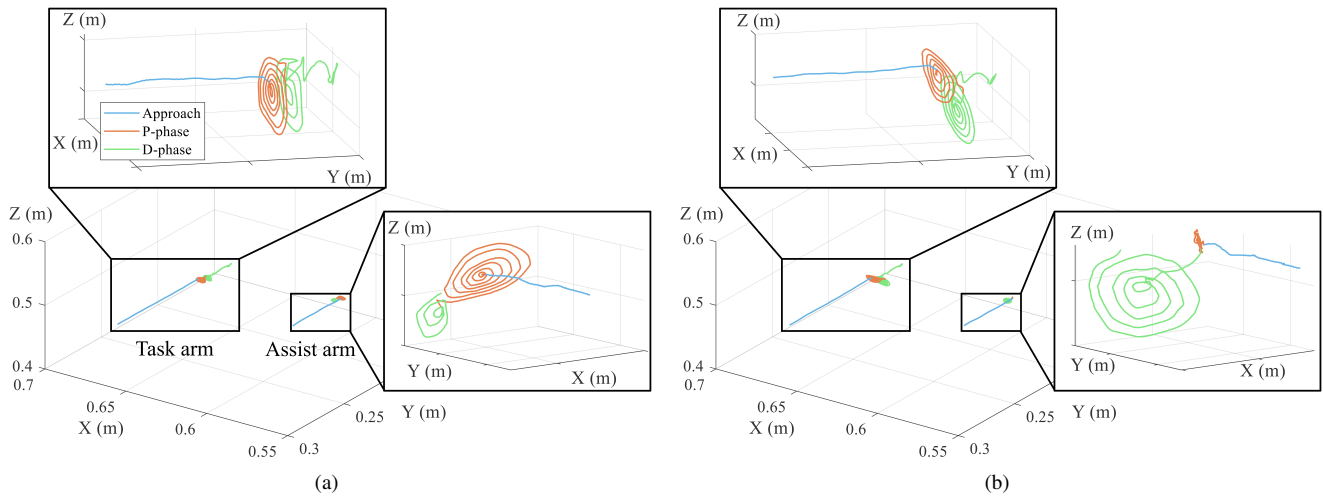


Fig. 7: Trajectories of both manipulators: (a) bi-spiral with async mode, and (b) uni-spiral with async mode.

of each joint. Although there are no significant changes in the required joint torque for both spiral trajectories, the joint velocity of the task arm is significantly reduced since the required search speed (i.e., v_p) is decreased. Therefore, it can be concluded that this reduction in joint velocity ultimately leads to a decrease in the average joint power.

D. Experiments for D-phase

The goal of the D-phase is to complete the given layered peg-in-hole assembly by guiding the peg to the hole, while maintaining a secure connection between the peg and the through-hole. In this experiment, it is assumed that the P-phase has already been completed. Under this condition, two control modes are evaluated: sync-mode and async-mode. In the sync-mode, both manipulators generate identical spiral trajectories. In contrast, in the async-mode, only the task arm generates the spiral trajectory, while the assist arm compliantly follows the motion of the task arm. In both modes,

the spiral trajectories, defined by (1), are parameterized with $p = 1.0$ mm and $v = 8.0$ mm/s.

Table II summarizes the result of the D-phase comparing the two modes. Across 25 trials, the async-mode achieves a 100 % success rate. In contrast, two failure cases are observed in the sync-mode. This degradation can be attributed to the motion of the assist arm, which becomes an additional disturbance from the perspective of the task arm. Although both manipulators are commanded with identical spiral trajectories, their motions do not perfectly coincide due to control errors. As a result, in one failure case, the unexpected disturbance caused by this discrepancy satisfies the detection criteria, leading the task arm to incorrectly detect the hole location (i.e., a false positive). In the other case, the disturbance gradually induces a significant orientation error between the peg and the hole, causing a jamming situation during insertion.

Based on this comparative experiment, it can be concluded

that the async-mode is preferable since the compliant behavior of the assist arm effectively suppresses unexpected disturbances caused by the relative motion of the manipulators.

E. Representative Examples

Fig. 7 illustrates the overall trajectories of both manipulators during the given layered peg-in-hole assembly. In Fig. 7(a), the bi-spiral and the async-mode are applied in the P-phase and the D-phase, respectively. The bi-spiral trajectory is generated with parameters $\{p_p, p_h, v_h, v_p, \Delta\theta\} = \{0.521 \text{ mm}, 0.479 \text{ mm}, 5.21 \text{ mm/s}, 4.79 \text{ mm/s}, \pi\}$. Since the values of pitch and search speed are nearly identical, both manipulator generates almost equal spiral trajectories. In contrast, since the uni-spiral is employed in the P-phase, the assist arm remains almost stationary as shown in Fig. 7(b).

The demonstration video is available at <https://youtu.be/nfNiaZm7hW4>

VI. CONCLUSION

As the main contribution, this paper introduces the layered peg-in-hole assembly and proposes a corresponding search strategy utilizing the dual manipulator system. In contrast to the conventional peg-in-hole assembly involving two workpieces (i.e., peg/hole), the layered peg-in-hole assembly includes one more workpiece, called the through-hole. Since the two mating parts, the peg and the through-hole, are not fixed, these non-fixed objects should be simultaneously manipulated during the assembly. Therefore, the dual manipulator system becomes necessary unless a task-specified device is implemented.

To complete the layered peg-in-hole assembly, the peg must penetrate the through-hole to establish a secure connection between the peg and the hole. Thus, this paper considers that the layered peg-in-hole consists of two consecutive peg-in-hole assemblies. The validity of the proposed search strategy is demonstrated through the real-robot experiment, which replicates the packaging process of the industrial coil.

The first step, termed the preliminary phase (P-phase), aims to assemble the peg and the through-hole by actively engaging both manipulators with the bi-spiral search motion. Through the optimization problem, the bi-spiral motion reduces the required joint power up to 36.8 % compared to the uni-spiral motion.

During the following step, termed the Definitive phase (D-phase), the peg is guided to the hole by maintaining the secure connection with the through-hole. To achieve this physical constraint, the two different motions, the sync/async modes, of the assist arm are evaluated. With the async-mode, the success rate achieves 100 % due to the compliant behavior of the assist arm, which effectively mitigates the effects of unexpected external disturbances arising from the relative motions of both manipulators.

Even though the layered peg-in-hole assembly can be completed with the proposed search strategy, this paper primarily focuses on the search task and places less emphasis on the insertion task. Furthermore, the experimental setup involves the shallow through-hole, which may neglect the impact of

the assist arm's motion during the insertion task. However, various real-world assembly scenarios might require the peg to pass through a thicker through-hole where frictional effects cannot be ignored. Consequently, future work will involve a comprehensive investigation into the motion of both manipulators during insertion tasks and the development of an accurate contact state estimation method to enhance the dual manipulator system for the peg-in-hole assembly.

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