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Autonomous Distributionally Robust Virtual Energy Storage Services based on Parked Electric Vehicles

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***Abstract*— We propose a novel model of a virtual energy storage system (ESS) that leverages the aggregate battery capacity of parked and idling electric vehicles (EVs). Such an energy service is offered to a community of prosumers as a temporary energy buffer and managed by a parking lot manager (PLM), which absorbs the risks arising from the unreliability of the EV-based ESS due to the arrival and departure of EVs. Hence, from the prosumers' perspective, such a virtual storage service behaves deterministically. Both the PLM and the prosumers act as self-interested agents that optimize their own objectives, subject to operational constraints, leading to a non-cooperative game. To deal with the uncertainty of prosumers' renewable net generation and EVs' arrivals/departures, we use a data-driven distributionally robust approach, showing that a tractable reformulation can be obtained, where the equilibrium solutions can be computed as a variational inequality. Numerical simulations based on real data illustrate the behavior of the proposed model.**

I. Introduction

Storage service based on electric vehicles (EVs) is a new paradigm that enhances the operational level of the modern grid [1]. In some studies, it is commonly known as the *vehicle-to-building* paradigm (V2B) [2], while from the perspective of service-based economics, it is known as a variant of, i.e., EV-based, of *virtual* ESS (VESS) [3]. It considers parking lots not only as hubs for EVs' charging, but also as potential temporary energy storage systems (ESS), constituted by the aggregate battery capacity of parked, thus idling, EVs. On such premises, the development of energy management models for optimally leveraging the flexibility of parked EVs is currently of utmost interest. Several energy schedule approaches have been proposed for charging and using EVs as a temporary ESS, ranging from model predictive control [4] and learning-based methods [5] to rule-based [6] and dynamic programming methods [7].

Two aspects heavily influence EV-based ESSs: on the one hand, EVs are characterized by stochasticity, since the behavioural patterns of owners arriving in and leaving from parking lots are non-deterministic. On the other hand, the relationship between EV-based ESS providers and their users – mostly prosumers – is non-cooperative, due to the conflicting interests and objectives these agents have. In this context, data-driven robust techniques have shown great potential in addressing challenges related to uncertainty in smart grids; theoretical results towards this direction are often based on stochastic and robust optimization [8]–[12], combined, for our purposes, with variational inequalities and operator theory concepts for stochastic Nash equilibrium seeking [13], in settings with self-interested agents. We leverage these aspects to propose a novel model formulation and resolution approach that differs from the aforementioned works.

We design a novel VESS management scheme for an EV parking lot serving a prosumers' community and a grid retailer. The *parking lot manager* (PLM) aggregates idle EV batteries as a temporary ESS and mediates with prosumers. Since EV arrivals/departures render physical capacity unreliable, the PLM absorbs this risk to offer reliable storage services while earning a profit. Prosumers face inaccurate demand and renewable forecasts, so uncertainty affects operation. Ensuring reliable ESS provision under self-interested PLM and prosumers leads to a game-theoretic model. Uncertainty in demand, generation, and EV availability is captured via Wasserstein ambiguity sets centered at data-driven nominal distributions. PLM and prosumers independently tune their risk aversion through the ambiguity radius, preserving operational feasibility and out-of-sample guarantees. Optimizing against worst-case distributions hedges uncertainty without the conservatism of robust optimization or the risk-neutrality of sample-average approximation [14]. We derive a tractable reformulation of the resulting distributionally robust Nash equilibrium and validate it with real-world data.

Simulation source code, data, and supplementary material are publicly available at `github.com/nicomignoni/icra26-dro-evs`.

A. Notation and Preliminaries

We denote the *positive part* function by $[\cdot]_+ = \max\{0, \cdot\}$. Given a vector $\mathbf{x} \in \mathbb{R}^n$, $[\mathbf{x}]_+ = \mathrm{col}([x_i]_+)_{i=1}^n$. Given a proposition P, we define $\mathbb{1}_P \in \{0, 1\}$ such that $\mathbb{1}_P = 1$ if P is verified and $\mathbb{1}_P = 0$ otherwise. Given a vector $\mathbf{x} \in \mathbb{R}^n$ and a set $\mathcal{X} \subseteq \mathbb{R}^n$, the ($\ell_1$ norm-based) *projection* of $\mathbf{x}$ onto $\mathcal{X}$ is denoted as $\mathrm{proj}_{\mathcal{X}}(\mathbf{x}) = \mathrm{argmin}_{\mathbf{y} \in \mathcal{X}} ||\mathbf{y} - \mathbf{x}||_1$. Let $\mathcal{K} = \{1, \dots, K\}$ be a time horizon of size $K \in \mathbb{N}$.

Let $\mathsf{P}(\Xi)$ be the set of probability distributions supported on $\Xi \subseteq \mathbb{R}^m$, and $\mathsf{M}(\Xi)$ those on Ξ with bounded first moment. Consider agents $i \in \mathcal{A}$ with unknown uncertainty

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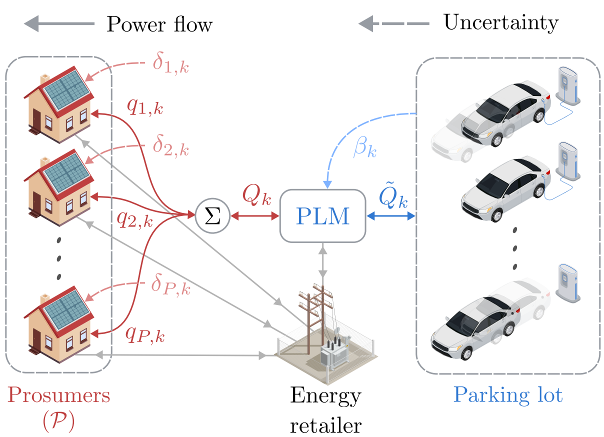

Fig. 1. Overview of the energy community, with the prosumers, PLM, retailer, and parking lot.

distributions $\mathbb{P}_i$, estimated from i.i.d. samples $\hat{\boldsymbol{\xi}}_{i\sigma} \in \Xi_i$. The empirical distribution is $\hat{\mathbb{P}}_i = \frac{1}{|\mathcal{S}_i|}\sum_{\sigma\in\mathcal{S}_i} \mathbb{1}(\boldsymbol{\xi} = \hat{\boldsymbol{\xi}}_{i\sigma})$ where $\mathcal{S}_i$ is the sample index set. The Wasserstein ambiguity set is $\mathbb{B}_{\epsilon_i}(\hat{\mathbb{P}}_i) = \{\mathbb{Q} \in \mathsf{M}(\Xi_i) \mid d_W(\hat{\mathbb{P}}_i, \mathbb{Q}) \le \epsilon_i\}$, where $d_W(\cdot,\cdot)$ is the 1-Wasserstein metric [15]. Each agent selects $\mathbf{x}_i \in \mathcal{X}_i \subseteq \mathbb{R}^{n_i}$ so that

$$\min_{\mathbf{x}_i\in\mathcal{X}_i} \max_{\mathbb{Q}\in\mathbb{B}_{\epsilon_i}(\hat{\mathbb{P}}_i)} \mathbb{E}_{\mathbb{Q}}[J_i(\mathbf{x}_i, \mathbf{x}_{-i}, \boldsymbol{\xi}_i)] \text{ s.t. } \mathbf{x}_i \in \Omega_i(\mathbf{x}_{-i}), \quad (1)$$

for all $i \in \mathcal{A}$, where $\mathbf{x}_{-i} = \mathrm{col}(\mathbf{x}_j)_{j\neq i}$ and $N = \sum_{i\in\mathcal{A}} n_i$.

Definition 1 (DRGNE [16]): Given samples $\hat{\boldsymbol{\xi}}_i \in \Xi_i^{|\mathcal{S}_i|}$, $\mathbf{x}^* = \mathrm{col}(\mathbf{x}_i^*)_{i\in\mathcal{A}} \in \mathcal{X} = \prod_{i\in\mathcal{A}} \mathcal{X}_i$ is a distributionally robust generalized Nash equilibrium of (1) if, for all $i \in \mathcal{A}$,

$$\mathbf{x}_i^* \in \operatorname*{argmin}_{\mathbf{x}_i\in\mathcal{X}_i\cap\Omega_i(\mathbf{x}_{-i}^*)} \max_{\mathbb{Q}\in\mathbb{B}_{\epsilon_i}(\hat{\mathbb{P}}_i)} \mathbb{E}_{\mathbb{Q}}[J_i(\mathbf{x}_i, \mathbf{x}_{-i}^*, \boldsymbol{\xi}_i)]. \quad (2)$$

Each agent thus builds a private ambiguity set from local data and radius ϵ_i, extending standard DRO models that use a common ambiguity set [15].

II. The Energy Community Model

Set $\mathcal{P}$ indicates the prosumers' index set, while index 0 is used to denote quantities associated with the PLM. An overview of the energy community is provided in Fig. 1.

A. Energy Retailer

The energy retailer is assumed to be capable of injecting (absorbing) any amount of energy into (from) the prosumers' community and the PLM, at any time $k \in \mathcal{K}$. Such an assumption is realistic when the size of the infrastructure underlying the energy community is far smaller than the one characterising the retailer, e.g., the national grid operator. At a given time $k \in \mathcal{K}$, the retailer is willing to sell energy from prosumers at price $\pi_{1,k} \in \mathbb{R}_{\ge 0}$. Conversely, quantity $\pi_{2,k} \in \mathbb{R}_{\ge 0}$ represents the price the retailer offers to pay the prosumer if the latter injects energy back into the grid.

Assumption 1: $\pi_{1,k} > \pi_{2,k}$ holds for all $k \in \mathcal{K}$.

The net cost (revenue) for buying energy from (selling energy to) the retailer is given by function $R_k : \mathbb{R} \to \mathbb{R}$, defined as $R_k(z) = \max\{\pi_{1,k} z, \pi_{2,k} z\}$, for all $k \in \mathcal{K}$. for some energy amount $z \in \mathbb{R}$. Note that $R_k(\cdot)$ is convex for each $k \in \mathcal{K}$ as a consequence of Assumption 1.

B. Virtual Energy Storage Services

Energy storage services mitigate the need for private storage, which entails space, investment, and maintenance costs. Alternatively, a *Virtual Energy Storage Service* (VESS) provider owns the infrastructure and offers storage to prosumers for a fee. In *storage virtualization*, the physical system is not a dedicated battery but leverages devices whose primary function is not energy storage.

Here, these devices are batteries of parked, inactive EVs. From the prosumer viewpoint, this is equivalent to private storage; the provider's management of physical assets is transparent to them. Let $q_{ik} \in \mathbb{R}$ denote the energy prosumer $i \in \mathcal{P}$ injects ($q_{ik} > 0$) or withdraws ($q_{ik} < 0$) at time $k \in \mathcal{K}$. The state of charge $s_{ik} \in \mathbb{R}$ evolves as follows:

$$\forall i \in \mathcal{P}: \begin{cases} s_{ik} = s_{i,0} + \sum_{\kappa=1}^{k} q_{i\kappa} \ge 0, \quad \forall k \in \mathcal{K} & (3a) \\ s_{i,K} = s_{i,0} & (3b) \end{cases}$$

with $s_{i,0} \in \mathbb{R}_{\ge 0}$ being the initial condition for the i-th prosumer's VESS. In other terms, s_{ik} represents the amount of energy the i-th prosumer is entitled to take back from the VESS provider. Note that in (3b) we set the initial virtual storage of each prosumer to equal its final value, to keep consistency across the results obtained in time window $\mathcal{K}$. For notational convenience, let us define the aggregate prosumers' virtual energy state of charge at time $k \in \mathcal{K}$ as $S_k \in \mathbb{R}_{\ge 0}$ and the prosumers' aggregate net virtual energy flow as $Q_k \in \mathbb{R}$, i.e.,

$$S_k = \sum_{i\in\mathcal{P}} s_{ik}, \; Q_k = \sum_{i\in\mathcal{P}} q_{ik}, \quad \forall k \in \mathcal{K}. \quad (4)$$

In our setting, the VESS provider corresponds to the PLM, which is capable of aggregating the free space of the parked EV's batteries to create an energy storage buffer that can be offered to prosumers as an energy storage service. Such a buffer is the physical energy storage system (PES) whose state of charge is denoted with $b_k \in \mathbb{R}$ and is characterized by the following linear dynamics:

$$0 \le b_k = b_0 + \sum_{\kappa=1}^{k} Q_\kappa \le \beta_k, \quad \forall k \in \mathcal{K} \quad (5)$$

where $b_0 \in \mathbb{R}_{\ge 0}$ is the initial condition of the PES state-of-charge, while $\beta_k \in \mathbb{R}_{\ge 0}$ is the PES capacity, which depends on the amount of parked EVs that offer energy space through their batteries.

Ideally, the balance condition $S_k = b_k$ should be satisfied, i.e., the collective amount of VESS prosumers' use and the actual amount of physical storage should be equal. However, since b_k should be bounded by the uncertain quantity β_k, it is challenging to ensure that such a condition is met beforehand. The role of the PLM is acting as a mediator between the VESS and the PES resulting from aggregating parked EVs. Specifically, it decides an admissible VESS flow, $\tilde{Q}_k \in \mathbb{R}$, under the balance constraint

$$Q_k = \tilde{Q}_k, \quad \forall k \in \mathcal{K}. \tag{6}$$

Note that, from (3), the dynamics of the VESS, namely the left-hand side of (6), solely depend on the prosumers' decisions. This implies the presence of a coupling constraint to achieve consensus between the prosumers and the PLM. In the following sections, we characterize the PLM and the prosumer model in more detail.

C. Parking Lot Manager

The virtual energy storage service price per unit of energy that the PLM charges the i-th prosumer at time k is $p_k \in \mathbb{R}_{\geq 0}$. Such a quantity can be interpreted as a contract between the prosumers' community and the PLM for the VESS service. The uncertainty in the PLM's model is expressed by the PES capacity β_k, which depends on the EVs' arrival and departure patterns. It is reasonable to assume that β_k itself is upper bounded, i.e., it exists a $\beta_{\max} \in \mathbb{R}_{\geq 0}$ such that $\beta_k \leq \beta_{\max}$ holds[1]. Thus, the support associated with the PLM's uncertainty is $\Xi_0 = [0, \beta_{\max}]$. The PLM's decision variable is $\tilde{Q}_k$, for all $k \in \mathcal{K}$, representing the admissible amount of aggregate VESS energy flow. Such a variable is determined to minimize an objective function $J_0 : \mathbb{R}^K \times \mathbb{R}^K \to \mathbb{R}$, defined as

$$J_0(\tilde{\mathbf{Q}}, \boldsymbol{\beta}) = \sum_{k \in \mathcal{K}} \pi_{1,k}[\tilde{b}_k - \beta_k]_+ - p_k \tilde{b}_k \tag{7}$$

where $\tilde{\mathbf{Q}} = \mathrm{col}(\tilde{Q}_k)_{k\in\mathcal{K}}$, $\boldsymbol{\beta} = \mathrm{col}(\beta_k)_{k\in\mathcal{K}}$, and $\tilde{b} \in \mathbb{R}$ corresponds to the PES dynamics in (5) from the perspective of the PLM. This state-of-charge of the PLM follows the dynamics:

$$\tilde{b}_k = b_0 + \sum_{\kappa=1}^{k} \tilde{Q}_\kappa \geq 0, \quad \forall k \in \mathcal{K} \tag{8}$$

One can verify that $Q_k = \tilde{Q}_k \implies b_k = \tilde{b}_k$, i.e., the PES state-of-charge dynamics from the perspective of the prosumers and the PLM coincide when reaching an equilibrium. J_0 represents the total cost over the time horizon $\mathcal{K}$ for the PLM. The term $\pi_{1,k}[\tilde{b}_k - \beta_k]_+$ in (7) corresponds to the additional cost the PLM incurs if the PES state-of-charge exceeds the total capacity β_k. In fact, differently from (5), Equation (8) does not impose an upper bound. In other words, it corresponds to what the PLM would pay if the realization of β_k was such that the upper bounds in (5) could not be satisfied. Thus, this term does not represent an actual cost that the PLM would pay in a nominal operational state, but rather a collateral cost due to unpredicted and inconvenient realizations of the uncertainty. Finally, term $-p_k\tilde{b}_k$ expresses the revenues for the provided service.

[1]The total number of spots in a parking lot is known by the PLM. However, the maximum capacities of the EVs that can be parked are generally not known, being dependent on the specifics of the current available models. Nonetheless, one can impose a reasonable maximum value for β_k.

Thus, the PLM aims at solving the following distributionally robust problem for the time window $\mathcal{K}$:

$$\begin{aligned} \underset{\tilde{\boldsymbol{Q}}}{\text{minimize}} \ \max_{\mathbb{Q}\in\mathbb{B}_{\epsilon_0}(\hat{\mathbb{P}}_0)} \ & \mathbb{E}_{\mathbb{Q}}\left[J_0(\tilde{\mathbf{Q}}, \boldsymbol{\beta})\right] && (\mathcal{P}_{\text{PLM}}) \\ \text{subject to } \ & \tilde{b}_k = b_0 + \sum_{\kappa=1}^{k} \tilde{Q}_\kappa \geq 0, \quad \forall k \in \mathcal{K} && \text{(cf. (8))} \\ & Q_k = \tilde{Q}_k, \quad \forall k \in \mathcal{K} && \text{(cf. (6))} \end{aligned}$$

It can be shown that the problem ($\mathcal{P}_{\text{PLM}}$) is convex: the objective J_0 is piecewise linear, while the constraints are constituted by linear equalities and inequalities.

D. Prosumers

Apart from the possibility of using a virtual energy storage system, prosumer $i \in \mathcal{P}$ is characterized by the uncertain quantity $\delta_{ik} \in \mathbb{R}$, i.e., the net difference between autonomous energy generation and demand at time $k \in \mathcal{K}$. Such a term is bounded by physical limits of the i-th prosumer's infrastructure, represented by $\delta_i^{\max} \geq 0$ and $\delta_i^{\min} \leq 0$. Clearly, δ_{ik} is uncertain, for all $k \in \mathcal{K}$, since the autonomous generation is obtained by renewable means, such as wind or photovoltaic energy. The demand depends on the prosumers' behaviour, which, although often characterized by common and periodic patterns, is usually unpredictable. As such, its support is $\Xi_i = [\delta_i^{\min}, \delta_i^{\max}]$, for all $i \in \mathcal{P}$.

As introduced in Section II-B, prosumers can leverage VESS capability, whose state-of-charge is given by s_{ik}, by injecting and drawing q_{ik}, which corresponds to the decision variable of prosumer i. As such, prosumers' decisions must ensure a net-zero flow. Thus, we distinguish between two cases: 1) $q_{ik} - \delta_{ik} > 0$: The surplus is sold to the retailer; 2) $q_{ik} - \delta_{ik} < 0$: the needed amount is bought to restore the balance[2]. Therefore, each prosumer $i \in \mathcal{P}$ choses q_{ik} aiming at minimizing individual objective $J_i : \mathbb{R}^K \times \mathbb{R}^K \to \mathbb{R}$, defined as:

$$J_i(\mathbf{q}, \boldsymbol{\delta}) = \sum_{k\in\mathcal{K}} p_k s_{ik} + R_k(q_{ik} - \delta_{ik}). \tag{10}$$

with $\mathbf{q} = \mathrm{col}(q_k)_{k\in\mathcal{K}}$ and $\boldsymbol{\delta} = \mathrm{col}(\delta_k)_{k\in\mathcal{K}}$. Note that the term $p_k s_{ik}$ expresses the virtual energy service fee paid to the PLM, while $R_k(q_{ik} - \delta_{ik})$ expresses the cost (revenue) for balancing out the power flow by buying (selling) energy from (to) the retailer. Note that, differently from J_0, the prosumers' objective is a local function, i.e., it bears no dependence on the PLM decision.

In summary, the distributionally robust problem that each prosumer wants to solve is the following:

$$\begin{aligned} \underset{\mathbf{q}}{\text{minimize}} \ \max_{\mathbb{Q}\in\mathbb{B}_{\epsilon_i}(\hat{\mathbb{P}}_i)} \ & \mathbb{E}_{\mathbb{Q}}\left[J_i(\mathbf{q}, \boldsymbol{\delta})\right] && (\mathcal{P}_{\text{PRS}}) \\ \text{subject to } \ & s_{ik} = s_{i,0} + \sum_{\kappa=1}^{k} q_{i\kappa} \geq 0 \quad \forall k \in \mathcal{K} && \text{(cf. (3a))} \\ & s_{i,K} = s_{i,0} && \text{(cf. (3b))} \\ & Q_k = \tilde{Q}_k \quad \forall k \in \mathcal{K} && \text{(cf. (6))} \end{aligned}$$

[2]Recall the sign convention for q_{ik} as in (3).

The prosumer optimization problem in ($\mathcal{P}_{\text{PRS}}$) is also convex with respect to their own local decision variable.

III. Distributionally Robust Generalized Nash Equilibrium Problem

Agents in the energy community (Section II) minimize individual objectives subject to coupling constraints. The model is generally non-robust to uncertainty in prosumers' demand and renewable generation, as well as in the PLM's storage state of charge and capacity (e.g., due to EV arrivals/departures). Moreover, the underlying probability distributions are unknown and hard to estimate accurately without restrictive assumptions. Neglecting these factors may impair operation.

We therefore adopt the distributionally robust GNEP in (1), with shared constraint $Q_k = \tilde{Q}_k$ for all $k \in \mathcal{K}$, ensuring agreement between prosumers and the PLM. This accounts for the distributional ambiguity of the stochastic parameters. For uncertainties β_k and δ_{ik}, $i \in \{0\} \cup \mathcal{P}$, each agent has $|\mathcal{S}_i|$ i.i.d. samples over $\mathcal{K}$. A Wasserstein ball centered at the empirical distribution $\hat{\mathbb{P}}_i$ (Section I-A) is built from these samples, yielding heterogeneous, data-driven ambiguity sets and preserving sample privacy. The radius ϵ_i, $i \in \{0\} \cup \mathcal{P}$, may be fixed or time-varying in k, allowing arbitrary prediction models. For the remainder, we impose the following standing assumption.

A. Distributionally Robust Reformulation

Let us introduce the auxiliary variables w_i, $v_{ik\sigma j}$, $y_{ik\sigma} \in \mathbb{R}$, for all $i \in \{0\} \cup \mathcal{P}$, with $\mathbf{v}_i = \text{col}(v_{ijk\sigma})_{k\in\mathcal{K},j=1,2,\sigma\in\mathcal{S}_i}$ and $\mathbf{y}_i = \text{col}(y_{ik\sigma})_{k\in\mathcal{K},\sigma\in\mathcal{S}_i}$. Finally, let us introduce g_i, defined as

$$g_i = \epsilon_i w_i + \frac{1}{|\mathcal{S}_i|}\sum_{\sigma\in\mathcal{S}_i}\sum_{k\in\mathcal{K}} y_{ik\sigma}, \quad \forall i \in \{0\} \cup \mathcal{P} \tag{12}$$

Theorem 1: The distributionally robust reformulations of the PLM and each prosumer's problems are as follows:
i) PLM's Reformulation: Let $u_{0,jk\sigma} \in \mathbb{R}$ be the following scalar expression

$$u_{0,jk\sigma} = (p_k - \mathbb{1}_{j=2}\pi_{1,k})\tilde{b}_k + \hat{\beta}_{k\sigma}\mathbb{1}_{j=2}\pi_{1,k} + \left(\hat{\beta}_{k\sigma} - \frac{\beta^{\max}}{2}\right) v_{0,jk\sigma} + y_{0,k\sigma} \tag{13}$$

for all $k \in \mathcal{K}$, $j = 1,\dots,3$ and $\sigma \in \mathcal{S}_0$. The game in (1) admits the following finite-dimensional convex reformulation

$$\begin{aligned}
&\underset{\tilde{Q},w_0,\mathbf{v}_0,\mathbf{y}_0}{\text{minimize}}\ g_0 && (\mathcal{P}_{\text{PLM}}^{\text{dro}})\\
&\text{subject to}\\
&|v_{0,jk\sigma} + \mathbb{1}_{j=2}\pi_{1,k}|_\infty \le w_0 \quad \forall k\in\mathcal{K}, \forall j=1,2, \forall\sigma\in\mathcal{S}_0 && \text{(i)}\\
&\frac{\beta^{\max}}{2}|v_{0,jk\sigma}| \le u_{0,jk\sigma} \quad \forall k\in\mathcal{K}, \forall j=1,2, \forall\sigma\in\mathcal{S}_0 && \text{(ii)}\\
&\tilde{b}_k = b_0 + \sum_{\kappa=1}^{k}\tilde{Q}_\kappa \ge 0 \quad \forall k\in\mathcal{K} && \text{(iii)}\\
&Q_k = \tilde{Q}_k \quad \forall k\in\mathcal{K} && \text{(v)}
\end{aligned}$$

ii) Prosumers' Reformulation: Let $u_{ik\sigma j} \in \mathbb{R}$ be a scalar expression, defined as follows

$$u_{ijk\sigma} = \hat{\delta}_{ik\sigma}\pi_{jk} + y_{ik\sigma} - p_k s_{ik} - \pi_{jk}q_{ik} + \left(\hat{\delta}_{ik\sigma} - \frac{\delta_i^{\max} + \delta_i^{\min}}{2}\right) v_{ijk\sigma} \tag{14}$$

for all $k \in \mathcal{K}$, $j = 1,\dots,2$ and $\sigma \in \mathcal{S}_i$. For all prosumers $i \in \mathcal{P}$, the distributionally robust game in (1) admits the following finite-dimensional exact convex reformulation:

$$\begin{aligned}
&\underset{\mathbf{q}_i,w_i,\mathbf{v}_i,\mathbf{y}_i}{\text{minimize}}\ g_i && (\mathcal{P}_{\text{PRS}}^{\text{dro}})\\
&\text{subject to}\\
&|v_{ijk\sigma} + \pi_{jk}|_\infty \le w_i \quad \forall k\in\mathcal{K}, \forall j=1,2, \forall\sigma\in\mathcal{S}_i && \text{(i)}\\
&\frac{\delta_i^{\max} - \delta_i^{\min}}{2}|v_{ijk\sigma}| \le u_{ijk\sigma} \quad \forall k\in\mathcal{K}, \forall j=1,2, \forall\sigma\in\mathcal{S}_i && \text{(ii)}\\
&s_{ik} = s_{i,0} + \sum_{\kappa=1}^{k} Q_\kappa \ge 0 \quad \forall k\in\mathcal{K} && \text{(iii)}\\
&s_{i,1} = s_{i,K} && \text{(v)}\\
&Q_k = \tilde{Q}_k \quad \forall k\in\mathcal{K} && \text{(vi)}
\end{aligned}$$

Theorem 1 adapts [15, Theorem 4.2]; the proofline is reported in the supplementary material.

B. Equilibrium Seeking

The reformulations provided by Theorem 1 yield a deterministic generalized Nash Equilibrium problem. Based on the definitions in ($\mathcal{P}_{\text{PLM}}^{\text{dro}}$) and ($\mathcal{P}_{\text{PRS}}^{\text{dro}}$), such a problem can be compactly expressed as follows

$$\underset{\mathbf{x}_i}{\text{minimize}}\ g_i(\mathbf{x}_i) \text{ subject to } \mathbf{x}_i \in \mathcal{X}_i \cap \Omega_i(\mathbf{x}_{-i}) \tag{15}$$

for all $i \in \{0\} \cup \mathcal{P}$. For our specific case, let $n_i = 1 + K + 5KS_i$, for all $i \in \mathcal{P}$, so that the decision vectors for the PLM and the i-th prosumer are $\mathbf{x}_0 = [\tilde{\mathbf{Q}}, w_0, \mathbf{v}_0, \mathbf{y}_0]$ and $\mathbf{x}_i = [\mathbf{q}_i, w_i, \mathbf{v}_i, \mathbf{y}_i]$, the latter for all $i \in \mathcal{P}$. Finally, $\mathbf{x} = \text{col}(\mathbf{x}_i)_{i\in\{0\}\cup\mathcal{P}}$. Set $\mathcal{X}_i \subseteq \mathbb{R}^{n_i}$, for all $i \in \{0\} \cup \mathcal{P}$, constitute the local constraint set, defined as follow

$$\mathcal{X}_0 = \left\{\mathbf{x} \in \mathbb{R}^{n_0} \mid (\mathcal{P}_{\text{PLM}}^{\text{dro}}, \text{i})\text{-}(\mathcal{P}_{\text{PLM}}^{\text{dro}}, \text{v})\right\} \tag{16a}$$

$$\mathcal{X}_i = \left\{\mathbf{x} \in \mathbb{R}^{n_i} \mid (\mathcal{P}_{\text{PRS}}^{\text{dro}}, \text{i})\text{-}(\mathcal{P}_{\text{PRS}}^{\text{dro}}, \text{vi})\right\}, \quad \forall i \in \mathcal{P} \tag{16b}$$

where the local constraints introduced by the DRO reformulation have been included. Moreover, the shared constraints set is $\Omega_i(\mathbf{x}_{-i}) = \left\{\mathbf{x} \in \mathbb{R}^{n_i} \mid Q_k = \tilde{Q}_k,\ \forall k \in \mathcal{K}\right\}$ for all $i \in \{0\} \cup \mathcal{P}$.

C. GNE as Variational Inequality

The advantage brought by the DRO reformulation is the ability to compute an equilibrium for (15) leveraging the tools of monotone operator theory [17]. Specifically, the non-cooperative game in (15) can be cast as a variational inequality VI$(\mathbf{F}, \mathcal{Z})$, where $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$ is the so-called pseudogradient mapping, while $\mathcal{Z} \subseteq \mathbb{R}^n$. The definition of the VI problem is as follows:

Definition 2: A vector $\mathbf{z}^* \in \mathcal{Z}$ is said to be a solution for the VI if and only if $\inf_{\mathbf{z}\in\mathcal{Z}} \mathbf{F}(\mathbf{z}^*)^\top(\mathbf{z} - \mathbf{z}^*) \ge 0$.

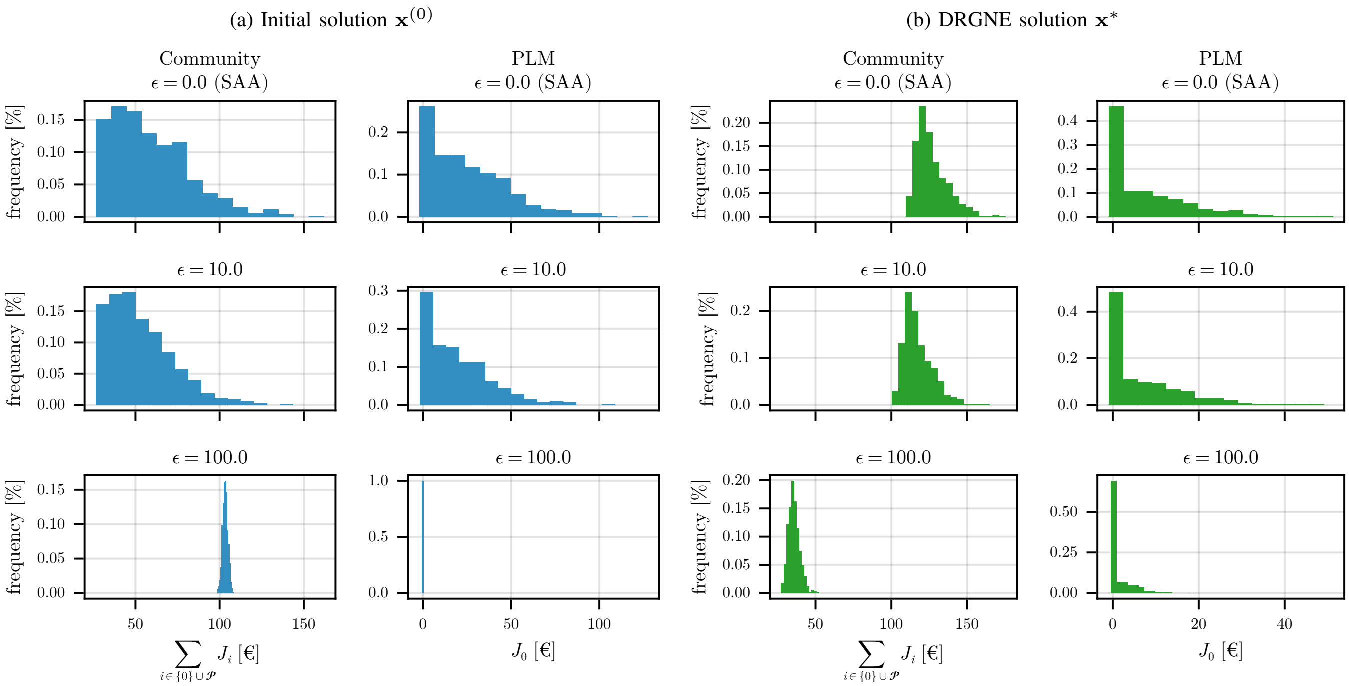

Fig. 2. Comparison between the out-of-sample performance of initial solution $\mathbf{x}^{(0)}$ (a) and the DRO Nash equilibrium $\mathbf{x}^{*}$ (b), for different values of the Wasserstein radius ϵ. Case $\epsilon = 0.0$ corresponds to the SAA approach.

A comprehensive review of monotone VI is provided in [18]. In our case, VIs can be employed for equilibrium seeking since, if $\mathbf{F}$ is the corresponding pseudogradient mapping and $\mathcal{Z}$ the set of agents' constraints [19], the solution $\mathbf{z}^{*}$ of the VI is a Nash equilibrium for (15) [20, Theorem 3.9]. The solutions of VIs that correspond to Nash equilibria are commonly referred to as *variational equilibria*.

In our setting, $n = K + \sum_{i \in \{0\} \cup \mathcal{P}} n_i$, $\mathbf{z} = \operatorname{col}(\mathbf{x}, \boldsymbol{\mu})$, with set $\mathcal{Z} = \bigcap_{i \in \{0\} \cup \mathcal{P}} \mathcal{X}_i \cap \mathbb{R}^K$ and the mapping $\mathbf{F}$ is given by:

$$\mathbf{F} : \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\mu} \end{bmatrix} \mapsto \begin{bmatrix} \operatorname{col}(\mathbf{G}_i)_{i \in \{0\} \cup \mathcal{P}} \\ \tilde{\mathbf{Q}} - \mathbf{Q} \end{bmatrix} \tag{17}$$

where $\boldsymbol{\mu} \in \mathbb{R}^K$ is the Lagrange multiplier vector associated with the shared constraint in $\Omega_i(\mathbf{x}_{-i})$. Finally, the agents' mapping $\mathbf{G}_i$ is defined as follows

$$\mathbf{G}_i = \begin{bmatrix} (2\mathbb{1}_{i \in \mathcal{P}} - 1)(\boldsymbol{\mu} + \rho(\mathbf{Q} - \tilde{\mathbf{Q}})) \\ \epsilon_i \\ \mathbf{0} \\ \mathbf{1}_{KS_i} / |\mathcal{S}_i| \end{bmatrix}, \quad \forall i \in \{0\} \cup \mathcal{P}.$$

Lemma 1: The mapping $\mathbf{F}$ is monotone, with $\operatorname{lip} \mathbf{F} = (P+1)K(\rho^2 + 2)$.

The proof is reported in the supplementary material. We can thus employ standard iterative schemes to solve $\operatorname{VI}(\mathbf{F}, \mathcal{X})$. Let $\operatorname{alg}_{\mathcal{X}}^{\mathbf{F}} : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathcal{X}$ be an equilibrium-seeking method mapping the current iterate $\mathbf{z}$ to $\operatorname{alg}_{\mathcal{X}}^{\mathbf{F}}(\mathbf{z}, \chi)$. For a suitable step-size χ, the iterates converge to $\mathbf{z}^{*}$. Strong monotonicity yields faster rates, while many methods ensure convergence under mere monotonicity. In most cases, admissible step-sizes depend on the Lipschitz constant of $\mathbf{F}$.

IV. Numerical Results

We refer the reader to the repository mentioned in Section I for details on the simulation setup. The game VI has been solved using `Monviso.jl` [21].

Equilibrium-seeking problems depend on the initial solution, in our case $\mathbf{x}^{(0)}$ and $\boldsymbol{\mu}^{(0)}$. A common and reasonable choice for the Lagrange multiplier is $\boldsymbol{\mu}^{(0)} = \mathbf{0}$, which can be interpreted as the stage where each agent does not have any economic incentive to choose a decision variable that meets the shared constraint, i.e., $\tilde{\mathbf{Q}} = \mathbf{Q}$. Moreover, the choice for an initial solution $\mathbf{x}^{(0)}$ follows a similar rationale: a sensible choice would be the *ideal* solution, where all agents can disregard the dependency on the shared constraints. Specifically, the agents' initial solution $\mathbf{x}_i^{(0)}$ is evaluated by solving ($\mathcal{P}_{\text{PLM}}^{\text{dro}}$) and ($\mathcal{P}_{\text{PRS}}^{\text{dro}}$), disregarding the shared constraint $Q_k = \tilde{Q}_k$. where $\mathcal{X}_0, \mathcal{X}_i$ are defined as in (16). In the following, we discuss the out-of-sample performance for different values of ϵ_i, and the relationship between the initial and equilibrium solutions. Without loss of generality, we consider $\epsilon_i = \epsilon$, for all $i \in \{0\} \cup \mathcal{P}$. The out-of-sample performance is reported in Fig. 2. OOS performance is measured by the value that the objective function takes when evaluated against the realization of the stochastic term $\boldsymbol{\xi}_i \neq \hat{\boldsymbol{\xi}}_i$. In a sense, calculating $\mathbf{x}^{(0)}$ (and subsequently $\mathbf{x}^{*}$) can be interpreted as the *training* phase, using a statistical machine learning terminology, with the OOS evaluation being the *testing* phase. Figure 2a illustrates the OOS for the initial solution $\mathbf{x}^{(0)}$, while Fig. 2b reports the OOS at the equilibrium point $\mathbf{x}^{*}$. Histograms represent the normalized frequency of the indicated objective values. Both plots are arranged in a grid: the left columns represent the OOS performance of the entire community, in terms of total cost $J_0 + \sum_{i \in \mathcal{P}} J_i$; the right column, instead, reports the PLM cost/revenue J_i. We are interested in the total costs since they provide an overview of the degree of efficiency of the energy market, at least restricted to the analyzed energy community. Similarly, we focus on the PLM performance, since it constitutes the additional component of the community.

The rows of both Figures 2a and 2b report the OSS performance for different values of the Wasserstein radius ϵ, i.e., 0, 10, and 100. The first is equivalent to finding a stochastic Nash equilibrium of $(\mathcal{P}_{\text{PLM}})$ and $(\mathcal{P}_{\text{PRS}})$ via sampled average approximation (SAA), since, for $\epsilon = 0$, the Wasserstein ball collapses into the empirical distribution. For our purposes, this case serves as a comparative approach. Radii $\epsilon = 10.0$ and $\epsilon = 100.0$ have been chosen to analyze the performance for *medium* and *large* radius values. We highlight that the OOS performance has been evaluated with the same distributions used during training, but with a larger variance. This simulates the fact that, for real applications, the true distribution can strongly deviate from the empirical one, justifying the need for distributionally robust approaches. Let us start from the initial solution, i.e., Fig. 2a: the case $\epsilon = 10.0$ is indeed more robust than the SAA approach, as the right tail of the distribution gets flatter in the second row plot. As expected, the case $\epsilon = 100.0$ provides a degenerate, extremely robust solution, in the sense that a much larger average is observed compared to the previous two cases, with the PLM performance degenerating to the null objective (i.e., no ESS service provided). Figure 2b can be interpreted as the shift induced by the equilibrium-seeking process on the OSS performance of the initial solution: both the SAA and $\epsilon = 10.0$ cases create a shift to the right in mean, i.e., an increase in cost for the community at large. The PLM performance is more robust with respect to the initial solution, with the $\epsilon = 10.0$ case comparable to the SAA approach. Case $\epsilon = 100.0$, instead, seems to improve the initial OOS performance, on average, despite being worse at the initial solution. What can be inferred from the overall performance is that the equilibrium-seeking process does influence the OSS at the Nash solution, possibly shifting the average (albeit maintaining the overall dispersion). This is comparable to the economic efficiency loss that characterizes Nash equilibria in economic settings, where the drawback of assuming the worst behaviour of the other agents is reflected in the worsening of the overall community performance.

V. Conclusions and Future Work

In this work, we proposed a game-theoretic distributionally robust virtual storage model for parking lot management (PLM) that efficiently utilizes EVs as distributed energy storage devices, while managing uncertainties in prosumers' demand and generation, as well as in EV arrivals/departures through a distributionally robust approach.

Future work will focus on integrating the EV owners as independent entities in the community model, thus augmenting it to capture more complicated relations, while maintaining computational tractability and out-of-sample performance.

References

[1] Y. Noorollahi, A. Golshanfard, A. Aligholian, B. Mohammadi-ivatloo, S. Nielsen, and A. Hajinezhad, "Sustainable energy system planning for an industrial zone by integrating electric vehicles as energy storage," *Journal of Energy Storage*, vol. 30, p. 101553, 2020.

[2] S. Nazari, F. Borrelli, and A. Stefanopoulou, "Electric vehicles for smart buildings: A survey on applications, energy management methods, and battery degradation," *Proceedings of the IEEE*, vol. 109, no. 6, pp. 1128–1144, 2020.

[3] D. Zhao, H. Wang, J. Huang, and X. Lin, "Virtual energy storage sharing and capacity allocation," *IEEE transactions on smart grid*, vol. 11, no. 2, pp. 1112–1123, 2019.

[4] Y. Shen, Y. Li, D. Liu, Y. Wang, J. Sun, and S. Sun, "Energy management strategy for hybrid energy storage system based on model predictive control," *Journal of Electrical Engineering & Technology*, vol. 18, pp. 3265 – 3275, 2023.

[5] C. Zeng, D. Ye, N. Wang, C. Feng, and C. Yang, "Robot-based automatic charging for electric vehicles using incremental learning and biomimetic control," in *2025 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2025, pp. 15 479–15 485.

[6] H. Tan, Y. Yuan, H. Yan, S. Zhong, and Y. Yang, "Human preference-aware rebalancing and charging for shared electric micromobility vehicles," in *2024 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2024, pp. 9608–9615.

[7] J. A. Taylor, "Optimal energy management and storage sizing for electric vehicles with dual storage," *IEEE Transactions on Control Systems Technology*, vol. 31, pp. 872–880, 2023.

[8] M. Fochesato, F. Fabiani, and J. Lygeros, "Generalized uncertain Nash games: Reformulation and robust equilibrium seeking," *European Control Conference (ECC)*, pp. 1–6, 2023.

[9] F. Fele and K. Margellos, "Probably approximately correct Nash equilibrium learning," *IEEE Transactions on Automatic Control*, vol. 66, no. 9, pp. 4238–4245, 2021.

[10] F. Fabiani, K. Margellos, and P. J. Goulart, "Probabilistic feasibility guarantees for solution sets to uncertain variational inequalities," *Automatica*, vol. 137, p. 110120, 2022.

[11] G. Pantazis, F. Fele, and K. Margellos, "A priori data-driven robustness guarantees on strategic deviations from generalised Nash equilibria," *IFAC Automatica*, pp. 1–13, 04 2024.

[12] ——, "On the probabilistic feasibility of solutions in multi-agent optimization problems under uncertainty," *European Journal of Control*, vol. 63, pp. 186–195, 2022.

[13] F. Fabiani and B. Franci, "On distributionally robust generalized Nash games defined over the wasserstein ball," *Journal of Optimization Theory and Applications*, vol. 199, no. 2, pp. 298–309, 10 2023.

[14] A. Ben-Tal, L. Ghaoui, and A. Nemirovski, "Robust optimization," *Princeton Series in Applied Mathematics*, 08 2009.

[15] P. Mohajerin Esfahani and D. Kuhn, "Data-driven distributionally robust optimization using the wasserstein metric: Performance guarantees and tractable reformulations," *Mathematical Programming*, vol. 171, no. 1, pp. 115–166, 2018.

[16] M. Aghassi and D. Bertsimas, "Robust game theory," *Mathematical programming*, vol. 107, no. 1, pp. 231–273, 2006.

[17] E. K. Ryu and W. Yin, *Large-scale convex optimization: algorithms & analyses via monotone operators*. Cambridge University Press, 2022.

[18] F. Facchinei and J.-S. Pang, "Finite-dimensional variational inequalities and complementarity problems," *Springer-Verlag New York*, 2003.

[19] G. Scutari, D. P. Palomar, F. Facchinei, and J.-S. Pang, "Convex optimization, game theory, and variational inequality theory," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 35–49, 2010.

[20] F. Facchinei and C. Kanzow, "Generalized nash equilibrium problems," *Annals of Operations Research*, vol. 175, no. 1, pp. 177–211, 2010.

[21] N. Mignoni, R. R. Baghbadorani, R. Carli, P. M. Esfahani, M. Dotoli, and S. Grammatico, "`monviso`: A Python package for solving monotone variational inequalities," in *2025 European Control Conference (ECC). IEEE*, 2025.