2026 IEEE International Conference on Robotics and Automation (ICRA 2026)
June 1-5, 2026. Vienna, Austria

A New Repetitive Control Framework for Robot Manipulators: Optimal Controller Design and Stability Analysis

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Abstract— **This paper provides a new repetitive control framework for robot manipulators with periodic reference signals. We first take the inverse dynamics (ID) approach to a robot manipulator to transform its nonlinear input/output behavior into an equivalent linear time-invariant (LTI) system, for which the conventional repetitive control strategy is employed. To facilitate an optimal controller synthesis and an associated stability analysis, we next derive the so-called delay-feedback system. We then provide a linear matrix inequality (LMI)-based optimal controller synthesis procedures for minimizing the H_∞ norm from the disturbance to the tracking error. We next established operator-theoretic stability assertions in terms of the monodromy operator. In particular, a necessary and sufficient condition for the exponential stability of the delay-feedback system is derived. Finally, experiment comparisons are given to demonstrate the overall developed arguments.**

I. Introduction

As robot manipulators have been widely employed in industrial applications, numerous control strategies have been developed, as discussed in [1], [2]. These approaches are typically derived using properties of the Lagrangian formulation describing the dynamics of robot manipulators. To avoid directly handling the nonlinear differential equations in the Lagrangian formulation, the inverse dynamics (ID) approach is adopted in [3], since it acts as a feedback linearization and transforms the nonlinear dynamics into an equivalent linear time invariant (LTI) system. The effectiveness of ID-based control methods has been demonstrated through simulation and experimental studies [4], [5], showing accurate trajectory tracking. Moreover, the effects of modeling errors and disturbances on stability and performance resulting from the ID approach are analyzed in [6], [7]. Related optimal and robust performance analyses for Euler–Lagrange type systems have also been actively investigated in recent studies, including H_∞ optimal disturbance estimation and generalized H_2 tracking control for uncertain Euler–Lagrange equations [8], as well as robust balancing control of biped robots under external forces [9].

To address the stability of robot manipulators with periodic reference and disturbance signals, operator-based arguments on the repetitive control (RC) framework [10] are given in [11]. More precisely, the RC framework is based on the internal model principle [12], in which the corresponding repetitive controller generates harmonic signals at the frequencies $\omega = 2n\pi/L$ $(n = 0, \pm 1, \pm 2, \dots)$ for the period L of the reference and disturbance signals by taking the time-delay $\exp(-sL)$ in the associated feedback loop, to attain the zero steady-state tracking error. The effectiveness of the RC framework for robot manipulators has also been discussed in several works, e.g., flexible manipulators [13] and finite dimensional passive controller [14].

However, the above studies on the RC framework do not consider any optimizations for the overall closed-loop systems, although some arguments on the optimality are partially considered in [15]. A new method for designing the low-pass filter involved in the RC framework is also given in [16], but no optimality is discussed in that study. Despite recent progress in induced-norm-based optimal control for systems [17]–[21], such optimality considerations have not been incorporated into existing repetitive control frameworks. To address this issue, this paper develops controller synthesis procedures that minimize the H_∞ norm from disturbance to tracking error. Since the H_∞ norm coincides with the L_2 induced norm, the resulting controller minimizes tracking error energy, leading to rapid disturbance attenuation and improved transient performance.

To enable optimal controller synthesis within the RC framework for the ID approach, a delay feedback system is first derived. LMI based synthesis methods are then developed to minimize the H_∞ norm while accounting for the effect of the time delay $\exp(-sL)$ on stability. An operator theoretic necessary and sufficient condition for exponential stability is established via the monodromy operator [22], and comparative experiments are presented to validate the proposed approach.

The contributions of this study can be described as follows.

- The so-called delay-feedback system is derived with respect to the RC framework for the ID approach to robot manipulators.

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This study was carried out with the support of ´R&D Program for Forest Science Technology (RS-2024-00403460)´ provided by Korea Forest Service(Korea Forestry Promotion Institute).

This study was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education(RS-2025-25428747)

This study was supported by the Ministry of Trade, Industry and Energy (MOTIE, South Korea), through the Robot and Robot Service Digital Twinization Framework Technology, under Grant RS-2024-00424974.

- The LMI-based H_∞ optimal synthesis procedures are provided.
- The exponential stability condition for the delay-feedback system are established by using the monodromy operator.

The organization of this paper is as follows. In Section II, we revisit the conventional ID approach to robot manipulators with periodic reference signals and formulate the problem definition. The repetitive control framework for the ID approach to robot manipulators and the delay-feedback system are provided in Section III. The LMI-based synthesis procedure for an H_∞ optimal controller is given in Section IV. The exponential stability for the delay-feedback system are characterized in Section V. The theoretical validity and the practical effectiveness of the developed arguments are demonstrated through some experiment results in Section VI. Finally, some concluding remarks are given in Section VII.

We use the notations in this paper as follows. The notations $\mathbb{R}^\nu$ and $\mathbb{R}_+$ denote the sets of ν-dimensional real vectors and non-negative real scalars, respectively. The notation $|\cdot|$ denotes the Euclidean norm of a finite-dimensional real vector. The notation $\|\cdot\|$ represents the L_2 norm of a hybrid continuous/discrete-time signal, whose distinction will be clear from the context. The notation $\rho(\cdot)$ denotes the spectral radius of an operator $(\cdot)$. The symbol $\succ$ is used to indicate the positive definiteness of a symmetric matrix, that is, $(\cdot) \succ 0$ means that $(\cdot)$ is symmetric and all its eigenvalues are strictly positive. A function $\zeta : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is strictly increasing and satisfies $\zeta(0) = 0$, and a function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{KL}$ if $\beta(r, t) \in \mathcal{K}$ for each fixed $t \in \mathbb{R}_+$ and $\beta(r, t)$ is strictly decreasing in t for each fixed $r \in \mathbb{R}_+$.

II. Inverse Dynamics Approach to Robot Manipulators and Problem Definition

The dynamic equation of a robot manipulator is given by

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau + \tau_d \tag{1}$$

where $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $V(q, \dot{q}) \in \mathbb{R}^n$ is the Coriolis and centrifugal vector, $G(q) \in \mathbb{R}^n$ denotes the gravitational torque vector, $q(t) \in \mathbb{R}^n$ is the generalized coordinate vector, $\tau(t) \in \mathbb{R}^n$ is the control input, and $\tau_d(t) \in \mathbb{R}^n$ is the disturbance torque.

For this manipulator, we consider the trajectory tracking problem described by

$$q(t) \to q_d(t) \quad (t \to \infty) \tag{2}$$

where $q_d(t) \in \mathbb{R}^n$ is the reference trajectory. As one of the effective methods to address this problem, we employ the inverse dynamics (ID) approach [3], which is given by

$$M(q)(\ddot{q} - u) + V(q, \dot{q}) + G(q) = \tau \tag{3}$$

where u is an auxiliary control input to stabilize the resulting closed-loop systems. Substituting (3) into (1) yields

$$\ddot{e} = u + M^{-1}(q)\tau_d =: u + w \tag{4}$$

where w represents the disturbance and e is the tracking error defined as

$$e(t) := q_d(t) - q(t) \in \mathbb{R}^n \tag{5}$$

Combining (4) and (5) together with defining the output $y(t) := e(t)$ admits the representation

$$P : \begin{cases} \dot{x} = Ax(t) + Bu(t) + Bw(t) \\ y = Cx(t) \end{cases} \tag{6}$$

where $x(t) := \begin{bmatrix} e^T(t) & \dot{e}^T(t) \end{bmatrix}^T \in \mathbb{R}^{2n}$, and

$$A := \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad C := \begin{bmatrix} I & 0 \end{bmatrix}.$$

Practical robot manipulators are often inevitably subject to uncertainties and disturbances, and thus designing a stabilizing controller alone might not lead to a satisfactory tracking performance. We also note the fact that such components are usually regarded as involve in biasing and/or periodic components and the reference signal q_d is often taken by a periodic signal in many practical robot manipulators.

This motivates us to consider the repetitive control (RC) framework [10] for achieving high tracking accuracy in such practical operating conditions. In other words, assuming that the period of the reference signal is an integer multiple of that of the disturbance or vice versa, a time-delay is contained in the feedback loop of the RC framework; the denominator of the transfer function for the ideal repetitive controller is given by $1 - e^{-sL}$ with the period L of exogenous signals. Based on the internal model principle [12], this repetitive controller could attain the zero steady-state error with respect to such exogenous signals. Even though the RC framework for robot manipulators is dealt with deeply in [13], [14], the time-delay could lead to the instability of the closed-loop systems and somewhat conservative assertions on the stability are only given in those studies. Furthermore, no optimal control framework is provided in [13], [14].

In connection with the above issues, this paper is concerned with developing a new RC framework for robot manipulators to tackle the following problem.

Problem 1: Develop a new analysis and design method of the RC framework for robot manipulators equipped with the ID approach.

III. Repetitive Controller and Corresponding Closed-Loop Systems

On the basis of the ID approach to robot manipulators given by (6), this paper introduces to design the repetitive controller [10], [11] and derives the corresponding closed-loop systems, as a preliminary step to proceed to an optimal controller synthesis.

We first decompose the control input by

$$u = u_{\mathrm{tc}} + u_{\mathrm{rc}} \tag{7}$$

where the tracking control u_{tc} is given by

$$u_{\mathrm{tc}} = Kx = -K_{\mathrm{p}}e - K_{\mathrm{d}}\dot{e} \tag{8}$$

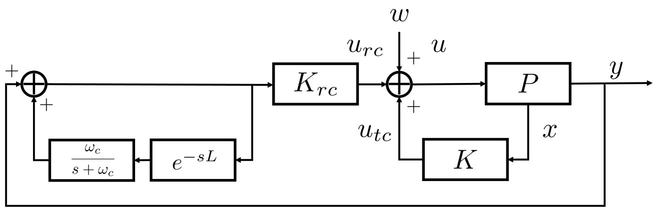


Fig. 1. The controller formulation using ID with RC for robot manipulators.

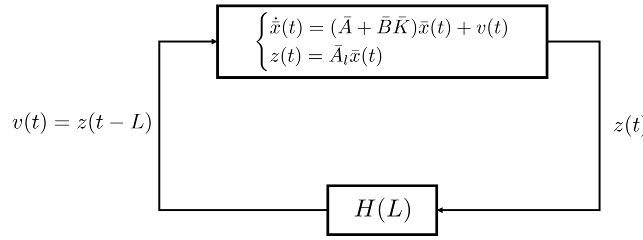


Fig. 2. The delay-feedback system Σ.

and $u_{\rm rc}$ is generated by the repetitive controller. For the simplicity of the arguments, assume that q_d and τ_d (and thus w) are periodic functions with the period L. Then, the repetitive controller [10] generates signals with the frequencies of $\frac{2n\pi}{L}$ $(L = 0,\ \pm 1,\ \pm 2, \dots)$. To this end, the transfer function of the ideal repetitive controller has its denominator by $1 - e^{-sL}$. However, such a transfer function has infinitely many poles along the imaginary axis and thus its direct implementation could lead to instability in practical situations. To deal with this issue, a low-pass filter is often taken, by which the repetitive controller would be given by

$$H_{\rm rc}(s) = \frac{U_{\rm rc}(s)}{Y(s)} = \frac{1}{1 - \frac{\omega_{\rm c}}{s+\omega_{\rm c}} e^{-sL}} K_{\rm rc} \tag{9}$$

as shown in Fig. 1, where $H_{\rm rc}(s)$ is the transfer function of the repetitive controller, $\omega_{\rm c}$ is the cutoff frequency of the low-pass filter, $K_{\rm rc}$ is the corresponding controller gain, and $U_{\rm rc}(s)$ and $Y(s)$ denote the Laplace transforms of the associated control input $u_{\rm rc}(t)$ and the system output $y(t)$, respectively.

For a tractable analysis for the closed-loop system shown in Fig. 1, we note that the state-space representation of (9) is given by

$$H_{\rm rc}: \begin{cases} \dot{x}_{\rm rc}(t) = \omega_{\rm c}\big(-x_{\rm rc}(t) + x_{\rm rc}(t-L) + e(t-L)\big) \\ u_{\rm rc}(t) = K_{\rm rc}\big(x_{\rm rc}(t) + e(t)\big) \end{cases} \tag{10}$$

By substituting (8) and the second equation of (10) into (7), we obtain

$$\begin{aligned} u(t) &= K_{\rm rc}x_{\rm rc}(t) + K_{\rm rc}e(t) - K_{\rm p}e(t) - K_{\rm d}\dot{e}(t) \\ &= (K_{\rm rc} - K_{\rm p})e(t) - K_{\rm d}\dot{e}(t) + K_{\rm rc}x_{\rm rc}(t) \end{aligned} \tag{11}$$

Next, we define the augmented state vector as

$$\bar{x}(t) := \begin{bmatrix} x^T(t) & x_{\rm rc}^T(t) \end{bmatrix}^T \tag{12}$$

Combining (10)–(12) together with (6) leads to the overall closed-loop system described by

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_{\rm l}\bar{x}(t-L) + \bar{B}w(t) + \bar{B}u(t) \\ u(t) = \bar{K}\bar{x}(t) \end{cases} \tag{13}$$

where

$$\bar{A} = \begin{bmatrix} A & 0_{2n\times n} \\ 0_{n\times 2n} & -\omega_{\rm c}I_n \end{bmatrix}, \quad \bar{A}_{\rm l} = \begin{bmatrix} & 0_{2n\times 3n} & \\ \omega_{\rm c}I_n & 0 & \omega_{\rm c}I_n \end{bmatrix},$$
$$\bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \bar{K} = \begin{bmatrix} K_{\rm rc} - K_{\rm p} & -K_{\rm d} & K_{\rm rc} \end{bmatrix}$$

The representation of (13) is a sort of delay-differential equations (DDEs) [23] and the relevant stability analysis has been developed by using the arguments on Lyapunov-Krasovskii functionals (LKFs) [24]. However, such arguments often lead to only conservative conditions for the stability of DDEs. For a more tractable stability condition, we introduce the delay-feedback system Σ as shown in Fig. 2, which is different but equivalent to (13) and is given by

$$\Sigma: \begin{cases} \dot{\bar{x}}(t) = (\bar{A} + \bar{B}\bar{K})\bar{x}(t) + \bar{B}w(t) + v(t) \\ z(t) = \bar{A}_{\rm l}\bar{x}(t) \\ v(t) = z(t-L) \end{cases} \tag{14}$$

where $H(L)$ is the pure delay described by $H(L): v(t) = z(t-L)$. This feedback-delay representation facilitates an optimal controller design and allows us to establish a necessary and sufficient condition for the stability in terms of the operator-based arguments [22], and the details will be provided in the following sections.

IV. Optimal Controller Design for Repetitive Control Framework

This section provides an optimal control scheme in the ID approach to robot manipulators described by (13). One could regard the linear quadratic regulator (LQR) [25] as one of the most efficient optimal controllers, but it inherently focuses only on the quadratic cost minimization. In contrast, it is obvious from (14) that the delay-feedback system Σ is affected by the uncertainty term w. To take into account the effects of w in a sophisticated fashion, this subsection is devoted to providing optimal controller synthesis procedures through the linear matrix inequality (LMI)-based approach. As a preliminary step to proceed to such synthesis procedures, we provide the simplified version of (14) by

$$\begin{cases} \dot{\bar{x}}(t) &= \bar{A}\,\bar{x}(t) + \bar{B}u(t) + \bar{B}w(t) \\ z_{\rm r}(t) &= E\bar{x}(t) \end{cases} \tag{15}$$

where $z_{\rm r}$ denotes the regulated output to be minimized and the weighting matrix E is determined depending on desired control specifications.

- Method: H_∞ optimal controller synthesis

Because an H_∞ optimal controller is for minimizing the L_2-induced norm from w to $z_{\rm r}$, implementing this controller could allow us to attain fast settling time in the presence of uncertainties and disturbances. Furthermore, the energy of the tracking error would be minimized by the H_∞ optimal controller, and thus we could expect quick convergence of the tracking error to the zero.

With these characteristics in mind, we give the synthesis procedure of an H_∞ optimal controller as follows. For a pre-specified constant $\gamma\ (>0)$, there exists a controller $u(t) = \bar{K}\bar{x}(t)$ such that the H_∞ norm of the closed-loop system obtained by connecting (15) and the controller is less than γ, if and only if the decision variable matrices X and L exist such that

$$\mathcal{X} \succ 0, \quad \begin{bmatrix} \mathcal{A}^T + \mathcal{A} & \mathcal{B} & \mathcal{C}^T \\ \mathcal{B}^T & -\gamma I & 0 \\ \mathcal{C} & 0 & -\gamma I \end{bmatrix} \prec 0 \tag{16}$$

where $\mathcal{X},\ \mathcal{A},\ \mathcal{B}$ and $\mathcal{C}$ are given by

$$\mathcal{X} := X,\ \mathcal{A} := \bar{A}X + \bar{B}L,\ \mathcal{B} := \bar{B},\ \mathcal{C} := EX \tag{17}$$

If these LMIs are feasible, then the corresponding state-feedback gain $\bar{K}$ is given by $\bar{K} = LX^{-1}$.

Since the above LMI-based condition corresponds to a necessary and sufficient condition for the existence of a controller ensuring that the H_∞ norm of the resulting closed-loop system is less than γ, taking γ smaller in the constraint of (16) leads to an H_∞ optimal controller.

V. Stability Analysis

Even though the preceding section is concerned with designing the H_∞ optimal controller, the corresponding arguments do not take into account the stability with respect to the time delay. In connection with this, we clarify whether the delay-feedback system Σ determined through the optimal controller synthesis proposed in the preceding section is stable by deriving an operator-based necessary and sufficient condition. More precisely, this section introduces an exponential stability condition for the delay-feedback system Σ. We derive a necessary and sufficient condition ensuring that $\xi(t) \to 0$ as $t \to \infty$ with an exponential rate, where

$$\xi(t) := \begin{bmatrix} \bar{x}^T(t) & v^T(t+\cdot) \end{bmatrix}^T,$$

and $v(\tau+\cdot)$ denotes the segment of $v(t)$ for $\tau \le t < \tau+L$. In connection with this, we introduce the following definition.

Definition 1: The delay feedback system Σ is said to be exponentially stable if there exist constants $c, \alpha > 0$ such that

$$\|\xi(t)\| \le ce^{-\alpha t}\|\xi(0)\|, \quad t \ge 0, \\ \forall \xi(0) \in \mathbb{R}^{3n} \oplus (L_2[0,L))^{3n}. \tag{18}$$

Here,

$$\|\xi(t)\| := \left(|\bar{x}(t)|^2 + \int_\tau^{\tau+L} |v(\theta)|^2\, d\theta \right)^{1/2}. \tag{19}$$

It follows from (18) that $\xi(t) \to 0$ as $t \to \infty$ whenever Σ is exponentially stable. Since $x(t)$ is an element of $\xi(t)$, this also implies that $x(t) \to 0$ as $t \to \infty$. We further note that Definition 1 ensures this convergence without imposing any additional restriction on the initial state $\xi(0)$.

To characterize exponential stability, we consider the monodromy operator of Σ defined by

$$\mathbf{H} := \begin{bmatrix} A_\mathrm{d} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}, \tag{20}$$

where $\bar{x}_k := \bar{x}(kL)$ and

$$\begin{aligned} A_\mathrm{d} &= e^{(\bar{A}+\bar{B}\bar{K})L}, \\ (\mathbf{D}v)(\tau) &= \int_0^\tau \bar{A}_\mathrm{l} e^{(\bar{A}+\bar{B}\bar{K})(\tau-\theta)} v(\theta)\, d\theta, \\ \mathbf{B}v &= \int_0^L e^{(\bar{A}+\bar{B}\bar{K})(L-\tau)} v(\tau)\, d\tau, \\ (\mathbf{C}\bar{x}_k)(\tau) &= \bar{A}_\mathrm{l} e^{(\bar{A}+\bar{B}\bar{K})\tau} \bar{x}_k. \end{aligned} \tag{21}$$

With this operator, the system evolution over one period is described as

$$\begin{bmatrix} \bar{x}_{k+1} \\ v((k+1)L+\cdot) \end{bmatrix} = \mathbf{H} \begin{bmatrix} \bar{x}_k \\ v(kL+\cdot) \end{bmatrix}. \tag{22}$$

Note that

$$\mathbf{H} : \mathbb{R}^{3n} \oplus (L_2[0,L))^{3n} \to \mathbb{R}^{3n} \oplus (L_2[0,L))^{3n},$$

since $A_\mathrm{d} : \mathbb{R}^{3n} \to \mathbb{R}^{3n}$, $\mathbf{B} : (L_2[0,L))^{3n} \to \mathbb{R}^{3n}$, $\mathbf{C} : \mathbb{R}^{3n} \to (L_2[0,L))^{3n}$, and $\mathbf{D} : (L_2[0,L))^{3n} \to (L_2[0,L))^{3n}$. Thus, $\mathbf{H}$ captures the hybrid continuous and discrete time behavior of Σ.

On the basis of the monodromy operator, we obtain the following result.

Theorem 1: The delay feedback system Σ is exponentially stable if and only if

$$\rho(\mathbf{H}) < 1,$$

where $\rho(\mathbf{H})$ denotes the spectral radius of $\mathbf{H}$.

It is clarified from Theorem 1 that $\xi(t) \to 0$ as $t \to \infty$ if and only if $\rho(\mathbf{H}) < 1$. Moreover, this condition is computationally feasible since $\rho(\mathbf{H})$ can be evaluated with arbitrary accuracy using the numerical procedure in [26].

VI. Experiment Results

This section presents experimental results to verify the theoretical and practical effectiveness of the overall arguments developed in this paper.

The experiments are conducted by using the robot manipulator and the periodic reference trajectory shown in Fig. 3–(a) and (b), respectively. The model information of the robot manipulator is given in Table I, and the reference trajectory is described by

$$\begin{cases} q_{d1} = 0.2\sin(\pi t), \\ q_{d2} = -\dfrac{\pi}{4} + 0.2\sin(\pi t), \\ q_{d3} = 0.4\sin(\pi t), \\ q_{d4} = -\dfrac{3}{4}\pi + 0.1\sin(\pi t), \\ q_{d5} = 0.1\sin(\pi t), \\ q_{d6} = \dfrac{\pi}{2} + 0.1\sin(\pi t), \end{cases} \tag{23}$$

For controller synthesis, we take the common weighting matrix by $E = \begin{bmatrix} 5I_6 & 1I_6 & 1I_6 \end{bmatrix}$.

The control parameters are determined through the synthesis procedures proposed in the preceding section. In other words, we obtain the H_∞ optimal controller.

For comparisons, we take other two conventional control approaches. The first controller '$\bar{u} = \bar{K}x$' with respect to (14) is obtained by taking the standard quadratic regulator [25] for the LTI system (15). The second controller is associated with the advanced low-pass filter synthesis [16] but no optimization is considered in that study. For the notational simplicity, 'the H_∞ optimal controller' obtained through the proposed argument is denoted by the 'Method: H_∞', while the first and second conventional controllers are denoted by the 'LQR [25]' and the complex-coefficient filter (CC-filter) [16], respectively.

For the Method: H_∞, LQR [25] and CC-filter [16], we note that the spectrum radii of the corresponding monodromy operators are given by 0.936, 0.958 and 0.972, respectively (where the computations are carried out by using the arguments in [26]). Because they are smaller than 1, we can see from Theorem 1 that all the four control approaches stabilize the delay-feedback system Σ.

The experiment results for the tracking error e are shown in Fig. 4. We can observe from Fig. 4 that the magnitudes of the tracking error e under the Method: H_∞ is quite smaller than those under the conventional LQR [25] and CC-filter [16] for all the joints. For a quantitative analysis, the root mean square (RMS) and peak values of the tracking error e obtained from all the four approaches are shown in Tables II and III, respectively.

We can see from Table II that the RMS values for the Method: H_∞ is significantly smaller than those for the LQR [25] and CC-filter [16]. More interestingly, the RMS values of e for the Method: H_∞ are always smaller than those for the other methods under the same joints. These observations clearly demonstrate the effectiveness of the Method: H_∞ in reducing the L_2 norm (or equivalently the RMS) of the tracking error e.

The RMS values of the torque inputs used in the experiments are given in Table IV. It can be confirmed from this table that the RMS values for the Method: H_∞ are also smaller than those for the two existing methods. This clearly indicates that the proposed method achieve higher energy efficiency than the conventional two methods.

To summarize, the stability conditions introduced in this paper are validated through the experiment results. These experiment results also highlight that the Method: H_∞ successfully improve the tracking accuracy with higher energy efficiency of robot manipulators in the RC framework, compared to the existing LQR [25] and CC-filter [16]. Furthermore, the theoretical implications of the proposed control method associated with suppressing the L_2 norm of the tracking error are verified the experiment results.

TABLE I

MODEL INFORMATION OF ROBOT MANIPULATOR.

Joint	1	2	3	4	5	6
Mass [kg]	5.618	3.229	3.588	1.226	1.667	0.736
Length [m]	0.333	0.166	0.150	0.160	0.124	0.088

TABLE II

RMS VALUES OF THE POSITION ERROR e IN EXPERIMENTS 10^{-2}[rad].

Joint	1	2	3	4	5	6
Method: H_∞	0.85	0.74	0.69	1.82	1.75	6.23
LQR [25]	2.65	2.19	2.15	6.87	7.07	7.22
CC-filter [16]	4.90	4.20	4.12	6.93	8.02	7.25

TABLE III

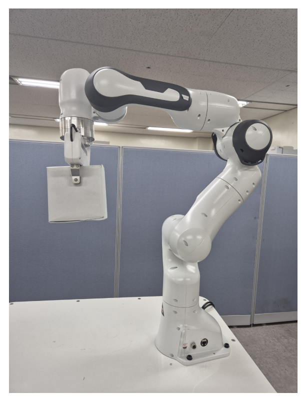
PEAK VALUES OF THE POSITION ERROR e IN EXPERIMENTS 10^{-2}[rad].

Joint	1	2	3	4	5	6
Method: H_∞	7.45	6.48	8.76	4.58	12.42	16.22
LQR [25]	4.45	4.81	5.50	5.92	13.05	12.48
CC-filter [16]	5.15	5.71	7.04	7.50	15.14	13.80

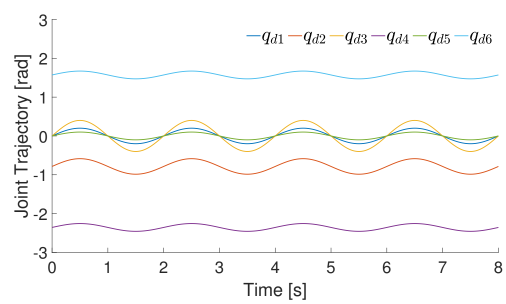
TABLE IV

RMS VALUES OF THE TORQUE INPUT τ IN EXPERIMENTS $[Nm]$.

Method	Method: H_∞	LQR [25]	CC-filter [16]
RMS	2.166	4.189	7.152



(a) Robot manipulator.

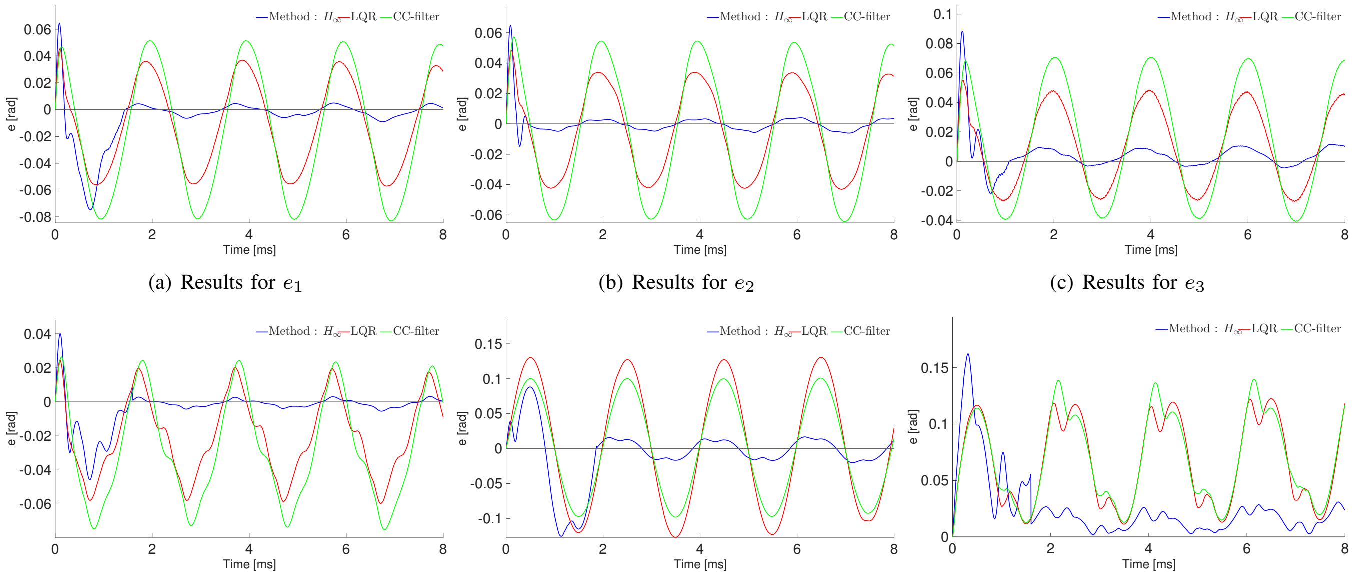


(b) Reference trajectories.

Fig. 3. Experiment environments.

VII. CONCLUSIONS

This paper developed an optimal control framework and an operator-based stability conditions for robot manipulators with periodic reference signals. The developed argu-



(a) Results for e_1 (b) Results for e_2 (c) Results for e_3

(d) Results for e_4 (e) Results for e_5 (f) Results for e_6

Fig. 4. Results for the joint position error e in experiments.

ments were based on applying the repetitive control (RC) approach [10] to the inverse dynamics treatment [3] of robot manipulators. We first introduced the so-called delay-feedback system, to facilitate an optimal controller synthesis and an associated stability analysis in a tractable fashion. We next derived linear matrix inequality (LMI)-based optimal controller synthesis methods for minimizing the H_∞ norm from the disturbance to the tracking error, respectively. We next established operator-theoretic stability conditions by using the monodromy operator [22]. More precisely, a necessary and sufficient condition for the p-type exponential stability of the delay-feedback system was derived. The theoretical validity and the practical effectiveness of the overall arguments developed in this paper were demonstrated through comparative experiments of a robot manipulator; the developed control methods achieved improved tracking accuracy with higher energy efficiency than the conventional methods [16], [25].

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