

# Local Linearized Cosserat Rod Model for Contact Force Estimation in Flexible Medical Instruments and Continuum Robots

Christoph Eyberg  
 Fraunhofer Institute for  
 Manufacturing Engineering and  
 Automation (IPA)  
 Mannheim, Germany  
 christoph.eyberg@ipa.fraunhofer  
 .de

Johannes Horsch  
 Fraunhofer Institute for  
 Manufacturing Engineering and  
 Automation (IPA)  
 Mannheim, Germany

Thomas Bauernhansl  
 Fraunhofer Institute for  
 Manufacturing Engineering and  
 Automation (IPA)  
 Stuttgart, Germany

Jens Langejürgen  
 Fraunhofer Institute for  
 Manufacturing Engineering and  
 Automation (IPA)  
 Mannheim, Germany

**Abstract**— Knowledge of instrument contact forces can lead to safer medical interventions. We present a formulation of the frequently used Cosserat rod model that is linearized around the measured shape for efficient model-based contact force estimation in flexible instruments and robots. Validation on instruments' deflection in an endovascular intervention use case resulted in an average force estimation error of only 14 %.

**Keywords**— Modeling, Control, and Learning for Soft Robots; Medical Robots and Systems; Force and Tactile Sensing

## I. INTRODUCTION

Flexible robotic systems and compliant passive instruments play a critical role in medical procedures such as endovascular therapy – the gold standard for treating vascular diseases like strokes and ischemic heart disease, which account for one in three deaths in Western societies. During these interventions, catheters and guidewires are navigated through the vascular system under fluoroscopy guidance. Despite their compliance, these instruments can still damage vessel walls [1]. Accurate knowledge of instrument-vessel interaction forces can help avoid dangerous loads, enable haptic feedback in robot-assisted interventions (which lack native tactile sensation), and support higher levels of autonomy through model-based control [2, 3]. Sensor-based approaches using FBG fibers [4, 5] provide good results but

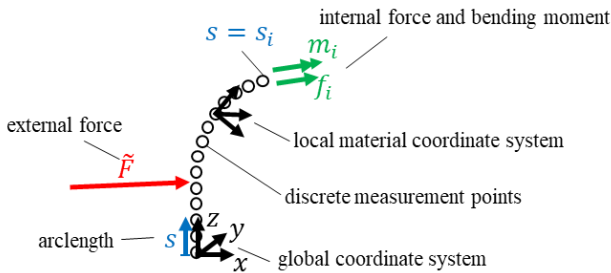


Fig. 1. Bending moments induced by any external forces can be calculated at each measurement point independently using static equilibrium. For simplicity, we chose the local coordinate system at the tip of the rod as the global coordinate system.

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increase the cost of single-use instruments, alter mechanical properties, and complicate instrument exchange during procedures. Existing model-based approaches typically rely on iterative optimization of nonlinear Cosserat rod models using, e.g., Extended Kalman filters [6], which are computationally expensive and may struggle with convergence in highly tortuous 3D geometries [7]. We propose transforming the nonlinear force-deflection relationship  $\mathbf{p} = g(\mathbf{F})$  of Cosserat rod theory into a linearized system  $\mathbf{p} = \mathbf{A}(\mathbf{p}^*)\mathbf{F}$  using the measured pose  $\mathbf{p}^*$  to facilitate contact force retrieval. For known contact points this allows us to estimate contact forces by matrix inversion, for unknown contact points it facilitates the optimization process. We also propose integration of physically motivated explicit constraints into this optimization process and validate our methods under realistic OR conditions using interventional guidewire and fluoroscopy imaging system.

## II. METHODS

Starting from the measured 3D centerline  $\mathbf{p}_i(s_i)$  of the instrument (obtained, e.g., from fluoroscopy), we compute tangent vectors and local rotation matrices  $R_i$  in the Darboux frame. For a force  $\tilde{\mathbf{F}}$  at arclength position  $\tilde{s}$ , the internal bending moment at each point follows from static equilibrium (see Fig. 1). Applying Hooke's law with stiffness matrix  $K_i$  and the Cosserat kinematic relations, we derive a direct mapping from force to second derivative of position

$$\ddot{\mathbf{p}}_i = \lambda_i R_i \hat{\mathbf{e}}_3^T K_i^{-1} R_i^T (\widehat{\mathbf{p}_i - \tilde{\mathbf{p}}}) \tilde{\mathbf{F}} \quad (1)$$

where  $\lambda_i$  denotes the values of a step function at  $\tilde{s}$ . Using two-fold trapezoidal integration independently from  $\tilde{\mathbf{F}}$  over measurement points denoted as  $J^2$  we get the linear relationship

$$\mathbf{p}_i - \mathbf{p}_{0_i} = J^2 (\lambda_i R_i \hat{\mathbf{e}}_3^T K_i^{-1} R_i^T (\widehat{\mathbf{p}_i - \tilde{\mathbf{p}}})) \tilde{\mathbf{F}} = \mathbf{A}_i \tilde{\mathbf{F}} \quad (2)$$

where  $\mathbf{A}$  is constructed entirely from measured pose quantities and known material properties and  $\mathbf{p}_{0_i}$  is the resting shape. For known contact positions, the force is obtained directly via least-squares inversion. For multiple

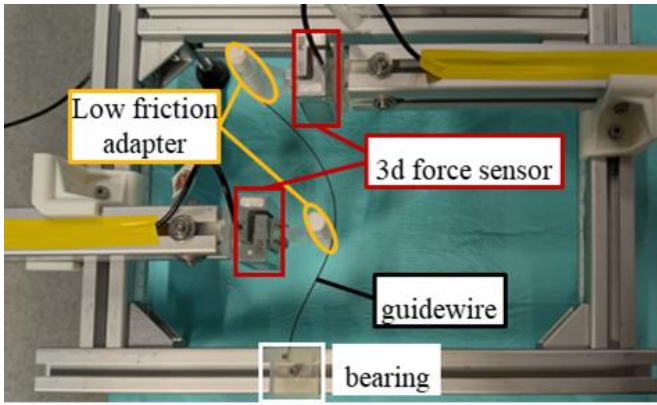


Fig. 2. Testbed setup that was used to apply forces on the guidewire. The testbed was placed on an OR table, and deflection was measured using fluoroscopy imaging.

simultaneous forces, the superposition principle applies directly. The Jacobian with respect to force position can be retrieved from (2), enabling gradient-based search for unknown contact locations  $s$  along the arclength with

$$\mathbf{p}_i - \mathbf{p}_i^\gamma = \mathbf{A}_i \Delta \mathbf{x}^\gamma \quad (3)$$

where  $\mathbf{x}^\gamma = [\mathbf{F}^\gamma, s^\gamma]$  is the best estimate at step  $\gamma$  and  $\mathbf{p}_i^\gamma$  the corresponding instrument shape in linear space. To address the inherent ill-posedness of the inverse problem, we enforce explicit physical constraints on the solution space: (i) point forces only, (ii) minimum spacing  $\Delta s_{\min}$  between forces, (iii) tangential forces bound by a friction coefficient  $|\mathbf{F}_t| \leq \mu |\mathbf{F}_n|$ , (iv) no cancelling consecutive tangential forces, and (v) minimum number of forces. Constraints (ii)–(iv) are enforced via an adaptive constraint matrix  $\mathbf{C}$  that reduces the degrees of freedom of the update vector  $\Delta \mathbf{x}$  when necessary, by substituting  $\mathbf{A}_i^c = \mathbf{A}_i \mathbf{C}$  and updating  $\Delta \mathbf{x}^\gamma = \mathbf{C} \Delta \mathbf{x}^{c,\gamma}$ .

### III. EXPERIMENTAL VALIDATION

A guidewire (Radiofocus™, 0.035", 200 mm straight section) was mounted in a testbench with two sensor-equipped arms (K3D40 3D force sensors, 2 N, 0.5% accuracy) that acted on the guidewire and induced a deflection as depicted in Fig. 2. The testbench was placed on a radiotransparent OR table and imaged with an Artis Zeego fluoroscopy system ( $\sim 0.25$  mm/px resolution). Centerlines were extracted, smoothed (Savitzky–Golay), and registered to the undeformed model at the tip, yielding  $\sim 532$  nodes per experiment. The forces that were randomly applied onto the guidewire ranged from 8 to 577 mN. Force positions ranged from 0 to 139 mm measured from the tip. Either one or two forces were applied for each deflection in 23 test cases giving a total of 32 estimated forces. A statistical summary of force estimation errors is given in Table I. On average forces were calculated within 190 ms and 37 optimization steps (median 111 ms and 28 steps). Our 14 % relative error meets or improves upon published methods: 15–31 % [5] (FBG-based), 27 % [6] (image-based, actuated rod), 17–23 % [7] (image-based, proximal force only). Qiao et al. (FBG-based) [4] report 6–10 % but are limited to small deflections.

TABLE I. MEAN AND FIVE NUMBER SUMMARY OF ABSOLUTE FORCE ESTIMATION ERRORS

Error	$\mu$	min	$Q_I$	$Q_{II}$	$Q_{III}$	max
Force magnitude relative [%]	14	0	5	12	19	64
Force magnitude [mN]	26	0	4	10	37	117
Force position [mm]	4	0	1	1	5	29
Normal force [mN]	24	0	4	10	33	109
Tangential force [mN]	43	1	8	22	68	182

### IV. CONCLUSION AND OUTLOOK

The local linearization of the Cosserat rod model transforms contact force estimation from a nonlinear optimization problem into a linear system that can be solved efficiently and stably even for multiple forces and large deformations without reducing measurement accuracy compared to the state of the art. The approach works directly on pose measurements from clinical imaging systems without requiring additional sensors. While the concept can be directly extended to 3D deformations and pre-shaped instruments, experimental validation of these scenarios remains subject to future work. Also, the achieved accuracy remains above the 5% target for haptic feedback systems [2] and the gradient-based optimization does not yet meet clinical real-time requirements. One dominant error source – poor observability of tangential forces from shape – may be mitigated through stronger orientation constraints or regularization. Future work includes combination of the linear model with linear regressors for global optimality guarantees and improved real-time capability, and cross-timestep consistency checks for robust contact point identification.

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