

A 2-DoF Ankle Rehabilitation Platform Based on an Inclined Dual-Cylinder Mechanism

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Abstract—This paper presents a novel ankle rehabilitation platform based on an inclined dual-cylinder mechanism that provides 2-DoF motion through geometric coupling, without complex multi-link structures. Two cylinders sharing a 9° inclined contact surface are driven by two stepper motors, enabling simultaneous dorsiflexion/plantarflexion and inversion/eversion of up to 18° in each axis. The platform provides both a passive mode, which follows predefined trajectories, and an active mode, which captures user intent through center-of-pressure estimation using a force-sensing resistor-based insole. A Particle Swarm Optimization-tuned PD controller is used in both modes, achieving an RMS tracking error below 0.35° in experimental validation. An IMU-integrated gamification environment further demonstrates the feasibility of the platform as an interactive active training system.

I. INTRODUCTION

The ankle joint plays a crucial role in enabling multi-planar movements necessary for shock absorption, propulsion, and balance during gait. Ankle sprains are among the most common musculoskeletal injuries, and chronic ankle instability develops and persists in approximately 40% of patients following the initial injury [1]. Conventional rehabilitation methods based on elastic bands and balance boards are fundamentally limited in their ability to objectively quantify joint range of motion or systematically monitor recovery progress, underscoring the need for more precise and structured rehabilitation tools.

Robotic rehabilitation systems have been widely recognized as a promising approach capable of delivering repeatable and precisely controlled therapy. Among them, parallel ankle rehabilitation robots (PARRs) are particularly well suited for ankle rehabilitation, as their platform-based design inherently restricts unnecessary shank movement during training. This helps prevent secondary injuries while enabling targeted mobilization of the ankle joint [2]. Although electrically driven systems provide bidirectional control, they typically rely on redundant actuator configurations to enhance motion fidelity, which increases both structural and control complexity [3]. Consequently, existing PARRs share a common limitation of depending on a large number of actuators and complex mechanism structures, which represents the major barrier to broader clinical adoption.

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To address these limitations, this paper proposes a compact ankle rehabilitation platform based on a novel non-coaxial tilting mechanism that achieves 2-DoF ankle motion, including dorsiflexion/plantarflexion(D/P) and inversion/eversion (I/E), using two actuators.

II. MECHANISM DESIGN

The core of the proposed mechanism consists of two cylinders that share a 9° inclined contact surface, on which a tapered roller bearing is seated, as illustrated in Fig. 1. The lower cylinder is driven by a stepper motor mounted at its base, rotating by angle θ_1 about the lower cylinder axis. The upper cylinder is independently driven by the second stepper motor mounted at its top, rotating by angle θ_2 about the upper cylinder axis. As θ_1 and θ_2 are independently commanded, the bearing at the inclined interface undergoes a relative rotation of $\delta = \theta_2 - \theta_1$. This relative rotation, combined with the geometric constraint of the 9° inclined interface, continuously determines both the tilt direction and angle of the footplate, achieving up to 18° in each direction, which is twice the 9° inclination angle of the interface.

The overall system configuration is illustrated in Fig. 1a. The platform consists of the inclined dual-cylinder mechanism, two NEMA 17 stepper motors with 40:1 worm gear reductions and integrated encoders, and a force-sensing resistor (FSR) insole for center-of-pressure estimation and pressure distribution measurement.

III. CONTROL FRAMEWORK

The control framework supports passive and active rehabilitation modes, as illustrated in Fig. 2. The two modes differ only in how the desired footplate orientation is generated; both subsequently follow a common control path consisting of inverse kinematics and a PD controller, implemented on an ESP32-based embedded architecture with encoder-based position monitoring. The PD controller gains are optimized

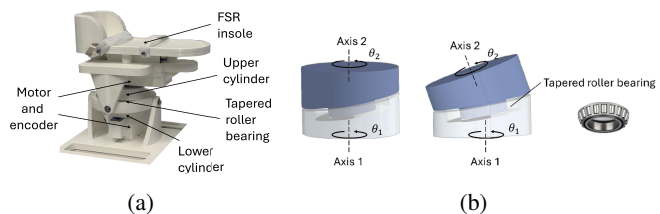


Fig. 1: The proposed ankle rehabilitation platform. (a) System overview. (b) Inclined dual-cylinder mechanism.

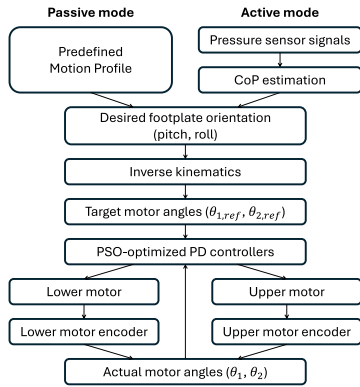


Fig. 2: Block diagram of the proposed control framework.

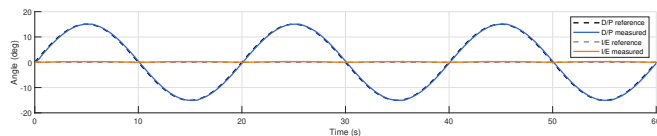
offline using Particle Swarm Optimization (PSO) [4]. The footplate pose is described by a DH parameter-based forward kinematics model. The inverse kinematics analytically computes δ and θ_1 from the desired footplate orientation, with the final solution selected by selecting the solution closest to the current motor position to ensure smooth motor trajectories.

In passive mode, predefined trajectories specify the desired footplate orientation, while in active mode the desired orientation is derived from real-time CoP computed from the weighted average of FSR measurements. The resulting CoP deviation is translated into a desired footplate orientation and processed through the same control path, enabling active patient participation throughout the rehabilitation session.

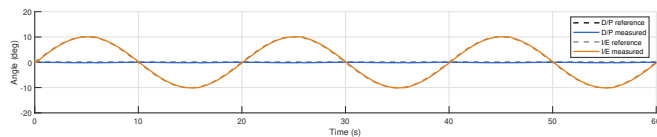
IV. EXPERIMENTAL RESULTS



(a) Lateral view of dorsiflexion/plantarflexion (D/P) motion. (b) Frontal view of inversion/eversion (I/E) motion.



(c) Dorsiflexion/plantarflexion motion tracking results.



(d) Inversion/eversion motion tracking results.

Fig. 3: Validation of passive mode motion control.

Single-axis tracking experiments were conducted in passive mode, with sinusoidal reference trajectories of $\pm 15^\circ$ for dorsiflexion/plantarflexion motion and $\pm 10^\circ$ for inversion/eversion motion. Fig. 3 shows sequential motion snapshots ((a), (b)) and the corresponding motor tracking results for

TABLE I: Motor Tracking Performance (RMS Error)

Experiment	D/P RMS	I/E RMS
Dorsiflexion/plantarflexion (D/P) motion	0.3409°	0.1911°
Inversion/Eversion (I/E) motion	0.1056°	0.2253°

both axes ((c), (d)). The RMS errors summarized in Table I confirm a tracking precision within 0.35° precision across the full motion range, and a maximum footplate tilt of 18° in each direction covers the minimum 15° required for functional gait, meeting the clinically relevant range of motion for ankle rehabilitation.

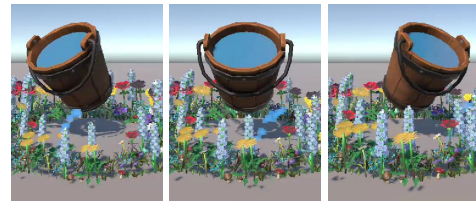


Fig. 4: Active mode VR-based rehabilitation interface featuring a virtual watering task, where users restore wilted flowers through ankle-controlled interaction.

In active mode, CoP-based footplate control driven by voluntary plantar pressure shifts was validated alongside an IMU-integrated gamification environment, as shown in Fig. 4. User interaction with on-screen targets through ankle motion was used to verify the feasibility of the proposed platform as an interactive active rehabilitation system.

V. CONCLUSION

This paper presented a 2-DoF ankle rehabilitation platform based on an inclined dual-cylinder mechanism that realizes dorsiflexion/plantarflexion and inversion/eversion using two stepper motors, achieving structural simplicity without compromising clinically relevant motion range or control precision. The proposed platform integrates a DH-parameter-based analytical inverse kinematics and a CoP-based active control scheme within a unified control framework, supporting rehabilitation stages from early passive joint mobilization to user-participatory active training. Future work will focus on clinical pilot studies to validate the therapeutic efficacy of the proposed platform in patient populations.

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