

Over-actuation in Soft Robots: Towards Active Variable Stiffness & Viscoelasticity

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Abstract—Continuum soft robots (cSR) represent a particular class of the highest compliant deformable robots made of elastomers, typically driven by embedded pneumatic chambers. While their structural compliance is appealing for many tasks, most existing research on variable stiffness at a constant generalized position $q \in \mathbb{R}^n$ relies on empirical evidence rather than a formal method that enhances the properties of cSR. Consequently, few sound advances have been reported regarding closed-loop (control-based) variable stiffness and furthermore for variable viscoelasticity for tracking in cSR, nor have they fully exploited the ease with which soft robots incorporate additional actuation inputs. In this extended abstract, the control of the fundamental behavioral structural viscoelasticity is addressed through commanding redundancy of actuation during motion. To this end, $r \in \mathbb{R}^m$, $m > n$ redundant pneumatic chambers are introduced into the cSR such that n chambers are used for motion tracking, while the remaining $r = m - n$ are available to allocate variations of stiffness in regulation tasks, and of viscoelasticity in tracking tasks.

Index Terms—Variable Stiffness; Modeling, Control, and Learning for Soft Robots; Soft Robot Materials and Design; Robust/Adaptive Control.

I. INTRODUCTION

The arrival of compliant (deformable) bodies into the robotics arena was conceived to extend agency into unstructured environments where morphological adaptation is a requirement rather than a parasitic effect. Soft robotics emerged as the primary approach for this vision, yet the field currently faces a fundamental impasse: instead of mastering the pure continuum, researchers often rely on architectural workarounds. For instance, to achieve stiffness variation—one of the most prospective properties of soft bodies—prevalent methods rely on jamming, tendon-driven designs, or the addition of discrete rigid elements. These non-homogeneous designs introduce nonlinearities and computational barriers that obscure the fundamental physics of the continuum, leaving a gap where closed-form analytical models and stability proofs are frequently ignored.

The core challenge of stiffness variation arises during motion. While stiffness defines the equilibrium of a cSR, structural viscoelasticity governs the approach to it. Furthermore, ignoring these effects compromises the stability required for robust tracking in real-world tasks. Hereby, we utilize a Lagrangian modeling principle to encompass the mechanical

deformation of the highly viscoelastic elastomers, driven by pneumatic control, substantiated in the Lagrange modelling principle. By considering an over-actuated system with a non-square input matrix $B \in \mathbb{R}^{n \times m}$ arise, we establish an affine map between the cSR torque $\tau \in \mathbb{R}^n$ and $P \in \mathbb{R}^m$ pneumatic control inputs. This observation leads to a solution where forward viscoelasticity is injected throughout the controller τ while tracking a given smooth trajectory $q_d \in \mathcal{C}^2$. Notably, the resulting manifolds of viscoelasticity enable the robot to tune its physical pneumatic tension to the environment without requiring exact knowledge of the cSR dynamics.

II. PROBLEM STATEMENT AND CONTRIBUTION

Preliminaries: We consider the integral Lagrangian dynamic model [1] based on D’Alembert-Lagrange equation of a deformable toroidal section of constant curvature:

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) - \tau_{ve} = B(q)p + \tau_d, \quad (1)$$

where $q = (l, \phi, \kappa)^T \in \mathbb{R}^3$ and $\dot{q} \in \mathbb{R}^3$ denote the vector of generalized coordinates (length, azimuth, curvature) $\dot{q} \in \mathbb{R}^3$ its velocity [2]; $H(q) \in \mathbb{R}^{3 \times 3}$ and $C(q, \dot{q}) \in \mathbb{R}^{3 \times 3}$ represent the Inertia and Coriolis matrices, respectively, related by the skew-symmetric matrix $A = 2C(q, \dot{q}) - \dot{H}(q)$ which enables exploiting passivity in closed-loop. The terms $g(q)$ and τ_d account for gravity loads and Lipschitz disturbances. The endogenous term τ_{ve} represents the complex viscoelastic generalized forces:

$$\tau_{ve} = \int J_j^T(q) df_j(q, \dot{q}, E, \rho, \nu)$$

emerging from nonlinear phenomena including the geometric configuration q , deformation velocity \dot{q} , non-constant Young modulus E , density ρ , and material viscosity ν . Finally, the exogenous input $\tau = B(q)p$, in the right-hand side of (1), models physically the linear map of pneumatic control pressure vector p that captures the pneumatic actuators.

The Problem: Given the uncertainties in cSR dynamics due to the complexity of τ_{ve} and presented disturbances, the exact model is assumed to be unknown. Then, the problem stands for the design of a control law for a dual-purpose: achieving robust asymptotic tracking toward q_d while simultaneously modulating an active forward viscoelasticity $\tau_{ve}^a = \tau_{ve}^a(K^a, D^a, q_d, \dot{q}_d)$. Notice that, for the sake

of clarity and without loss of generality, we consider the widely-used, linear Kelvin-Voigt (K-V) model, [3] $\tau_{ve} = -K\bar{q} - D\dot{q}$, for $(K, D) \geq 0$, $\bar{q} = q - q_0$, and q_0 standing for the equilibrium in open loop. Then, since τ_{ve} is not measurable but it is customarily computable in terms of a given formulae, it is substantiated in the phenomenological nature of viscoelasticity.

Contribution: To vary τ_{ve} in closed loop, we need to certainly guarantee that the state q is in the vicinity of desired configuration q_d , thus it is required such a robust controller. Overall, we aim at showing that control design plus a forward term $\tau_{ve}^a = \tau_{ve}^a(K^a, D^a, q_d, \dot{q}_d)$ induces variation of closed-loop configuration viscoelasticity along a desired trajectory (q_d, \dot{q}_d) , without any knowledge of the cSR dynamics. To this end, let the robot controller be an integral sliding mode (ISM) $\tau_{ISM} = -K_d(\dot{q} - \dot{q}_r)$ where $K_d > 0$ and \dot{q}_r is a nominal reference, see [4] for full details, plus a forward τ_{ve}^a , such that the controller becomes an ISM plus active forward viscoelasticity τ_{ve}^a injection, i.e. $\tau = -K_d(\dot{q} - \dot{q}_r) - \tau_{ve}^a$. We show that it guarantees local exponential tracking with a model-free control structure, with variable viscoelasticity. The main result states that closed-loop error equation tracks locally exponentially with varying viscoelasticity in accordance to the prescribed τ_{ve}^a function. Furthermore, using the novel concept of Pneumatic Field Tension as the the net pneumatic field within chambers pressure, we quantify the internal stress induced by the concurrent action of the tracking and modulation inputs.

III. SIMULATIONS AND RESULTS

To evaluate the proposed framework, we consider a p-cSR with $m = 6$ actuators ($r = 3$ degrees of redundancy) using parameters $r_{ex} = 0.05$ m, $A_c = 7.854 \times 10^{-3}$ m, and $r_m = 0.04$ m. The simulations distinguish between three r tasks and three underlying cases (Case 1: Baseline; Case 2: Viscoelasticity in neutral configuration; Case 3: Viscoelasticity in error space).

- Task 1: Effects of Active Elasticity. Based on Case 2, we evaluate the system when robust control is absent. The active elasticity $\tau_{ve}^a = K_\phi^a \phi$ is forwarded in the azimuthal direction, resulting in behavior similar to a torsional spring anchored at a neutral azimuth (see Fig. 1a). Poster Fig. 3 compares the reaction for $K_\phi^a \in \{0, 1, 10\}$ N, while being subject to a unit step disturbance $\tau_d = [0, 1, 0]^T$ Nm applied at $t \in [1, 2]$ s, showing how the pressure field adapts to the programmed stiffness.
- Task 2: Regulation. Using Cases 2 and 3, the cSR is held at a constant pose while subjected to time-varying forward active elasticity. We observe that internal pressure modulates the structural tension while successfully maintaining the target position (see Fig. 1b). Poster Fig. 4 provides the profiles for $K^a(t) = \text{diag}(0.1t \text{ [N/m]}, 0.1t \text{ [N]}, 100t \text{ [Nm]})^T$.
- Task 3: This extends the comparison of all three Cases to a crown-like trajectory. By modifying both elasticity and viscosity at different sectors we evaluate the response

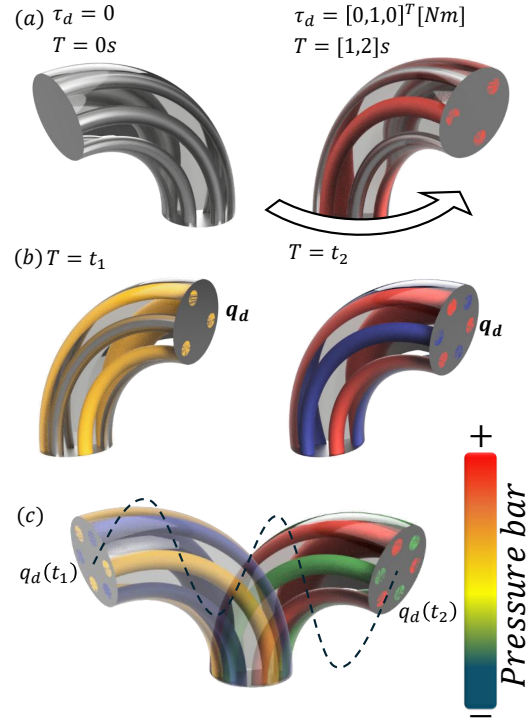


Fig. 1. Conceptual rendering of the tasks 1 (a), 2 (b), and 3 (c), where the color pressure bar indicates level of pressure of corresponding triplet of control chambers, for given open-loop (a), regulation (b) and tracking regime (c).

of the robot with abrupt changes of the Forward Active Viscoelasticity (see Fig. 1c). Please refer to Poster Fig. 5 to observe how pressure and Pneumatic Field Tension vary using parameters $K^a = [6000, 0.1, 8]^T$ and $D^a = [80, 0.01, 0.05]^T$ (in respective SI units).

IV. CONCLUSIONS

Redundancy of actuation is resolved to collocated control forces that varies viscoelasticity along the desired time-varying configuration, without exact knowledge of the cSR. The proposed method enables the robot to track trajectories while simultaneously “tuning” its physical pneumatic tension to the environment. Work is under way for experimental verification on a custom-made real-time testbed¹.

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