

# Development of Contextual Collision Risk Framework for Operational Envelope of Autonomous Navigation System

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**Abstract**—This paper introduces the ‘Contextual Collision Risk (CCR)’ method, a novel collision risk assessment approach for autonomous navigation systems. As global shipping traffic increases, marine traffic becomes more complex and busier, requiring advanced risk assessment methods for autonomous vessels. The CCR establishes the Operational Envelope (OE) of autonomous navigation systems by integrating the complexities of real-time marine traffic and the specific maneuvering capabilities of own ship. The central aspect of CCR involves setting up Reachable Velocities (RV), which includes the dynamics of own ship. Additionally, the Velocity Obstacle (VO) algorithm is implemented to identify potential collision risks from other vessels. By integrating RV and VO analyses, CCR provides a framework that effectively quantifies collision risk in congested maritime environments. To demonstrate the effects of CCR, we employ data from transoceanic voyages across various scenarios. Furthermore, simulations of collision avoidance maneuvers are conducted, focusing on high-risk situations such as near-misses and potential collisions. In particular, the analysis of transoceanic voyage data shows that CCR holds promise for enhancing navigation safety in complex maritime environments.

**Index Terms**—Collision Risk, Contextual Collision Risk (CCR), Operational Envelope (OE), Autonomous Navigation System, Traffic Complexity

## I. INTRODUCTION

Autonomous navigation systems are currently one of the hottest topics in the maritime industry. These advanced technologies are set to transform maritime operations by significantly enhancing efficiency and safety. However, as global shipping continues to expand, marine traffic becomes increasingly complex and congested. This evolution poses significant challenges for autonomous navigation systems, particularly in risk assessment and collision avoidance.

Risk assessment and collision avoidance on board ships have been left to the individual judgment of each navigator, with decisions made within the framework of Convention on the International Regulations for Preventing Collisions at Sea 1972 (COLREGs) [1]. These decisions often heavily depend on the navigator’s experience. However, the shift towards autonomous navigation systems brings a new dimension to this process. There is an increasing need for more accurate and quantitative risk assessments. Autonomous systems must not only assist navigators in making informed decisions but also can autonomously maneuver in the future.

To enhance the safety of autonomous vessels, [2] suggested the concept of an Operational Envelope (OE) as a tool to describe the cooperation between human operators and the system. Building on this concept, this paper delves

into a specific aspect of the OE, focusing on its application in scenarios where vessels encounter each other. Recognizing the gap in existing research for a comprehensive situational risk framework, we draw upon prior studies in maritime complexity and collision risk analysis.

The maritime complexity analysis has utilized Automatic identification system (AIS) data from many regions. For instance, [3] used the Vessel Conflict Ranking Operator (VCRO) model, and [4] introduced the conflict size concept to analyze complexity with AIS data. [5] and [6] clustered the fleets using AIS data, and derived algorithms by selecting Time to Closest Point of Approach (TCPA), Distance to Closest Point of Approach (DCPA), and others for regional collision risk assessment. Meanwhile, [7] proposed ship collision risk indices with AIS data, focusing on the risk assessment of vessels in certain regions. [8] and [9] considered DCPA and TCPA for risk assessment. [10] have introduced the Time-Varying Collision Risk (TCR) measure, which integrates the danger level of approaching ships and the difficulty of avoiding collisions. While this method improves upon traditional static models by considering the maneuverability of vessels, it remains primarily focused on pairwise ship encounters and does not fully adapt to dynamic, multi-ship scenarios. [11] proposed a ship-domain-based model for collision risk, which, although effective in reducing false alarms, relies heavily on predefined domains and lacks the flexibility needed to adjust to real-time maritime complexities.

Nonetheless, these methods encounter limitations in generating a universal, quantifiable metric that accurately reflects the real-time risk posed to own ship. Studies focusing on maritime complexity do not account for ship dynamics, and collision risk assessments involve numerous factors to define the operational envelope. Consequently, establishing criteria that are intuitively comprehensible becomes problematic.

The Contextual Collision Risk (CCR) framework addresses these challenges by providing quantitative and broadly applicable metrics that support the safe navigation of autonomous ships. This framework has the potential to evolve into a crucial indicator for a ship’s collision avoidance maneuvers, bridging the gap in current methodologies by integrating comprehensive risk assessments into real-time navigational decisions.

## II. CONTEXTUAL COLLISION RISK

This section introduces the Contextual Collision Risk (CCR) method, designed to assess contextually quantified collision risks in maritime navigation. The method integrates

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the dynamics of own ship with real-time information regarding surrounding vessels to calculate a risk assessment metric shown in Fig. 1. This metric guides autonomous systems in making informed navigational decisions.

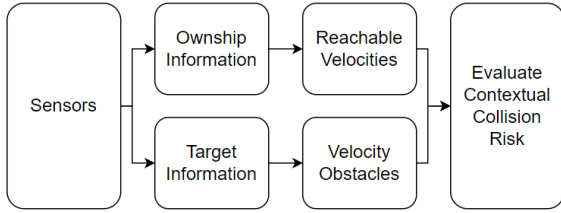


Fig. 1: Flow chart of the contextual collision risk

### A. Reachable Velocities

In [12], the velocities reachable by robot A at a given state over a given time interval  $\Delta t$  are computed by mapping the actuator constraints to acceleration constraints. This means the Reachable Velocities (RV) represent the potential future states of the object. The RV of own ship is generated by taking into account the dynamics of own ship as constraints instead. The vessel's status can be represented by its surge and sway velocity  $(u, v)$  and yaw rate  $(r)$ . The set of the vessel's acceleration vector,  $\mathbf{VA}(t)$ , at time  $t$  is defined as

$$\mathbf{VA}(t) = \{\mathbf{a} \mid \mathbf{a} = f(u, v, r, t)\}, \quad (1)$$

where  $\mathbf{a}$  represents the vector of each accelerations  $(\dot{u}, \dot{v}, \dot{r})$  and,  $f(u, v, r, t)$  represents the vessel dynamics model. Then, RV at time  $(t + \Delta t)$  can be defined as

$$\mathbf{RV}(t + \Delta t) = \{\mathbf{v} \mid \mathbf{v} = \mathbf{v}(t) \oplus \Delta t \cdot \mathbf{a}(t)\}, \quad (2)$$

where  $\oplus$  is the Minkowski vector sum operator, and  $\mathbf{v}(t)$  is the velocity vector of the vessel.

To generate RV effectively, it's essential to incorporate the precise dynamics of the vessel as constraints, enhancing the model's fidelity to real-world conditions. However, a more detailed dynamic model often increases computational demands, posing a challenge for real-time applications. It can be represented by various modeling methods. In [13], the dynamic window approach (DWA) was chosen to generate RV, and [14] takes into account environmental disturbances. Otherwise, [15] and [16] constant velocity field to consider desired velocities.

In this paper, a method that can efficiently perform real-time calculations while reflecting the characteristics of the vessel reasonably was chosen.

### B. Curvature Velocity Method

The Curvature Velocity Method (CVM) offers a compromise between computational speed and accuracy of dynamic representation [17]. Herein also, rather than computing the complicated vessel dynamics model every timestep, strategically simplifies this process. Setting predefined constraints

on possible accelerations derived from the vessel dynamics model and corresponding velocities, not only enhances computational efficiency but also preserves reasonable accuracy. The predefined constraints come from maneuvering test simulations using the dynamics model.

We define the constraints as the maximum acceleration  $(\dot{U}_{\max})$ , rotational acceleration  $(\dot{r}_{\max})$ , and yaw rate  $(r_{\max})$ , essential for steering the vessel towards the desired velocities  $(U_{\text{des}}, r_{\text{des}})$  after time  $t_{\text{des}}$ . The RV at any future timestep  $\Delta t$  is then determined using a case-based approach to ensure computational efficiency while adhering to the vessel's dynamic capabilities:

$$\mathbf{RV}(t + \Delta t) = \{\mathbf{v} \mid \mathbf{v} = \mathbf{v}(t) \oplus \Delta t \cdot g(\mathbf{v}(t))\},$$

where:

$$\begin{aligned} \mathbf{v}(t) &= (U(t), r(t)), \\ g(\mathbf{v}(t)) &= (g_U(U(t)), g_r(r(t))), \end{aligned} \quad (3)$$

$$g_U(U(t)) = \begin{cases} 0 & \text{if } U(t) = U_{\text{des}}, \\ \pm \dot{U}_{\max} & \text{otherwise,} \end{cases}$$

$$g_r(r(t)) = \begin{cases} 0 & \text{if } r(t) = r_{\text{des}}, \\ \pm \dot{r}_{\max} & \text{otherwise,} \end{cases}$$

where  $U(t)$  is the translational velocity, and  $r(t)$  is the rotational velocity of the vessel.

This formulation allows CVM to balance between real-time computational demands and the accuracy of dynamic modeling, making it an optimal choice for modeling RV within the constraints of maximum accelerations and achieving desired velocities. Fig. 2 shows how the set of reachable velocities varies depending on the evaluation method.

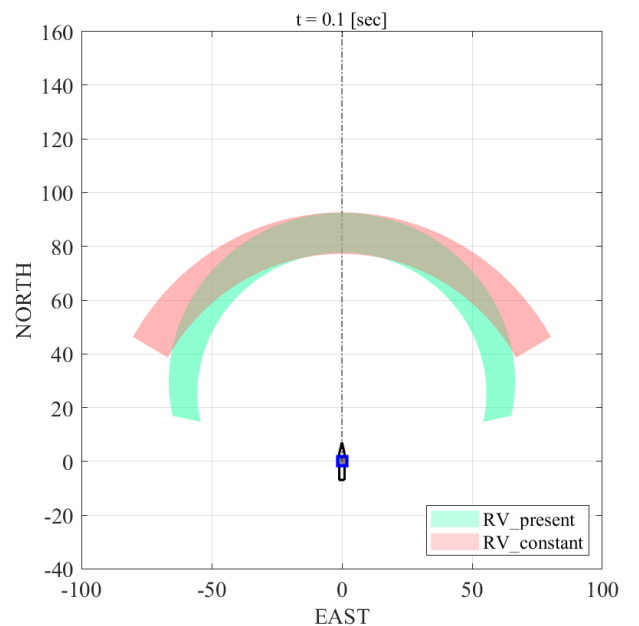


Fig. 2: Reachable velocities based on different methods

### C. Velocity Obstacle

The velocity obstacle (VO) method, introduced in [12], is an effective method for multiple dynamic objects. The VO method represents a set of velocities which may lead to colliding based on current status of the robot and objects. This is mathematically represented as:

$$VO_{AB} = \{v \mid (p_B - p_A) + v_{AB} \cdot t \in D_{obs}, \forall t \in [0, T]\}, \quad (4)$$

where  $p_A$  and  $p_B$  represent the positions of the autonomous vessel and the obstacle, respectively,  $v_{AB}$  is the velocity of the vessel  $A$  relative to the vessel  $B$ ,  $D_{obs}$  is the dimension of the obstacle, and  $T$  is the prediction time horizon for collisions in Fig. 3.

### D. Contextual Collision Risk

In formulating the Contextual Collision Risk (CCR) methodology, we adopt two pivotal rules for clarity:

- 1) **Vessel Behavior:** Vessels strive to maintain their existing course and velocity, adjusting only under critical conditions such as imminent collision threats or for necessary navigational corrections.
- 2) **Avoidance Strategy:** For collision avoidance, vessels favor directional changes over velocity adjustments, attributed to the elevated risks and operational costs linked with engine-powered speed modifications.

These rules are integral to the CCR's framework, guiding the allocation of risk scores within the reachable velocities (RV) spectrum. RV scope can be decided with vessel dynamics constraints and the scope of "desired velocities." Equation (3) involves desired velocities ( $U_{des}, r_{des}$ ) which can be decided with the maneuver capacity of the operator or autonomous navigation systems. Then from (3), the RV for CCR with the combination of each  $U_i$  and  $r_j$  can be represented as

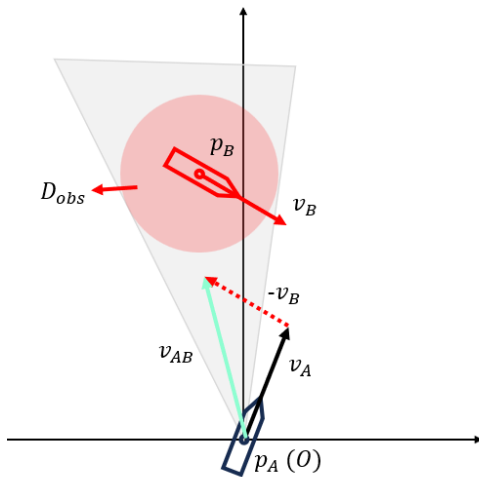


Fig. 3: The velocity obstacle

$$RV_{i,j}(t + \Delta t) = \{\mathbf{v} \mid \mathbf{v} = \mathbf{v}(t) \oplus \Delta t \cdot g_{i,j}(\mathbf{v}(t))\},$$

where:

$$g_{i,j}(\mathbf{v}(t)) = (g_{U_i}(U(t)), g_{r_j}(r(t))),$$

$$g_{U_i}(U(t)) = \begin{cases} 0 & \text{if } U(t) = U_i, \\ \pm \dot{U}_{\max} & \text{otherwise,} \end{cases}$$

$$g_{r_j}(r(t)) = \begin{cases} 0 & \text{if } r(t) = r_j, \\ \pm \dot{r}_{\max} & \text{otherwise,} \end{cases} \quad (5)$$

$$RV(t + \Delta t) = \bigcup_{i,j} RV_{i,j}(t + \Delta t). \quad (6)$$

Within the computed set of reachable velocities, each velocity vector is assigned a risk level using a cost function that embodies our preceding rules about vessel behavior and avoidance preferences. This function evaluates risk through a weighted sum of two Gaussian distributions:

$$C(\mathbf{v}) = \alpha_U \cdot \exp\left(-\frac{(U - U_{cur})^2}{2\sigma_U^2}\right) + \alpha_r \cdot \exp\left(-\frac{r^2}{2\sigma_r^2}\right),$$

$$\exists \mathbf{v} \in RV. \quad (7)$$

Here,  $C(\mathbf{v})$  denotes the cost for a velocity vector  $\mathbf{v}$  within the set of reachable velocities  $RV$ , where  $U_{cur}$  is the vessel's current speed over ground (SOG), aligning with the rule that vessels prefer maintaining their current speed. The yaw rate  $r$  is centered around zero, consistent with the preference for course alteration over speed adjustment when maneuvering to avoid collisions. The constants  $\alpha_U$  and  $\alpha_r$  are the weighting factors that balance the importance of matching the current SOG and maintaining a minimal yaw rate, with  $\sigma_U$  and  $\sigma_r$  as the respective standard deviations.

$$CCR = \sum_i C(\mathbf{v}_i) \quad \text{where } \mathbf{v}_i \in RV \cap VO_{obs}. \quad (8)$$

The value of CCR is determined as cumulate of  $C(\mathbf{v})$ , where  $\mathbf{v}$  is the one of velocities that lead to potential conflicts are flagged as risks.

## III. SIMULATION

This section discusses the validation protocols employed to assess the efficacy of the CCR methodology, featuring simulations of potential collision scenarios.

### A. Hardware-in-Loop Simulator

The Hardware-in-Loop simulator, including the navigation equipment used on the bridge of vessels, was configured as shown in Fig. 4. The decision module in the autonomous navigation system passes control command values to be passed to the control navigation equipment, the simulator passes the next ship's state based on those command values, and the simulation proceeds.

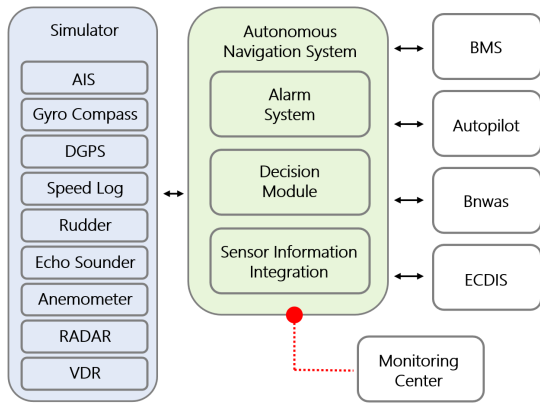


Fig. 4: Architecture of the Hardware-in-the-Loop Simulator

The simulation experiment was designed to validate the contextual collision risk framework. Two distinct scenarios were chosen to exemplify common maritime situations that a significant number of target ships might encounter, each embodying divergent risk profiles.

#### B. Scenario 1: Following the traffic flow

This is a scenario where all vessels are following the traffic flow in the same direction, simulating that own ship encounters target ships in the same direction in traffic flow. The initial simulation settings are shown in Fig. 5a. This scenario typically occurs in regions characterized by dense maritime traffic, including straits, and areas close to ports, where the convergence of numerous moving vessels raises the potential for such encounters.

Each target ship is simulated as a constant velocity linear motion from its initial positions. Own ship has also been set not to initiate collision avoidance maneuvers, allowing for the observation and analysis of variations in CCR.

#### C. Scenario 2: Crossing

This scenario simulates an encounter with a crossing traffic separation scheme, where target vessels navigate paths almost perpendicular to the own ship's trajectory. This scenario exemplifies a situation where a ferry, operating regular routes between two proximate landmasses, is required to navigate across a strait. Such a setting mirrors the operational conditions of a ferry regularly commuting between two closely situated landmasses, necessitating a transit through a strait. This operational scenario visually represented in Fig. 6a, providing a foundational setup from which to explore the interactions between the own ship and crossing traffic. Similar to the approach in Scenario 1, the design of vessel movements here is deliberately tailored to foreground the observation of the CCR trend.

#### D. Results & Discussion

In Fig. 5, own ship initiates from a standstill, gradually accelerating to reach an SOG determined by the decision module depicted in Fig. 4, which is calculated to be faster than that of the target ships for this scenario. As illustrated

in Fig. 5b and Fig. 5c, the reduction of the distance between own ship and the target ship leads to the increase of intersections between the RV of the own ship and the VOs of the target ships. This growing overlap, as evidenced in Fig. 5d, leads to a discernible escalation in the CCR over time, signifying a context-dependent increase in potential collision risk. Upon comparing the RV depicted in Fig. 5a with those in Fig. 5b and Fig. 5c, it becomes evident that the shape of the RV evolves in response to the changing state of the own ship. This variation in the RV configurations vividly demonstrates that the RVs generated through the curvature velocity method (CVM) accurately encapsulate the ship's motion characteristics, offering a dynamic and realistic representation of the vessel's potential navigational paths under different operational conditions.

Scenario 2 begins under similar conditions, yet, as depicted in Fig. 6a, the RV of own ship is immediately distinguished by shading. This shading serves as a visual cue for their overlaps with the VOs right from the start of the simulation, effectively marking the zones of potential collision risk. Differently from Scenario 1, Scenario 2 shows that the CCR exceeds 0.5 from the beginning. The initially high CCR provides numerical evidence indicating that this scenario is perceived as more dangerous than Scenario 1, even based on human intuition. This underscores the CCR framework's effectiveness in quantifying perceived risks, closely mirroring our instinctual assessments of navigational safety.

Based on the above results, the validation of the contextual collision risk framework for transoceanic voyages will be discussed in the next session.

## IV. TRANSOCEANIC VOYAGES

Herein, we discuss the application of the CCR method to actual voyage data, providing a real-world context for validation. This section delineates the validation protocols employed to assess the efficacy of the CCR methodology, featuring simulations of potential collision scenarios and the examination of empirical data from transoceanic voyages.

#### A. Voyages Specification

The contextual collision risk validation with the transoceanic voyages data was designed using voyage log data for a route from Zhoushan, China, to Singapore via the Taiwan Strait and Singapore Strait (Fig. 7). The vessel that conducted the navigation is a bulk carrier with an Length Overall (LOA) of 340 meters. The autonomous navigation system, similar to the architecture of the Hardware-in-the-Loop Simulator (HiLS), was installed on the bridge of an actual vessel. Unlike the simulator setup, the system on the vessel was capable of transmitting recommended control inputs to the navigation equipment, thereby enabling the tracking of the pre-planned route.

In the scenarios presented thereafter, the ship was operated by navigators, and the CCR was applied considering the operational envelope of the autonomous navigation system,

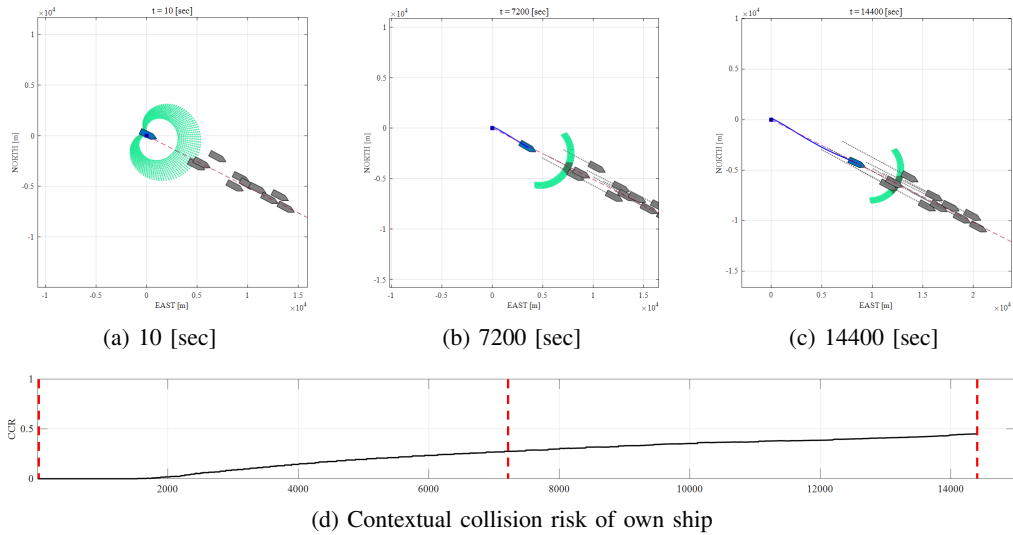


Fig. 5: Scenario 1 simulation

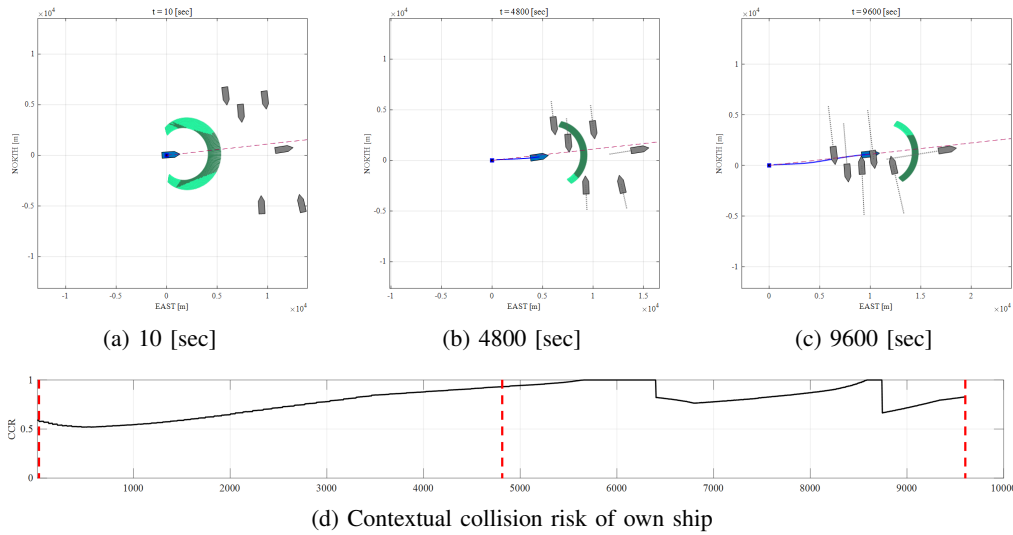


Fig. 6: Scenario 2 simulation

which was defined based on constraints decided in consultation with the navigators.

### B. Results & Discussion

Fig. 8 illustrates the changes in contextual collision risk value during a head-on encounter scenario. Fig. 8d depicts the trend of CCR alongside the Distance to Closest Point of Approach (DCPA) and the Time to Closest Point of Approach (TCPA), a critical factor in some of the former collision avoidance research. In Fig. 8d, the red vertical dashed lines indicate each timestep, while the red horizontal dashed lines mark the threshold for identifying risky target ships. These criteria were determined based on the navigator's interview. The risky target ships are determined when both the DCPA and TCPA fall below the specified thresholds.

The CCR framework has the potential to serve as an indicator for collision avoidance maneuvers. By comparing Fig. 8a and Fig. 8b, it can be observed that from the moment

a course change is initiated in Fig. 8b. The VO initially aligned with own ship's trajectory begins to shift towards the port side, leading to a gradual reduction in CCR values. The decision to maneuver own ship was made with the intent to increase the DCPA from the approaching head-on target ship. This change verifies that the own ship has successfully avoided the target ship through maneuvering, as also evidenced by CCR. Additionally, Fig. 8c confirms that the encounter situation has been resolved.

The situations introduced in Fig. 9 show encounters while navigating through the Taiwan Strait, applying the CCR framework to data from congested areas. In Fig. 9, we observed that the presence of a ferry crossing the strait contributes to the complexity of the waters as Fig. 9a shows overall higher CCR values compared to Fig. 9b. This result aligns with the method's intent to reflect increased navigational complexity and potential collision risk in cross-traffic scenarios. As designed in the CCR method, when

ships only navigating along traffic lanes shows lower CCR compared to ships crossing traffic lanes.

These instances also highlight the effectiveness of the contextual collision risk framework in busy maritime environments, providing quantifiable risk assessments contextually. While TCPA and DCPA effectively assess the risk of simple situations as demonstrated in Fig. 8d, their application becomes tricky and challenging in complex scenarios. In Fig. 9c and Fig. 9d, the number of hazardous target ships is chosen as a metric for risk assessment of complex situations. Here, the hazard target ship is decided when its TCPA and DCPA values fall below the predefined criteria. This approach shows a similar trend in Fig. 9c. However, when compared with Fig. 9d, CCR demonstrates that the number of hazardous ships cannot fully represent the risk of the current situation contextually. Since, TCPA and DCPA cannot including the status of each target ship such as position, speed, and course which can also affect the risk.

Fig. 10 depicts a scenario encountered while navigating from the Taiwan Strait to the South China Sea, where the



Fig. 7: Transoceanic voyage route (Zhoushan-Singapore)

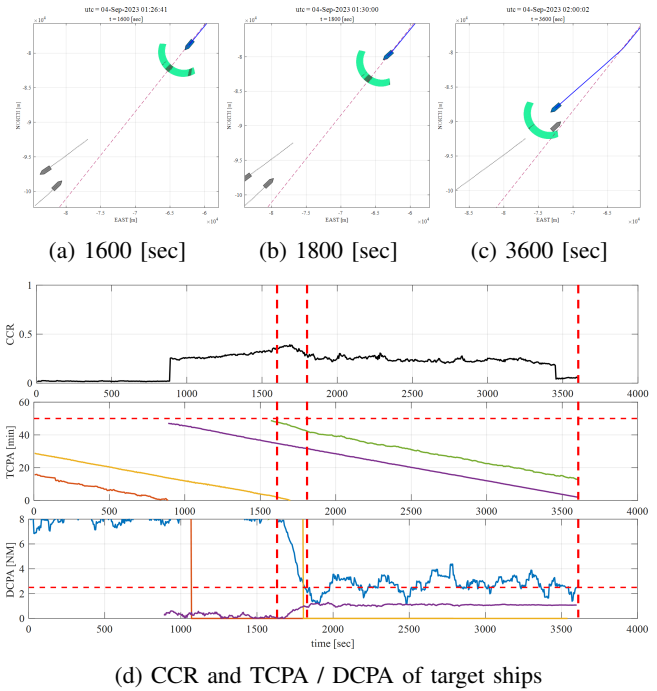


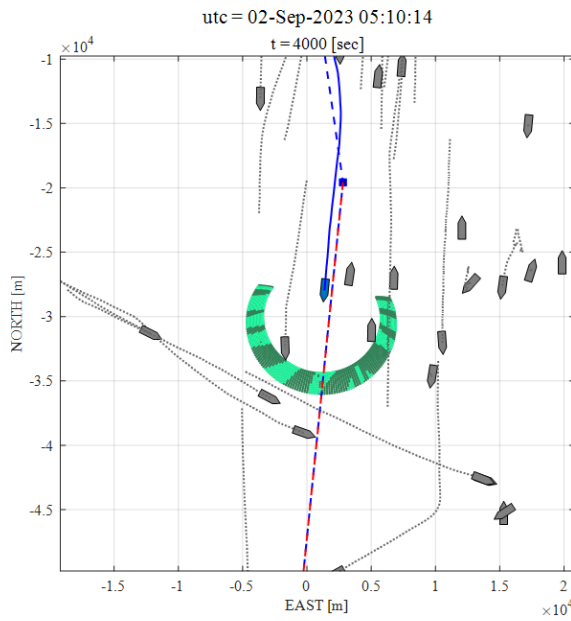
Fig. 8: Head-on situation

vessel had to navigate around fishing buoys equipped with AIS, placed by fishing vessels. In Figs.10a and 10b, the clusters of targets within the red circles, affected by waves and currents, exhibited random fluctuations in their Course Over Ground (COG) and SOG as recorded by AIS data. These fluctuations are suspected to have led to an unstable and unintended increase in CCR values. This observation suggests the need for further investigation into how obstacle classification factors influence CCR calculations, indicating a direction for future research within the CCR framework.

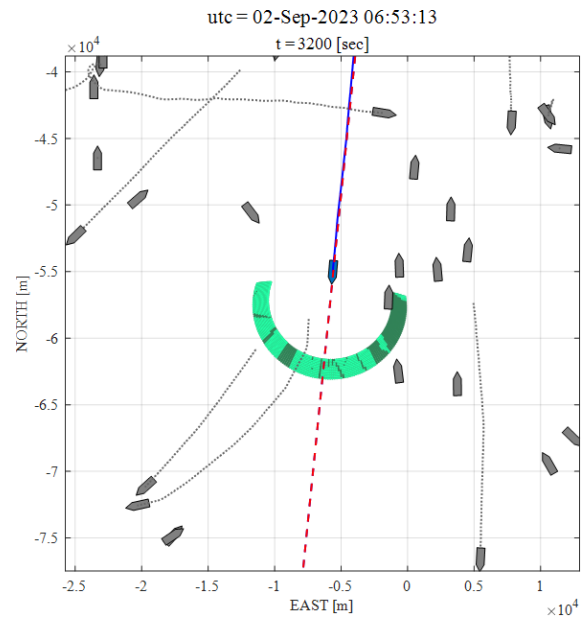
## V. CONCLUSIONS

In this paper, we propose a novel methodology for representing a generalized collision risk assessment through the Contextual Collision Risk (CCR) approach. A central aspect of this research is applying CCR to define the operational envelope and its potential in real-time collision avoidance scenarios. By providing a quantifiable risk assessment, CCR enables intuitive navigational decisions, thereby contributing to safer and more efficient autonomous navigation system operations. This work highlights the importance of advanced risk assessment in autonomous navigation and offers valuable insights for future enhancements in maritime safety and operational efficiency.

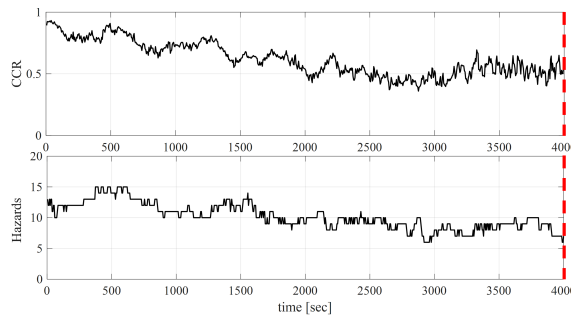
Looking forward, our research will focus on several areas for further development: 1) Refining the setting of Reachable Velocities (RV) to include broader factors such as environmental disturbances; 2) Enhancing the CCR's cost function to include COLREGs compliance, fuel efficiency, and obstacle classification for a more comprehensive risk assessment approach; 3) Specifying the thresholds for traffic-related Operational Envelope (OE) across different maritime



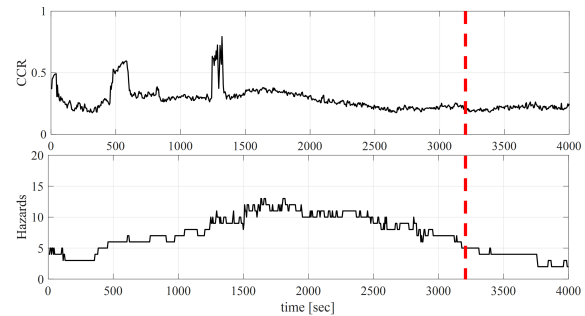
(a) Traffic with crossing



(b) Traffic without crossing



(c) CCR and number of hazard ships with crossing



(d) CCR and number of hazard ships without crossing

Fig. 9: Complex maritime situation during voyage

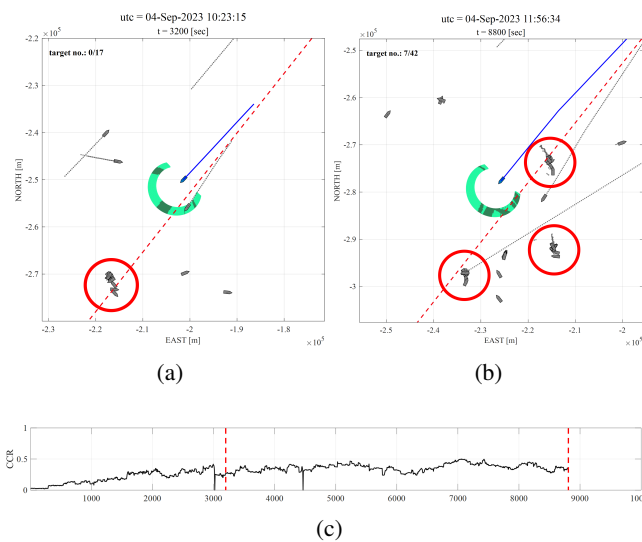


Fig. 10: Encounter with the Fishing Buoys

regions. These advancements are aimed at making CCR a more flexible and applicable tool across varied environmental and operational contexts, thereby enhancing its utility in the field of autonomous maritime navigation.

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