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Coalition Formation Game Approach for Task Allocation in Heterogeneous Multi-Robot Systems under Resource Constraints

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Abstract— **This paper studies a case of the multi-robot task allocation (MRTA) problem, where each unmanned aerial vehicle (UAV) is endowed with multiple but limited resources. Completing each task necessitates UAVs to combine different resources through coalition formation, which will incur various costs including flight cost, execution cost, and cooperation cost. To minimize the total cost while maximizing both task completion rate and resource utilization rate, we model the MRTA problem of the UAVs as a leader-follower coalition formation game. In this game, leader UAVs coordinate follower UAVs to fulfill task resource requisites. Meanwhile, follower UAVs select suitable coalitions to join based on the altruistic preference. Theoretical analysis confirms the existence of a Nash stable partition in the coalition formation game. To achieve this stable partition, we propose a coalition formation algorithm. Simulation experiments validate that the proposed algorithm outperforms existing methods for the MRTA problem under resource constraints in terms of both task completion rate and resource utilization rate.**

I. INTRODUCTION

As robotics and artificial intelligence technologies advance, an ever-growing fleet of unmanned aerial vehicles (UAVs) with diverse capabilities takes on roles once reserved for humans in hazardous environments [1]–[5]. However, the constraints on UAV capabilities and resources, coupled with the varied and intricate demands of tasks, often preclude a single UAV from dependably undertaking complex tasks. This reality calls for deploying multiple UAVs with different specializations to work together and tackle many complex tasks collaboratively. Consequently, the challenge of efficiently and effectively allocating tasks among these multiple UAVs has emerged as a critical issue that demands resolution.

In multi-robot systems (MRS), the task allocation problem is conceptualized as assigning a set of tasks to a group of robots by optimizing various metrics, e.g., task completion rate, task execution costs, and resource utilization rate [6], [7]. Following the taxonomy proposed in [8], the task allocation problem can be divided into eight categories based on task types, robot types, and allocation types, as illustrated in Fig. 1. This paper focuses on an multi-robot task allocation (MRTA) scenario, where each UAV is equipped with different quantities and types of resources, while each task requiring different types and amounts of resources. In such cases, multiple UAVs must form coalitions to perform tasks collaboratively. Consequently, the MRTA approach examined in this study is classified as the single-task robots and multi-robot tasks instantaneous assignment (ST-MR-IA), identifying the most beneficial coalition structure to enhance task completion rate and resource utilization rate.

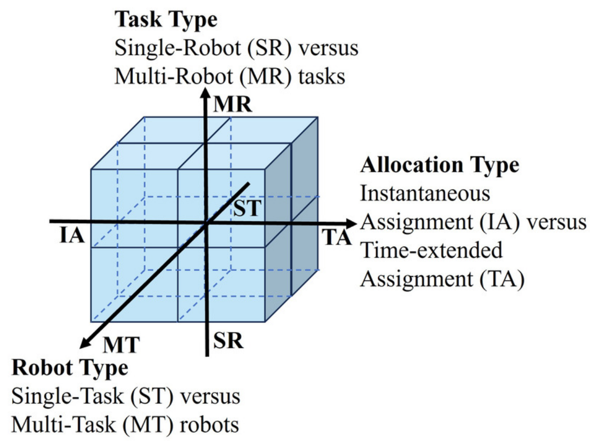

Fig. 1. Classification of multi-robot task allocation.

In this paper, we present a game-theoretic task allocation framework aimed at improving task completion rate and resource utilization rate in situations where tasks require a variety of resources, while robotic resources are finite. Initially, we formulate the problem of resource-constrained, heterogeneous multi-robot task allocation as a leader-follower coalition formation game, establishing a utility function that motivates robots to opt for tasks. Subsequently, by leveraging the characteristics of exact potential games, we demonstrate the existence of a Nash stable partition. We then present a coalition formation algorithm for task allocation and examine the complexity of the algorithm. The key contributions of this paper are as follows:

- We formulate the task allocation problem under resource constraints as a leader-follower coalition formation game, while proving that a Nash stable partition exists in the coalition formation game.
- We propose a coalition formation algorithm to address the task allocation problem under resource constraints in MRS, aiming to maximize task completion rate and resource utilization rate. Meanwhile, we analyze the complexity of the algorithm.

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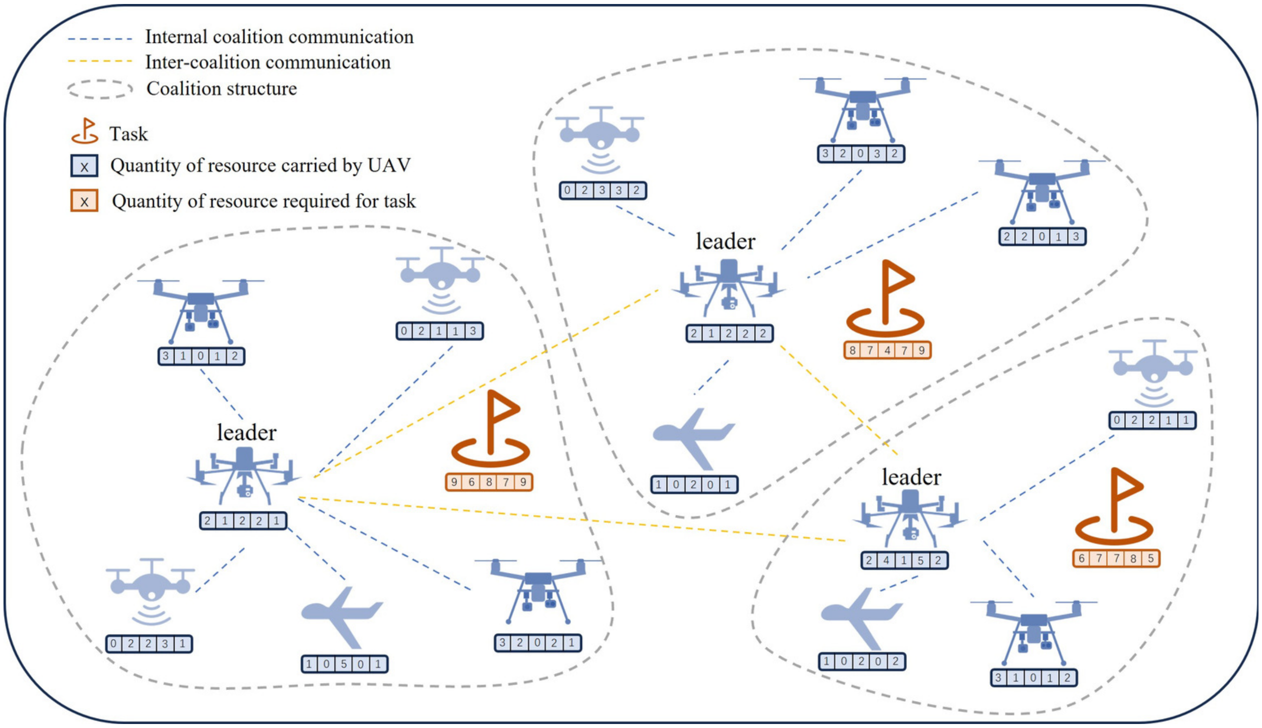

Fig. 2. Illustration of a leader-follower coalition formation game approach for multi-robot task allocation under resource constraints.

- The simulation experiments demonstrate that our approach outperforms previous work regarding task completion rate and resource utilization rate.

II. RELATED WORK

In recent years, game theory has been increasingly applied in MRS. Researchers have found that obtaining optimal task allocation results becomes increasingly difficult as the number and diversity of robots and tasks in scenarios increase. In [9], the authors demonstrate that solving the task allocation problem is NP-hard. Therefore, researchers continuously seek practical solutions to obtain high-quality approximate solutions [10]–[14]. Compared to other commonly used task allocation solutions, such as market auction methods and heuristic approaches, the game-theoretic approach based on coalition formation allows robots to negotiate task allocation results with other robots through local communication, effectively reducing communication overhead and demonstrating robustness.

The authors in [15] propose an algorithm based on the anonymous hedonic game to solve the task allocation problem for large-scale homogeneous robots, which demonstrates good scalability, adaptability, and robustness. Based on this work, the authors in [14] consider the scenario where each task has a minimum requirement for the number of robots needed to complete it and design a virtual utility function that allows each robot to choose a task that has not met the minimum requirement. According to [16], the authors propose a preference-driven method based on hedonic game to solve the task allocation problem for multi-robot systems in emergency rescue scenarios, and demonstrate that the algorithm can be effectively executed even when communication distances are insufficient. The study of [17] considers the energy consumption of UAVs during position scheduling and task execution and propose a task allocation algorithm based on coalition formation game, effectively reducing the flight loss of UAVs. The work in [18] proposes a reputation-based coalition formation mechanism. This mechanism enhances the efficiency of creating optimal coalitions by analyzing the cooperative behavior of robots, thereby excluding unreliable ones.

Table I compares our work with existing works based on different criterias. Although previous game-theoretic approaches based on coalition formation have been applied by researchers in task allocation, they cannot effectively address the issue when each task requires multiple types of resources and the robots have insufficient resources. The main reason is that in those approaches, robots are considered self-interested individuals who only care about their own costs, but not whether the tasks are completed. Therefore, we need to design a utility function that motivates robots to choose appropriate tasks in order to improve task completion rate and resource utilization rate. This challenging problem has not been well addressed in the existing works.

III. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider a scenario where many UAVs are deployed to perform multiple tasks. Each task requires various resources, with specific quantities needed for each type, each UAV possesses different amounts of resources. Due to resource constraints faced by individual UAVs, UAVs must form

TABLE I
COMPARISON WITH EXISTING WORKS

Works	Heterogeneous	Coalition	Resource-Constrained	Task Completion Rate	Resource Utilization Rate
[13]	✓	×	×	×	×
[14]	✓	✓	×	×	×
[15]	×	✓	×	✓	×
[16]	✓	✓	×	×	×
[17]	✓	✓	✓	✓	×
[18]	✓	✓	✓	×	✓
Ours	✓	✓	✓	✓	✓

Heterogeneous: Robots differ in attributes or functions; Coalition: Robots cooperate to perform tasks.

coalitions to undertake tasks. We define the set of UAVs as $\mathcal{R} = \{r_1, r_2, \ldots, r_I\}$, where r_i represents the i-th UAV in the collection, and I is the total number of UAVs. The vector $\mathcal{X}_{r_i} = [x_{r_i}^1, x_{r_i}^2, \ldots, x_{r_i}^{N_z}]$ represents the types and quantities of resources owned by UAV r_i, where N_z represents the number of types of resources. Similarly, we define the set of tasks as $\mathcal{T} = \{t_1, t_2, \ldots, t_J\}$, where t_j represents the j-th task, and J is the total number of tasks. The vector $\mathcal{X}_{t_j} = [x_{t_j}^1, x_{t_j}^2, \ldots, x_{t_j}^{N_z}]$ represents the types and quantities of resources required by task t_j. We define the set of coalitions as $\mathcal{S} = \{S_{t_1}, S_{t_2}, ..., S_{t_J}\}$. Each coalition executing a task is disjoint, meaning that $S_{t_p} \cap S_{t_q} = \varnothing$ for $p \neq q$. This indicates that a UAV cannot simultaneously be part of two different task coalitions. Fig. 2 shows a simple example of coalition formation game solving task allocation under resource constraints.

B. Cost Model of UAVs

The costs incurred by a UAV for task completion comprise flight cost, execution cost, and cooperation cost.

1) *Flight Cost*: Following the assumptions similar to those in [19], the flight cost for UAV r_i to perform task t_j can be represented as:

$$E_{i,j}^f = g_i \cdot e_i^f \cdot \frac{d_{i,j}}{v_i^f}, \tag{1}$$

where g_i represents the mass of UAV r_i, e_i^f denotes the energy consumed per unit of time during the flight, $d_{i,j}$ is the distance between UAV r_i and the the location of the task t_j that needs to be performed, and v_i^f is the average flight speed of UAV r_i.

2) *Execution Cost*: The execution cost for UAV r_i to perform task t_j is determined by the time spent executing the task and the communication time between the UAVs. We assume that once the leader UAV detects a task, it will proceed to the location of the task. The time spent executing the task τ_{t_j} is determined by the nature of the task itself, while the communication time among the UAVs in the coalition is defined as the ratio of the communication volume to the communication speed, where the communication speed is determined as follows:

$$\begin{aligned} &R_{i,j} = B_j \log_2 (1 + SNR), \\ &SNR = \frac{p_i G_{i,j}}{N_j},\ G_{i,j} = \frac{C_{i,j}}{d_{i,j}^{\delta}}, \end{aligned} \tag{2}$$

in which B_j is the communication bandwidth of the coalition S_{t_j}, SNR represents the signal-to-noise ratio, $G_{i,j}$ denotes the channel gain on the wireless channel link, p_i is the transmission power, δ is the path loss exponent, and N_j is the power spectral density of the noise [20]. For the coalition S_{t_j}, since each follower UAV will transmit its data to the leader UAV, the total amount of data is equal to $(|S_{t_j}| - 1) \cdot \lambda$, where $|S_{t_j}|$ represents the number of UAVs in S_{t_j} and λ represents the amount of data for communication. In particular, a coalition with only one UAV does not need to calculate the communication time. In this case, the total time to perform the task can be represented as:

$$T_{i,j} = \begin{cases} \tau_{t_j} + \frac{|S_{t_j} - 1|\lambda}{R_{i,j}}, & |\mathcal{S}_{t_j}| > 1, \\ \tau_{t_j}, & |\mathcal{S}_{t_j}| = 1. \end{cases} \tag{3}$$

The execution cost can then be expressed as:

$$E_{i,j}^c = T_{i,j} \cdot e_i^h, \tag{4}$$

where e_i^h is the energy consumed per unit of time by UAV r_i while hovering.

3) *Cooperation Cost*: The cooperation cost represents the cost of collaboration among the UAVs in a coalition and is related to the number of UAVs in the coalition. When considering the cooperation cost of a coalition, the marginal cost of a UAV joining the coalition increases as the size of the coalition grows, due to the increasing complexity of coordination and communication with the addition of more coalition members. Therefore, we consider defining it as a quadratic function:

$$\Xi_{S_{t_j}} = |S_{t_j}|^2, \tag{5}$$

where $\Xi_{S_{t_j}}$ is the cooperation cost for the coalition S_{t_j} executing task t_j.

In summary, the total cost function for UAV r_i performing task t_j within coalition S_{t_j} is given by

$$\mu_i(S_{t_j}) = \alpha_1 \cdot \tilde{E}_{i,j}^f + \alpha_2 \cdot \tilde{E}_{i,j}^c + \alpha_3 \cdot \tilde{\Xi}_{S_{t_j}}, \tag{6}$$

where α_1, α_2, α_3 represent the weights attached for corresponding cost. Since the factors influencing the total cost function have different dimensions, we use min-max normalization to map the flight cost, execution cost, and cooperation cost to the interval [0,1]:

$$\begin{aligned}\tilde{E}^f_{i,j} &= \frac{E^f_{i,j} - E^f_{\min}}{E^f_{\max} - E^f_{\min}},\\ \tilde{E}^c_{i,j} &= \frac{E^c_{i,j} - E^c_{\min}}{E^c_{\max} - E^c_{\min}},\\ \tilde{\Xi}_{S_{t_j}} &= \frac{\Xi_{S_{t_j}} - \Xi_{\min}}{\Xi_{\max} - \Xi_{\min}}.\end{aligned} \tag{7}$$

C. Problem Formulation

In this paper, we aim to determine a coalition partition to ensure the highest possible task completion rate while maximizing resource utilization rate. First, we define the task completion rate as:

$$\begin{aligned}D_{t_j} &= \frac{1}{N_z} \sum_{x^l \in \mathcal{X}_{t_j}} \theta(x^l_{t_j}),\\ \theta(x^l_{t_j}) &= \begin{cases} 1, & \text{if } x^l_{t_j} \leq \sum_{r_i \in \mathcal{S}_{t_j}} x^l_{r_i},\\ 0, & \text{if } x^l_{t_j} > \sum_{r_i \in \mathcal{S}_{t_j}} x^l_{r_i}.\end{cases}\end{aligned} \tag{8}$$

Next, we define the resource utilization rate, which is represented by the ratio of the actual resources used to the resources invested. The resource utilization rate of the coalition S_{t_j} can be expressed as follows:

$$W_{t_j} = \frac{1}{N_z} \sum_{k=1}^{N_z} \frac{\sum_{r_i \in \mathcal{S}_{t_j}} \omega^k_{r_i}}{\sum_{r_i \in \mathcal{S}_{t_j}} x^k_{r_i}}, \tag{9}$$

where $\omega^k_{r_i}$ represents the quantity of resource k actually used by UAV r_i when performing task t_j.

Consequently, the utility function of UAV r_i in the coalition S_{t_j} can be expressed as:

$$u_i(S_{t_j}) = \frac{D_{t_j} + W_{t_j}}{\mu_i(S_{t_j})}. \tag{10}$$

Our optimization problem can be formulated as:

$$\begin{aligned}&\max_{\{x_{ij}\}} \sum_{\forall r_i \in \mathcal{R}} \sum_{\forall t_j \in \mathcal{T}} u_i(S_{t_j}) x_{ij}\\ &\text{subject to}\\ &\quad S_{t_j} \cap S_{t'_j} = \varnothing, \quad \forall t_j, t'_j \in \mathcal{T},\\ &\quad |S_{t_j}| \geq 1, \quad \forall t_j \in \mathcal{T},\\ &\quad \sum_{\forall t_j \in \mathcal{T}} x_{ij} \leq 1, \quad \forall r_i \in \mathcal{R},\\ &\quad x_{ij} \in \{0,1\}, \quad \forall (r_i, t_j) \in \mathcal{R} \times \mathcal{T},\end{aligned} \tag{11}$$

where x_{ij} is a binary decision variable with a value of 1 if task t_j is assigned to UAV r_i; Otherwise, $x_{ij} = 0$.

IV. GAME-BASED COALITION FORMATION

A. Coalition Formation Game

To address the optimization problem discussed in the previous section, this section models the problem as a coalition formation game. Coalition formation games are a type of cooperative game where the players form coalition with each other to maximize their collective benefits [21]. The definition of coalition formation game is as follows:

Definition 1 (**Coalition Formation Game**). A coalition formation game is defined as a tuple $\mathcal{G}(\mathcal{R}, \succ)$, where $\mathcal{R}$ is the set of players, and $\succ := (\succ_1, \succ_2, \ldots, \succ_I)$ is a profile of preferences that defines the order of players' preferences over coalitions.

Next, we provide the definition of a Nash stable partition in the coalition formation game:

Definition 2 (**Nash Stable Partition**). The coalition partition set S is Nash stable if and only if no player is willing to unilaterally change strategies to improve their utility, i.e.,

$$S_{t^*} \succ_i S_{t_j}, \forall t_j \in \mathcal{T}, \forall r_i \in \mathcal{R}, \forall S_{t_j} \in \mathcal{S}, \tag{12}$$

where t^* is the task selected by UAV r_i after achieving a Nash stable partition.

The preference relation plays a crucial role in coalition formation games. The commonly used preference relations are self-interested preference and Pareto preference. Under self-interested preference, UAVs only consider their utility without concern for the collective. Under Pareto preference, a UAV will only choose to change strategies when the utility of all UAVs in its current coalition and the coalition being joined increases. Such strict constraints can hinder the formation of coalition structures. Given that the objective of this paper is to maximize the global task completion rate and resource utilization rate in task allocation, an altruistic preference relation is adopted, which prioritizes the overall benefits over individual interests and is defined as follows:

Definition 3 (**Altruistic Preference Relation**). For any two coalitions S_{t_p} and S_{t_q}, containing r_n, the preference relation of the UAV r_n can be expressed as:

$$\begin{aligned}&S_{t_p} \succ_n S_{t_q}\\ &\Leftrightarrow \sum_{r_i \in S_{t_p}} u_i(S_{t_p}) + \sum_{r_j \in S_{t_q} \setminus \{r_n\}} u_j(S_{t_q} \setminus \{r_n\})\\ &> \sum_{r_i \in S_{t_p} \setminus \{r_n\}} u_i(S_{t_p} \setminus \{r_n\}) + \sum_{r_j \in S_{t_q}} u_j(S_{t_q}).\end{aligned} \tag{13}$$

B. Analysis of Nash Stable Partition

In this subsection, we will analyze the existence of Nash stable partitions in the coalition formation game. Based on the altruistic preference relation, we demonstrate the existence of the stable coalition by introducing the existence of Nash equilibrium of the exact potential game.

Definition 4 (**Exact Potential Game**). Given a game, if exist a potential function Φ such that $\forall r_i \in \mathcal{R}, \forall S_{\Pi(i)}, S_{\Pi'(i)} \in \mathcal{S}$, where $\Pi(i)$ represents the task selected by UAV r_i and satisfying the condition that follows, we can call this game an exact potential game.

$$\begin{aligned}&U_i\left(S_{\Pi(i)}, S_{\Pi(-i)}\right) - U_i\left(S_{\Pi'(i)}, S_{\Pi'(-i)}\right)\\ &= \Phi\left(S_{\Pi(i)}, S_{\Pi(-i)}\right) - \Phi\left(S_{\Pi'(i)}, S_{\Pi'(-i)}\right).\end{aligned} \tag{14}$$

Notably, every exact potential game has at least one pure strategy Nash equilibrium [22], which will play a crucial role in our subsequent proofs.

Theorem 1. The coalition formation game based on the proposed altruistic preference relation can ensure the existence of Nash stable partitions.

Proof: First, we define the potential function as the global utility function, i.e.,

$$\Phi = \sum_{r_k \in \mathcal{R}} u_k(S_{t_k}, S_{-t_k}), \tag{15}$$

where S_{t_k} represents the task coalition chosen by r_k and S_{-t_k} represents the strategies of all UAVs except r_k. Once the UAV r_k leaves its current coalition S_{t_k} to join another coalition $S^{'}_{t_k}$, the change in the potential function can be expressed as:

$$\begin{aligned}
&\Phi(S^{'}_{t_k}, S_{-t_k}) - \Phi(S_{t_k}, S_{-t_k}) \\
&= u_k(S^{'}_{t_k}, S_{-t_k}) - u_k(S_{t_k}, S_{-t_k}) \\
&+ \sum_{r_i \in \mathcal{S}_{t_k}} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right] \\
&+ \sum_{r_i \in \mathcal{S}^{'}_{t_k}} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right] \\
&+ \sum_{r_i \in \mathcal{S}_{-t_k}} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right] \\
&+ \sum_{r_i \in \mathcal{D}_k, i \neq k} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right],
\end{aligned} \tag{16}$$

where $\mathcal{D}_k = \mathcal{R} \setminus \{S_{t_k} \cup S^{'}_{t_k}\}$.

The change in utility due to the strategy change of UAV r_n is represented as follows:

$$\begin{aligned}
&U_k(S^{'}_{t_k}, S_{-t_k}) - U_k(S_{t_k}, S_{-t_k}) \\
&= u_k(S^{'}_{t_k}, S_{-t_k}) - u_k(S_{t_k}, S_{-t_k}) \\
&+ \sum_{r_i \in \mathcal{S}_{t_k}} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right] \\
&+ \sum_{r_i \in \mathcal{S}^{'}_{t_k}} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right] \\
&+ \sum_{r_i \in \mathcal{S}_{-t_k}} \left[u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i})\right].
\end{aligned} \tag{17}$$

According to Equation (10), each UAV's utility function is independent of the coalitions performing other tasks. Therefore, we have

$$u_i(S_{t_i}, S^{'}_{-t_i}) - u_i(S_{t_i}, S_{-t_i}) = 0, \quad \forall r_i \in \mathcal{D}_k, i \neq k. \tag{18}$$

Through combining Equations(16)-(18), we can know that

$$\begin{aligned}
&\Phi(S^{'}_{t_k}, S_{-t_k}) - \Phi(S_{t_k}, S_{-t_k}) \\
&= U_k(S^{'}_{t_k}, S_{-t_k}) - U_k(S_{t_k}, S_{-t_k}).
\end{aligned} \tag{19}$$

The difference in the potential function can represent the change in the utility function of the UAV. According to Definition 4, it is known that the coalition formation game proposed based on our altruistic preference relation can guarantee the existence of a Nash stable partition. Hence, Theorem 1 is proved. □

Algorithm 1: Coalition Formation Algorithm

Input: UAVs $\mathcal{R}$ and tasks $\mathcal{T}$ parameters are set randomly;
Output: A Nash stable partition;

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Each UAV that discovers a task becomes the leader UAV, while the remaining UAVs are designated as follower UAVs. Let follower UAVs randomly select a task t_j, ∀t_j ∈ T to form an initial partition;
while l < l_max do
    for each UAV r_i ∈ R, each task t_j ∈ T do
        According to Equation (10), each leader UAV calculate the four utility values u_1, u_2, u_3, u_4 when UAV r_i leaves its current coalition S_{t_j} and joins the new coalition S'_{t_j};
        if u_1 + u_2 < u_3 + u_4 then
            UAV r_i leaves its current coalition S_{t_j} and joins the new coalition S'_{t_j};
        end
    end
    if the coalition structure S has not changed then
        l = l + 1;
    else
        l = 0;
    end
end
```

V. ALGORITHM IMPLEMENTATION AND RELATED ANALYSIS

In this section, we propose a coalition formation algorithm to achieve a Nash stable partition, which aims to improve task completion rate and resource utilization rate. Moreover, we also discuss the complexity of the algorithm.

A. Coalition Formation Algorithm

In this subsection, we describe the process of the coalition formation algorithm.

In the initialization phase, each UAV that identifies a task assumes the role of the leader UAV for the coalition associated with that task. At the same time, the remaining UAVs act as follower UAVs. These follower UAVs randomly join existing coalitions, establishing the initial configuration of the coalitions. Subsequently, during each iteration, follower UAVs compute their respective costs using Equation (8) and send this information to the leader UAVs of the separate coalitions. The leader UAVs then assess the utility values of each follower UAV by considering the task completion rate and resource utilization rate. Each follower UAV selects a more suitable coalition based on the altruistic preference relation outlined in Definition 3. This iterative process continues until the coalition structure remains unchanged for a sufficiently large number of steps l_{max}. It has reached a Nash stable partition at that point. The pseudocode for the detailed algorithm is provided in Algorithm 1.

B. Analysis of Algorithm Complexity

In this subsection, we will examine the complexity of the algorithm to gain a more profound understanding of the proposed coalition formation algorithm. As demonstrated in Theorem 1, our proposed coalition formation game algorithm iteratively converges to form a Nash stable partition. We define l_{max} as the maximum number of iterations for algorithm execution. In each iteration, every follower UAV will traverse J tasks, calculate its cost, and then send this information to their respective leader UAVs. The time complexity of performing this step is $O(I \cdot J)$. The leader UAV in each coalition then computes the utility values related to the number of UAVs in the coalition. Therefore, the time complexity of Algorithm 1 is $O(l_{max} \cdot I \cdot J^2)$.

VI. Simulation Results and Analysis

This section will design a simulation experiment to validate the effectiveness of the algorithm. Additionally, it will demonstrate the performance of the proposed algorithm framework under resource constraints through comparison with previous work.

TABLE II
SIMULATION PARAMETER SETTINGS

Parameters	**Value**	**Unit**
UAV Position	[0,1000]×[0,1000]	m
Task Position	[0,1000]×[0,1000]	m
Types of Resources	5	-
Quantity of Resources	[0,10]	-
UAV Mass	[10,15]	kg
Flight Energy per Unit Time	[1,3]	mW
Hover Energy per Unit Time	[1,3]	mW
Average Flight Speed	[20,30]	m/s
Communication Bandwidth	[5,15]	kHz
Transmission Power of UAV	[20,30]	dBm
Antenna Gain Constant	1	-
Attenuation Factor	3	-
Noise Power	[-60,-70]	dBm

A. Mission Scenario and Settings

In this subsection, we set the parameters for the UAVs and tasks. To ensure that critical information can be shared between UAVs, we assume that all communication between UAVs is connected. Each leader UAV can broadcast task information to all follower UAVs. Without loss of generality, the parameters characterizing the UAVs and the tasks are assigned randomly, while a comprehensive list of these parameters, along with their respective physical interpretations, is provided in Table II.

B. Analysis of Results

To investigate the impact of the number of UAVs and tasks on the convergence of the algorithm, we conduct 100 Monte Carlo simulations for scenarios with different numbers of UAVs and tasks. In the first group of experiments, we fix the number of tasks at 50 and vary the number of UAVs among $\{50, 70, 90, 110, 130, 150\}$. In the second group, we fix the number of UAVs at 90 and vary the number of tasks among $\{30, 40, 50, 60, 70, 80\}$. In the experiment, the maximum number of iterations l_{max} for algorithm execution was set to 100. Fig. 3 presents the results of these experiments through box plots. Fig. 3(a) shows that with a fixed number of tasks, the number of iterations increases linearly as the number of UAVs grows. Fig. 3(b) shows that with a fixed number of UAVs, the number of iterations increases slowly as the number of tasks grows. The growth in the number of iterations is more pronounced when increasing the number of UAVs compared to increasing the number of tasks. This is because adding more UAVs substantially increases the potential number of coalitions, resulting in a significant expansion of the search space. While increasing the number of tasks also contributes to problem complexity, its impact is less significant than that of increasing the number of UAVs.

Next, we compare the performance of our proposed algorithm with the methods presented in [17] and [18]. The research in both papers investigates the MRTA problem under resource constraints, achieving notable results in task completion rate and resource utilization rate. Owing to the more comprehensive UAV cost model considered in this paper than in [18] and [17], to ensure experimental fairness, we standardized the cost function to enhance the evaluation of algorithm performance. We conduct experiments by fixing either the number of UAVs or the number of tasks. The experimental results are presented using box plots, where the three sets of data near each x-axis coordinate correspond to the same coordinate. In the first set of experiments, the number of tasks is fixed at 50, and the number of UAVs varies among 50, 70, 90, 110, 130, 150. In Fig. 4(a) and Fig. 4(b), we demonstrate the changes in task completion rate and resource utilization rate as the number of UAVs varies. For all schemes, the increase in the number of UAVs leads to an improvement in the task completion rate, as a more significant number of UAVs results in a larger supply of various resources in the formed coalition, thereby increasing the task completion rate. However, the increase in the number of UAVs leads to a decrease in the resource utilization rate. This is because the more resources provided by UAVs, the more likely it is for irrational resource usage to occur, thus reducing the resource utilization rate. In the second set of experiments, we fix the number of UAVs at 90 and vary the number of tasks among 30, 40, 50, 60, 70, 80. From Fig. 5(a) and Fig. 5(b), we can observe that as the number of tasks increases, the available resources are insufficient to meet all tasks, leading to a continuous decrease in the task completion rate. Additionally, the increase in the number of tasks provides opportunities to utilize the idle resources of the UAVs, thereby improving the resource utilization rate.

Fig. 4 and Fig. 5 show that our solution outperforms the approaches described in [18] and [17] in terms of both task completion rate and resource utilization rate. This is because our solution employs a leader-follower coalition formation game, where the leader UAV considers the utility of each follower UAV by comprehensively evaluating both task completion rate and resource utilization rate. Guided

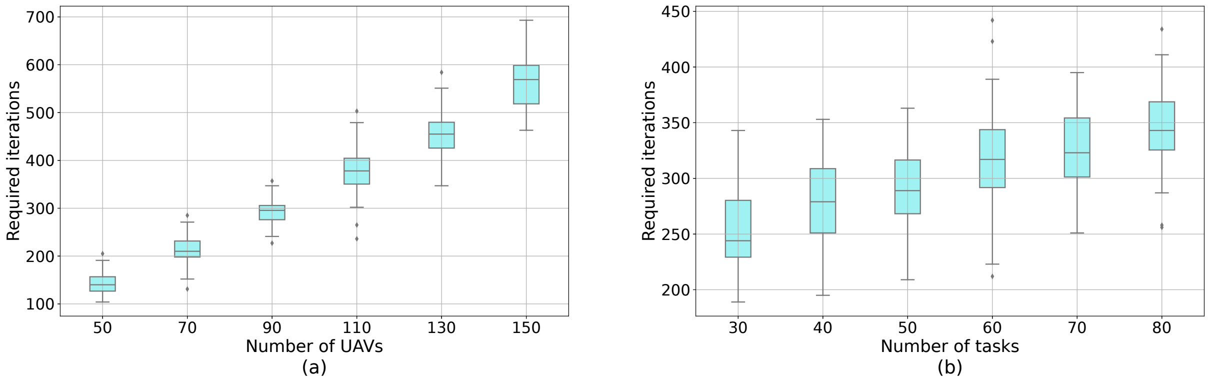

Fig. 3. Convergence performance of the coalition formation algorithm proposed in this paper.

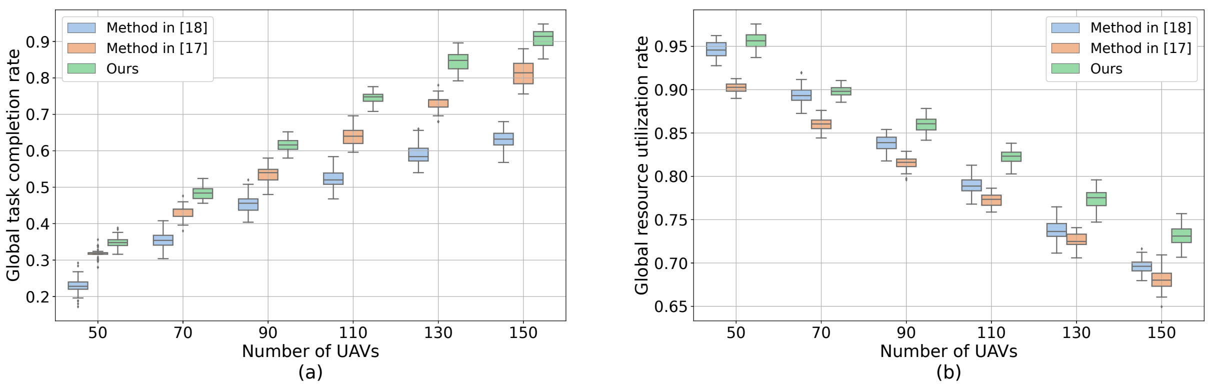

Fig. 4. Task completion rate and resource utilization rate vary with the number of UAVs.

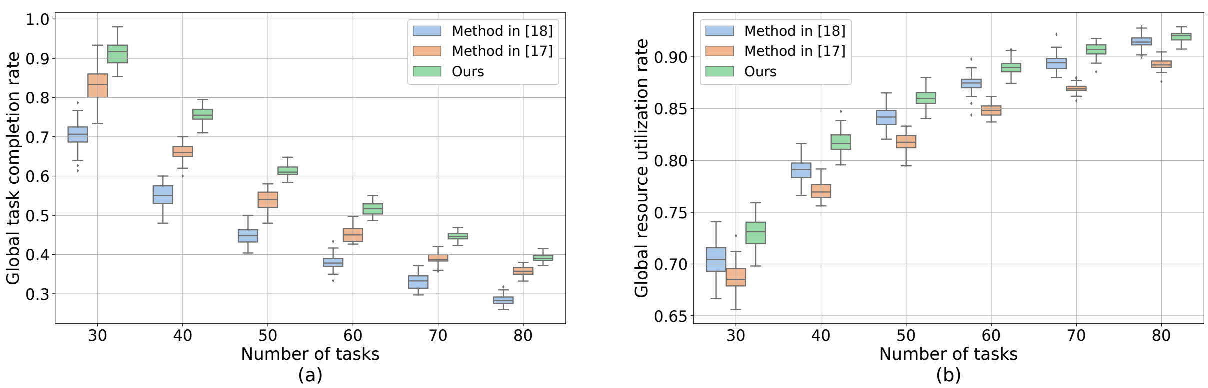

Fig. 5. Task completion rate and resource utilization rate vary with the number of tasks.

by the altruistic preference, follower UAVs select coalitions that maximize the overall benefit of the system rather than individual interests.

VII. CONCLUSIONS AND FUTURE WORKS

In this paper, we model the task allocation problem under resource constraints in MRS as a leader-follower coalition formation game problem. Our proposed coalition formation algorithm has proven effective in notably enhancing both task completion rate and resource utilization rate. By leveraging the characteristics of exact potential games, we have established the existence of Nash equilibria within our proposed framework. The validation of our algorithm through extensive simulation experiments shows that it outperforms existing methods for the MRTA problem under resource constraints in terms of both task completion rate and resource utilization rate.

In the future, we plan to expand our research to address common challenges in real-world environments, such as communication obstacles and robot malfunctions. Additionally, we intend to apply our research findings to specific practical scenarios, thereby enhancing the applicability and robustness of our methods in the real world.

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