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Energy-efficient Trajectory Planning with Media Transition for a Hybrid Unmanned Aerial-Underwater Vehicle

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***Abstract*— Vehicles capable of operating in more than one environment have been developed to solve real problems. Among them, the hybrid unmanned aerial-underwater vehicle (HUAUV) is receiving attention from the robotics community, mainly with a quadrotor-like configuration. However, this vehicle presents high energy consumption because of the larger mass required compared to the only aerial vehicle, limiting its autonomy. This work addresses the trajectory planning problem for a HUAUV. The method is based on Rapidly-exploring Random Trees (RRTs), a highly customizable planning technique. In addition, we propose two new heuristics to increase the energy efficiency of the hybrid vehicle. The first consists of biasing the tree expansion towards the environment with the lowest navigation cost, while the second one assigns estimated costs to nodes in the tree and chooses the least expensive trajectories. These techniques are evaluated in physically realistic simulation experiments performed in 135 scenarios. A comparative analysis of their performances is presented relative to the state of the art. We show that using efficient heuristics can significantly contribute to reducing energy consumption and even increase the average velocity in the missions performed by these vehicles.**

I. Introduction

HUAUVs are a class of mobile robots that can move through the air and underwater without any external action. Therefore, they can inspect partially submerged structures, monitor flooded ecosystems, and perform search and rescue tasks in hybrid environments. In recent years, advances have been made in the design [1], construction [2], [3], [4], modeling [5], and control of such vehicles [6], [7]. With these advances, systems with a higher level of autonomy could be developed. However, planning strategies for this class of vehicles are still understudied, with only a few investigations in the literature [8], [9], [10].

The authors of [8] utilized an improved teaching-and-learning-based optimization algorithm with Chebyshev collocation points to optimize the trajectory of a vertical navigating HUAUV, demonstrating the efficacy of the algorithm in this simplified problem through simulated results. Liang et al. [9] proposed a novel heuristic generalized extensive neighborhood search algorithm for a HUAUV [4] with online replanning. The author also applied the k-means technique in information clustering, while the heuristic GLNS algorithm was adopted to find the best path according to the path budgets and motion constraints. Wu et al. [10] presented an improved teaching-and-learning-based optimization algorithm for coaxial eight-rotor HUAUV trajectory, incorporating navigation error modeling, terrain matching, collision detection, and adjustment of cost functions. The authors used simulation results to validate the capability for multiphase tasks, including calculating the collision probability.

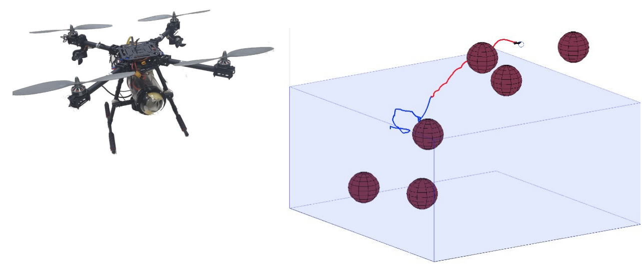

Fig. 1: Trajectory planning for a Hybrid Unmanned Aerial-Underwater Vehicle in unknown static cluttered environments.

In addition to the need for systems that support higher levels of autonomy, HUAUVs must also face another challenge. As these vehicles submerge, their electronic circuits, onboard computers, and batteries need waterproof protection, which significantly increases weight. As a result, their power-to-weight ratio is poorer than that of other vehicle classes, and for this reason, this work considers the study of the energy efficiency of these vehicles to be of utmost importance. Figure 1 illustrates the problem of planning trajectories for a HUAUV in a multi-media environment.

As discussed in [11], Robotics plays an important role in achieving a more sustainable society. As highlighted, allowing this new class of robots to reduce human presence in high-risk, high-cost activities can contribute directly to sustainability. However, studies such as [12] show the importance of considering the energy consumption of mobile robots for their wide-scale application.

We deal with these two issues related to HUAUVs: *planning systems* and *energy efficiency*. Our main contributions are two trajectory planning strategies designed to prioritize energy efficiency. In the first method, we introduce a bias to guide the planned trajectory toward the medium with the lowest energy consumption. In the second method, we incorporate energy consumption estimates for both media to optimize trajectory planning.

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The closest work in the literature is [13], which introduced an adaptation of the Closed-Loop Rapidly-exploring Random Tree (CL-RRT) [14] specifically tailored for smooth trajectory planning for HUAUVs, referred to in this work as Hybrid CL-RRT (HCL-RRT). Here, we build upon this prior work with a specific emphasis on enhancing energy efficiency, and we provide a comparative analysis of both approaches.

In Sec. II a formal definition of the problem is provided. Following, Sec. III introduces the vehicle model, the base trajectory planning algorithm, and both proposed heuristics. Then, in Sec. IV, the simulation environment, the experimental results, and their analysis are addressed. Finally, in Sec. V, we summarize this work and provide a brief discussion on future work.

II. Problem Formulation

Our problem involves planning a safe and energy-efficient trajectory for a HUAUV, formally defined as

Problem 1. *Let HUAUV be a system modeled with distinct dynamics for aerial and aquatic operation with their respective control laws [15], [13], navigating in an unknown environment $\mathcal{X}$ with static obstacles. We also assume the existence of a transition zone around the water surface at $z = 0$, in the range $-\mu \leq z \leq \mu$, for $\mu > 0$. Finally, given a cost function that relates the actions of the vehicle with the energy consumption along a trajectory, our main goal is to find a trajectory from the vehicle's starting point to a given goal ($\boldsymbol{r}_{goal}$)* online, *such that $\boldsymbol{x}(t) \in \mathcal{X}_{free}$ for all $t > 0$. The planner algorithm must also minimize energy consumption and allow for a smooth transition between environments.*

III. Methodology

A. Vehicle model

The dynamic model considered in this paper is the HUAUV proposed by [15], consisting of four upward-facing aerial thrusters, two downward-facing aquatic thrusters, and two forward-facing ones. In addition, these thrusters only act in one direction, relying on the restoring forces resulting from other physical principles, such as gravity and buoyancy, to achieve stability.

1) Power estimation: [16] presented a study of energy consumption for commercially available thrusters that could be used in HUAUVs. Based on their results, we obtain polynomial curves of energy consumption. These functions associate the energy power with the forces generated by the thrusters (Fig. 2).

B. Trajectory planning algorithm

RRTs [17] can accommodate dynamic and partially known environments. Our base trajectory planning approach (Fig. 3) is based on the CL-RRT [14] which allows continuous online planning. In this work, we introduce two novel heuristics to achieve energy-efficient vehicle navigation, which involves using estimated displacement costs for each medium and a biased tree expansion. We adopted the same symbols and representations proposed by [14] for details about the proposed heuristics.

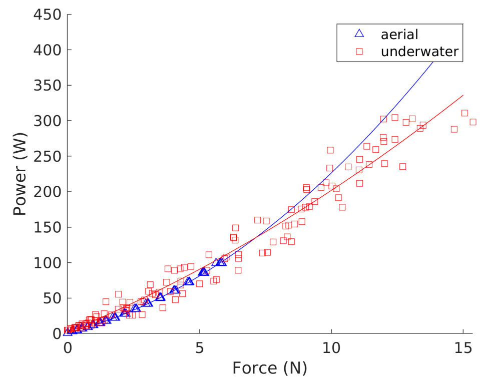


Fig. 2: Power consumption and polynomial regression curves for aerial and underwater thrusters. Both curves follow a second-degree polynomial trend. The underwater system presents higher consumption than the aerial system from null force until $7.5N$, while the aerial system assumes higher values for higher forces, widening the difference between the two systems.

The proposed heuristics are related to tree expansion. By controlling this, it is possible to add biases, as well as perform calculations that associate nodes and edges with estimated costs.

C. Water-Biased Expansion Heuristic (WBEH)

As verified in the energy consumption survey conducted by [16], the energy consumption per unit of force is higher for the aerial domain than for the aquatic domain. Furthermore, from the construction aspects of HUAUVs, it is known that these vehicles need to generate more force to keep the vehicle in the air than to keep it submerged in small velocities or hovering. Thus, the first proposed heuristic named WBEH drives the tree expansion to explore the aquatic environment than the aerial one. Thus, we expected that the planner would generate trajectories with lower amounts of force and, consequently, less energy consumption.

We bias the RRT sampling by using pseudorandom numbers in a certain interval. The interval can then be divided into two sampling subintervals: one for aquatic and the other for both environments. Thus, a threshold is set giving higher chances that the generated numbers fall into the aquatic sampling interval.

As shown on line 1 of the Algorithms 1 and 2, the `sampleRandomPoint()` function can take as argument the percentage of the pseudorandom generated numbers interval to be assigned to aquatic sampling.

D. Estimated Costs Expansion Heuristic (ECEH)

The second proposed heuristic named ECEH assumes that there are approximately constant average costs to travel between two points in the same environment or transition. The expansion function associated with this heuristic has a structure similar to WBEH. However, as presented in Algorithm 2 on line 11, we verified that a parent node - neighbor - in a certain radius of the new node - neighborhood

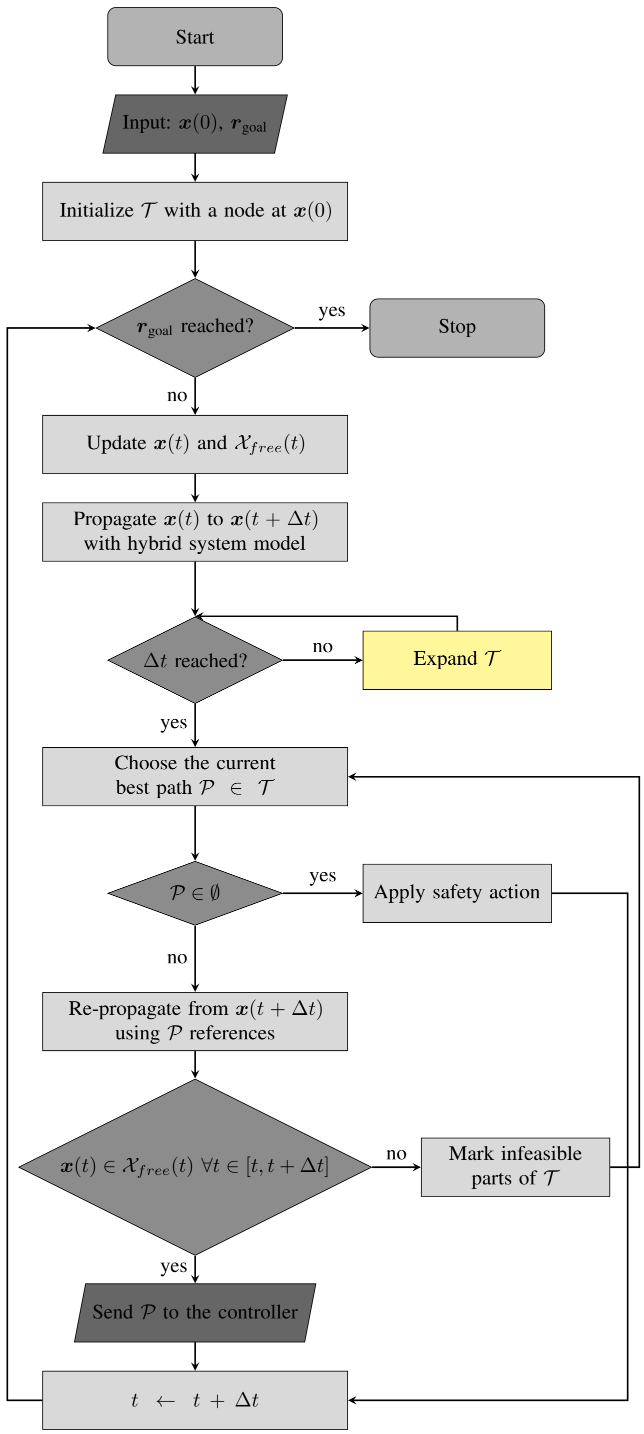


Fig. 3: Base trajectory planning procedure. The online planning procedure starts by checking if the vehicle has reached the goal region. If not, the algorithm will proceed to propagate a vehicle trajectory, expand the tree, and choose the best path in the tree to be followed. Safety measures are adopted if no safe path is found and infeasible parts of the tree are marked as unsafe. The tree expansion is a key process in this execution loop, where the proposed heuristics are applied, and is highlighted in yellow.

Algorithm 1 WBEH Algorithm for Tree Expansion

Require: Map, $segment\_length$, $water\_bias$

$\boldsymbol{p}_{sample} \leftarrow \texttt{sampleRandomPoint}(water\_bias)$
$\boldsymbol{p}_{closest} \leftarrow \texttt{getClosestNode}(\boldsymbol{p}_{sample})$
$\boldsymbol{p}_{dir} \leftarrow \boldsymbol{p}_{sample} - \boldsymbol{p}_{closest}$
$\boldsymbol{p}_{new} \leftarrow \boldsymbol{p}_{closest} + \hat{\boldsymbol{p}}_{dir} \times segment\_length$
if $\texttt{transition}(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
  $\boldsymbol{p}_{new} \leftarrow \texttt{verticalTransition}(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$
end if
if $\texttt{collision}(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
  return
else
  $\mathcal{T}.\texttt{addNode}(\boldsymbol{p}_{new})$
end if

- would generate a lower cost displacement, instead of simply adding the new node as a child of the closest node. After checking the best parent node, the new node is added to the tree as a child of it. Then, we verified which nodes in the neighborhood can be reached with a smaller cost. When a path that allows reaching any neighbors at a lower cost is found, rewiring is performed.

This approach would increase the number of steps for the expansion of the tree, which would probably lead to a longer execution time for each new insertion. However, this work seeks to evaluate the possibility of reducing energy consumption in missions performed by HUAUVs, and not necessarily reducing execution time. We assume that the more important energy consumption is related to the motor actuation.

The cost estimation can be obtained in different ways: from an experimental survey to consulting manufacturers' catalogs or even using generic models for motors. However, we considered that this survey could be carried out using a significant number in the order of thousands of small trajectories in simulation. The motor model used in the simulation should be consistent and close to the real one, as is the case.

IV. Experimental Results

In this section, we present the results of the experiments carried out in simulation using a realistic physical model. First, in Section IV-A, we describe the simulation tool. Then, in Section IV-B, we show the results and emphasize their main characteristics. And finally, in Section IV-C, the results are analyzed and discussed.

A. Simulation Environment

This work's simulation uses simplified versions of the world and map. Its world has only two kinds of space: spherical obstacles (occupied) and empty space. In addition, an air-water interface is a horizontal plane at $z = 0$ in the world frame. The map representation can be different from the world due to unknown obstacles, which are included in the map once their surface is inside the vehicle perception range.

Algorithm 2 ECEH Algorithm for Tree Expansion

Require: Map, $segment\_length$, $cost_{air}$, $cost_{uw}$, $cost_{trans}$, r

$\boldsymbol{p}_{sample} \leftarrow$ `sampleRandomPoint()`
$\boldsymbol{p}_{closest} \leftarrow$ `getClosestNode`$(\boldsymbol{p}_{sample})$
$\boldsymbol{p}_{dir} \leftarrow \boldsymbol{p}_{sample} - \boldsymbol{p}_{closest}$
$\boldsymbol{p}_{new} \leftarrow \boldsymbol{p}_{closest} + \hat{\boldsymbol{p}}_{dir} \times segment\_length$
if `transition`$(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
    $\boldsymbol{p}_{new} \leftarrow$ `verticalTransition`$(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$
end if
if `collision`$(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$ **then**
    return
else
    $min\_cost \leftarrow closest\_node.cost +$ `getCost`$(\boldsymbol{p}_{closest}, \boldsymbol{p}_{new})$
    $min\_cost\_node \leftarrow closest\_node$
    $neighbours \leftarrow$ `getNodesInRadius`$(\boldsymbol{p}_{new}, r)$
    for $node \in neighbours$ **do**
        if `transition`$(\boldsymbol{p}_{node}, \boldsymbol{p}_{new})$ **then**
            continue
        else
            $\boldsymbol{p}_{node} \leftarrow node.pos$
            $this\_cost \leftarrow closest\_node.cost +$ `getCost`$(\boldsymbol{p}_{node}, \boldsymbol{p}_{new})$
            if $this\_cost < min\_cost$ **then**
                $min\_cost \leftarrow this\_cost$
                $min\_cost\_node \leftarrow node$
            end if
        end if
    end for
    $\mathcal{T}$`.addNode`$(\boldsymbol{p}_{new})$
    `rewireTree`$(\boldsymbol{p}_{new}, neighbours)$
end if

Furthermore, the vehicle model described in Section III-A was adopted providing realistic dynamics and power estimation. The vehicle was considered to have null initial velocities to avoid collisions near the initial state. The model parameters used in the simulation were based on a HUAUV concept called Hydrone (Fig. 1), which is being developed at the Federal University of Rio Grande, Brazil. The vehicle parameters were previously presented in [16] and [13].

Table I presents the parameters associated with the simulation environment. The perception radius was set to a value that approximates the range of common sensors for both aerial and underwater perception, such as a stereo camera with good visibility. The world occupancy is the ratio between the total volume of obstacles and the world volume. The number of obstacles refers to how many spheres are in the world space. These values were chosen to represent large, but avoidable obstacles. Finally, the world dimensions show how the world space is distributed. These values were chosen to provide sufficient space for HUAUV maneuvers without overly increasing the $\mathcal{X}$ space.

TABLE I: Simulation Environment.

Parameter	Value
Perception radius	5 m
World occupancy	1%
Number of obstacles	10
World dimensions	20m x 20m x 20m

B. Quantitative Results

The objective of these experiments is to evaluate each heuristic showing their energy usage compared with the state-of-the-art algorithm HCL-RRT [13]. Hence, multiple scenarios were created, and each algorithm was evaluated for every scenario.

The results of 135 scenarios where all algorithms finished successfully were considered during this analysis. Figures 4 and 5 show three box plots, each with a notch in its box. The middle line of the box represents the median of the graph; the lower and upper lines of the box represent the first and third quartiles, respectively; the solid lines at the ends of the dashed line represent the tenth percentile and ninetieth percentile, below and above, respectively; and the notch represents the 95% confidence interval for the median.

In Fig. 4, one may note that the standard algorithm produced a larger interval for the average energy consumption. In addition, the distributions for the heuristics present significant differences regarding the state-of-the-art HCL-RRT, named Neutral in the figures.

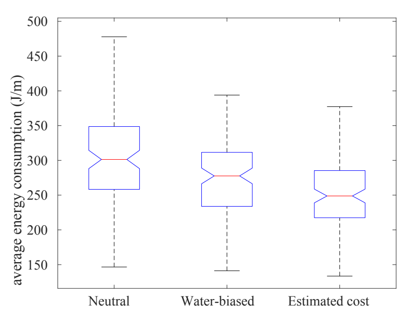


Fig. 4: Average energy consumption by trajectory planning algorithm.

In addition to the results for energy consumption, the results for average velocity are presented in Figure 5. As illustrated in the figure, the WBEH produced average speeds similar to the state-of-the-art algorithm HCL-RRT [13], whereas the ECEH was able to generate significantly higher velocity than the others.

Finally, the last evaluated perspective is the comparison of the algorithms for each scenario. In this situation, the performance of each algorithm concerning the others is separately assessed for each of the cases. Such results are presented in Table II.

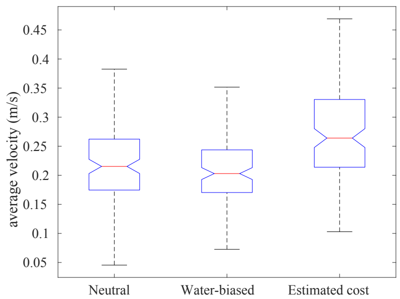


Fig. 5: Average velocity by trajectory planning algorithm.

TABLE II: Comparison between the energy consumption data of each algorithm for each scenario. The number of times each algorithm was the best (lowest consumption) or the worst (highest consumption) for the 135 scenarios considered.

Planning algorithm	Best	Worst
HCL-RRT	7 (5.18%)	82 (60.74%)
WCEH	39 (28.89%)	47 (34.81%)
ECEH	**89 (65.93%)**	**6 (4.44%)**

C. Analysis and Qualitative Results

1) Energy consumption results: The trajectory planners with the proposed heuristics were able to provide lower energy cost trajectories for the same scenario compared to the state-of-the-art HCL-RRT [13]. This is possible because physical principles were considered in a new approach to HUAUV multidomain planning.

The first of these principles consisted of biasing the vehicle, so it navigates through the medium that requires less energy to move. In this case, the aquatic environment was chosen because it requires less force for the vehicle to withstand its buoyancy than to support its weight in the air.

The second principle used for the heuristics was to estimate the trajectory's cost while it is planned. Once the average consumption for navigation in certain domains is known, it is possible to estimate the energy consumption for common operating situations. In this way, it is possible to estimate the least-cost trajectories that lead to the goal. An additional step was added to this heuristic, considering that the system can perform all the necessary steps for online operation, this planner calculates the realignment of trajectories according to their costs.

The lower energy consumption is related to two factors: traveling in a lower-cost domain and planning for a smoother and more direct trajectory. Traveling in a lower-cost medium may seem a straightforward solution; however, changing directions may waste vehicle momentum and contribute to higher overall energy consumption. Additionally, since the map is unknown, only planning for smooth, direct trajectories, regardless of the medium, may lead to detecting obstacles in the planned trajectory while not in an energy-friendly medium, and possibly leading to a higher energy consumption.

2) Velocity results: As presented in Section IV-B, the state-of-the-art HCL-RRT and WBEH algorithms had similar behaviors from the point of view of the average velocities between the start and end points of their trajectories. The ECEH algorithm presented higher average velocities than the other two algorithms. Thus, this algorithm can be classified as the best, since it significantly outperformed the others, both in energy consumption reduction and average speed increase. Both qualities are usually desired in missions with HUAUVs.

The data collected is close to a normal distribution. Therefore, they can be analyzed by testing the significance of the null hypothesis, the same as in the previous section. Through it, it can be concluded that for the significance level of 5%, both the HCL-RRT and WBEH algorithms can be considered to produce lower average velocities when compared to the ECEH algorithm. The performances of the planners regarding average speed can be better understood by analyzing Fig. 6.

Fig. 6a presents the trajectory for the state-of-the-art HCL-RRT algorithm. Through this, it is possible to observe that during the mission time, the trajectory uses more the aquatic and aerial environments.

Fig. 6b shows the results for the WBEH algorithm, where the vehicle navigates for longer in the water consuming less energy for its displacement. It also ends up being subjected to a condition of less maneuverability. This results in a lower average speed to reach the goal.

Finally, Fig. 6c presented qualitative results obtained using the ECEH algorithm. It produces trajectories that change direction with less intensity.Thus, the vehicle can follow the references passed more smoothly and end up reaching the goal in a more direct and faster way.

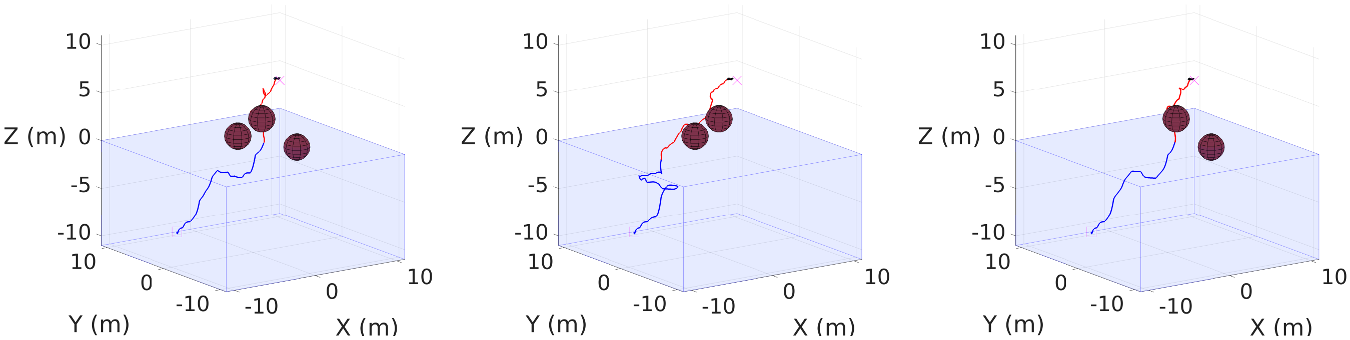
D. Results considerations

The results have yielded valuable insights into the behavior of multidomain trajectory planning. Initially, it was expected that planning based on geometric exploration would result in inefficient references for the model predictive trajectory generator. However, the significant impact of employing heuristics to modify planner properties is impressive.

Both heuristics were designed based on a fundamental principle of energy conservation: achieving motion with minimal effort. One approach biased the trajectory towards the lower-cost domain, while the other involved estimating energy consumption along a specific trajectory.

An unexpected behavior emerged where the estimated route not only led to energy savings but also resulted in faster-executed trajectories. It is important to note that these findings are influenced by the characteristics of the simulated scenario.

Finally, the results provide relevant evidence about the potential application of the planning techniques in computing trajectories for multidomain HUAUVs. The diverse range of scenarios examined demonstrates that the results obtained are significant and capable of generating lower-cost trajectories. The findings highlight the effectiveness and practicality of these techniques, particularly considering their algorithmic simplicity. Although further research is still required with



(a) HCL-RRT (32.8m). (b) WBEH (36.7m). (c) ECEH (30.8m).

Fig. 6: Final trajectory for one of the scenarios presenting only the detected objects. The detected obstacles are presented as dark red spheres, the underwater trajectory as a blue line, and the aerial trajectory as a red line.

a real robot, these findings offer promising insights for the adoption of these techniques in autonomous platform motion systems.

V. Conclusion and Future Work

This work addressed the problem of trajectory planning for HUAUV, focusing on energy efficiency. We propose two new heuristics to increase the energy efficiency of the vehicle in hybrid environments, one consisting of biasing the RRT expansion towards the domain with the lowest navigation cost - WBEH, and another assigning estimated costs to nodes in the tree, choosing the least expensive trajectories - ECEH. A comprehensive comparison was conducted between a state-of-the-art algorithm and the proposed heuristics, showing advantages int terms of energy consumption and velocity.

In future work, we aim to incorporate the influence of atmospheric or underwater currents into our trajectory planning algorithms to further enhance their adaptability and efficiency in real-world scenarios. Therefore, it could be studied if these algorithms would need to be adjusted for more variable conditions and if these adjustments would be enough to maintain their good performance. Another opportunity that was not investigated here refers to the use of the technique of node rewiring, used in the estimated cost algorithm. This technique could be employed in the other algorithms, and possibly contribute to more direct and agile trajectories.

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