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Opinion-based Strategy for Distributed Multi-Robot Task Allocation in Swarms of Robots

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Abstract— **Opinions of individuals in large groups evolve through interactions with neighbors and the environment, which can be modeled with opinion dynamics. In this paper, we propose a distributed opinion-based strategy for large-scale multi-robot task allocation utilizing the convergence behaviors of opinion dynamics. The strategy relies on the specialized opinion dynamics on the unit sphere for robot task selection. We investigate the convergence behaviors of opinion dynamics in the context of regions of attraction. Simulation results with a swarm of 200 homogeneous robots validate the effectiveness of our proposed strategy.**

I. Introduction

For homogeneous robots to collaborate on multiple tasks, they need to address the multi-robot task allocation problem. This problem can be modeled as a combinatorial optimization problem [1], it is challenging due to discontinuities in the solution space. To overcome this challenge, we propose an alternative approach by introducing opinions into robot states to represent task preferences. This models task allocation as a continuous-time convergence problem of opinions, eliminating the need for an explicit cost function.

Inspired by opinion dynamics in social networks, where individuals form consensus or dissensus through exchanging opinions [2], we leverage our previous work on convergence behaviors and stability [3]–[5] to propose an opinion-based strategy for distributed multi-robot task allocation. Robots' task preferences are modeled as vectors evolving continuously on the unit sphere, based on their prior information. Tasks are modeled as virtual robots with fixed opinions, participating in opinion exchanges with neighboring robots.

Driven by a specially designed interaction control, the robots can only communicate with local neighbors (no centralized communication or broadcasting) and their opinions converge to different stable equilibria, determining the task assignments among robots. Discussions on convergence behaviors based on regions of attraction are provided for stability guarantee. In simulations with 200 homogeneous robots and various initial scenarios, our strategy outperforms a local voting strategy, showing better performance and adaptivity in terms of average distance to selected tasks when robots have limited information about task locations.

Our major contributions in this paper are as follows: 1) Creating a scalable task allocation algorithm based on the convergence behaviors of specially designed opinion dynamics. 2) Providing detailed discussions on convergence behaviors with stability guarantee; 3) Validating the proposed strategy with comparisons to a local voting strategy.

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II. Background

A. Multi-Robot Task Allocation

Multi-robot task allocation, a key component for multi-robot coordination, determines the appropriate assignment between robots and tasks. In our earlier works [6], [7], we focused on a small multi-robot team where each robot can be assigned multiple tasks. Instead, in this paper, we consider a swarm of robots where each robot can participate in only one task, and each task requires cooperation between multiple robots. This can be categorized as "single-task robots, multi-robot tasks, instantaneous assignment (ST-MR-IA)" [1]. This is also known as *coalition formation*. Many existing decentralized approaches for robotic swarms require robots to know the current robot distribution to achieve the desired robot distribution, either by broadcast [8] or a consensus mechanism [9]. Our approach considers the scenario where robots do not know the current distribution but find the appropriate task assignments only based on the opinions/task selection of their neighbors.

Researchers have developed task allocation mechanisms for swarm robots that rely only on local sensor data and interactions between nearby neighbors [10]. These mechanisms are mainly threshold-based and probabilistic. In threshold-based methods [11], a robot engages in a task when its input value exceeds its threshold. Probabilistic methods [12] have robots perform tasks based on probabilities influenced by the environment. However, these methods mainly address foraging scenarios, where robots choose between only two tasks: resting and foraging

B. Opinion Dynamics

Individuals interact and exchange opinions on certain topics within social networks [13], [14]. While communicating with neighbors, individuals gradually form a consensus or dissensus based on the discussion results. Such convergence results then lead to proper decision-making within networks.

In literature, various opinion dynamics have been proposed to model different opinion formations. In [15], opinion states evolve in the Euclidean space and follow a linear update law based on the weighted average of neighbors' opinions. Opinion states have also been modeled on nonlinear manifolds such as the unit-sphere and the orthogonal group $SO(n)$, as in [16]–[18]. In [19], stable dissensus can be achieved based on linear/nonlinear consensus protocols on a signed graph.

Our proposed strategy works on unsigned graphs and enables richer convergence behaviors of multi-clustering other than consensus and bipartite dissensus only [3], [4], where the theoretical guarantee is discussed in [5].

Due to diverse convergence behaviors, opinion dynamics provide an alternative approach for solving task allocation problems, bypassing the need to solve combinatorial optimization problems. In [20], the task allocation problem between two options is addressed by applying opinion dynamics through decentralized switching transformations. Moreover, an opinion model has been applied in [21] to choose the shorter path of two available paths without explicit knowledge of travel time. However, to the best of our knowledge, no existing work on opinion dynamics has explored task allocation scenarios for more than two tasks.

III. Problem Formulation

Suppose an N_R number of robots are deployed in a 2D field with N_T tasks, which requires the robots to form N_T different groups. We label the tasks as $1, \cdots, N_T$, and the robots as $N_T+1, \cdots, N_T+N_R$. Denote the set of tasks as $\mathcal{V}_T = \{1, \cdots, N_T\}$ and the set of robots as $\mathcal{V}_R = \{N_T + 1, \cdots, N_T + N_R\}$. Define set $\mathcal{V} = \mathcal{V}_T \cup \mathcal{V}_R$. Denote the location of robot i (or task i) as $\boldsymbol{r}_i \in \mathbb{R}^2$. Denote by $\mathcal{N}_i$ the neighboring set of robot i (or task i) containing all the neighboring robots and tasks of i. The relationships between robots (or robots and tasks) in the field are described by an unsigned graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{E}$ is the set of all edges.

Assumption III.1. *Robots have communication limits and can share information with robots (or receive information from tasks) within communication range R_c, i.e. $j \in \mathcal{N}_i$ and $(i,j) \in \mathcal{E}$ if and only if $\|\boldsymbol{r}_j - \boldsymbol{r}_i\|_2 < R_c$.*

Assumption III.2. *The number of robots is significantly greater than the number of tasks, i.e. $N_R \gg N_T$.*

Assumption III.3. *The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ formed by the group of robots and tasks is unsigned, undirected, and connected, i.e. $(i,j) \in \mathcal{E}$ and $(j,i) \in \mathcal{E}$ for all $i \in \mathcal{V}, j \in \mathcal{N}_i$. There also exists a path connecting i,j for all $i,j \in \mathcal{V}$.*

The opinion of robot i towards different tasks is modeled by a unit-length vector $\boldsymbol{o}_i \in \mathbb{S}^{N_T-1}$, $i \in \mathcal{V}$ and $N_T \in \mathbb{Z}_+$, $N_T \geq 2$. Each dimension in the opinion $\boldsymbol{o}_i$ represents the preference towards one specific task, and the larger the element is the more likely the robots will choose the task. Because of the unit-length property, at most one element of $\boldsymbol{o}_i$ is 1, which means that robot i can select at most one task.

Define $\boldsymbol{o} = [\boldsymbol{o}_1^\intercal, \cdots, \boldsymbol{o}_N^\intercal]^\intercal \in \mathbb{R}^{N_T N_R}$ as the big vector containing all the opinion states. Each opinion state evolves on the surface of the unit sphere $\mathbb{S}^{N_T-1}$ according to the nonlinear dynamics [18], [22]

$$\dot{\boldsymbol{o}}_i = \begin{cases} \boldsymbol{0}, \ \forall i \in \mathcal{V}_T, \\ (\boldsymbol{I} - \boldsymbol{o}_i \boldsymbol{o}_i^\intercal)\boldsymbol{u}_i, \ \forall i \in \mathcal{V}_R, \end{cases} \tag{1}$$

where $\boldsymbol{I} \in \mathbb{R}^{N_T \times N_T}$ is the identity matrix and $\boldsymbol{u}_i = \boldsymbol{u}_i(\boldsymbol{o}) \in \mathbb{R}^{N_T}$ is a control input for robot i determined by $\boldsymbol{o}$. We call this control the *interaction control* since it characterizes the nature of robot interactions. Since tasks are treated as virtual robots with fixed opinions, $\dot{\boldsymbol{o}}_i = \boldsymbol{0}$ for $i \in \mathcal{V}_T$ in (1).

We consider the following sub-problems in this paper:

- **(Sub-problem 1)** Initialize the opinion states based on the information that robots or tasks have.
- **(Sub-problem 2)** Design an interaction control
$$\boldsymbol{u}_i = \sum_{j \in \mathcal{N}_i} f(\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)\boldsymbol{o}_j, \ \forall i \in \mathcal{V}_R, \tag{2}$$
where $f(\cdot) : [-1, 1] \mapsto \mathbb{R}$ is a function of $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j$ describing relationships between robots and their neighbors.
- **(Sub-problem 3)** Classify the equilibrium sets of the closed-loop opinion dynamics system (1) under the novel control input (2), and interpret the task assignments based on convergence and stability behaviors.

(Sub-problem 1) models the scenario when robots have *a priori* information, and is the first step of the proposed opinion-based strategy. Besides designing a proper opinion interaction control, **(Sub-problem 2)** is significant because it focuses on the evolution of novel opinion behaviors. Lastly, the equilibrium sets and the connections studied in **(Sub-problem 3)** explain how opinion behaviors can be utilized to solve the multi-robot task allocation problem.

IV. Opinion-based Strategy

In this section, we provide three types of initialization of opinion states based on the available information, and the interaction control design for the opinion dynamics in (1).

A. Initialization of Opinion States

Denote the basis vectors in $\mathbb{R}^{N_T}$ as $\{\boldsymbol{e}_k\}_{k=1}^{N_T}$ where only the k-th element of $\boldsymbol{e}_k$ is 1 while others are all 0. The basis vectors $\{\boldsymbol{e}_k\}_{k=1}^{N_T}$ all satisfy $\|\boldsymbol{e}_k\|_2 = 1$ for all $k = 1, \cdots, N_T$. The tasks' opinions $\{\boldsymbol{o}_i\}_{i \in \mathcal{V}_T}$ are initialized as

$$\boldsymbol{o}_i(0) = \boldsymbol{e}_i, \ \forall i \in \mathcal{V}_T, \tag{3}$$

which models tasks as virtual robots with fixed opinions for selecting themselves. The initialization of the robots' opinions is greatly determined by the information they have. We consider the following three scenarios:

- *No initial information*: Robots have no information about the tasks at time $t = 0$. Thus, they have no special preference for different tasks.
- *Random initial information*: Robots have random-generated information about tasks at time $t = 0$, which determines their preferences for different tasks.
- *Perfect initial information*: Robots have obtained perfect information about the tasks at time $t = 0$ such as distance. Then they can form preferences for different tasks using such information.

Next, we discuss the three scenarios respectively.

1) No Initial Information: Since the robots have no available information and also no preference for tasks to conduct, the opinions are initialized as follows

$$\boldsymbol{o}_i(0) = \left[\frac{1}{\sqrt{N_T}}, \cdots, \frac{1}{\sqrt{N_T}}\right]^\intercal \in \mathbb{S}_+^{N_T-1},\ i \in \mathcal{V}_R, \tag{4}$$

where each element of the opinion is of the same value $\frac{1}{\sqrt{N_T}}$ and such initialization obeys the unit-length constraint.

2) Random Initial Information: Robots have randomly generated information at $t = 0$, with initial opinions

$$\boldsymbol{o}_i(0) = \text{rand}(\mathbb{S}_+^{N_T-1}),\ i \in \mathcal{V}_R, \tag{5}$$

where $\text{rand}(\mathbb{S}_+^{N_T-1})$ denotes the random unit-length vector in $\mathbb{R}^{N_T}$ and all elements in $\text{rand}(\mathbb{S}_+^{N_T-1})$ is strictly positive.

3) Perfect Initial Information: Suppose when the robots are initially deployed in a field with multiple tasks, they have some distance information and use such information to determine preferences towards different tasks.

Denote the distance between robot i and task k as $d_{ik} \in \mathbb{R}_+$, where $i \in \mathcal{V}_R$ and $k \in \mathcal{V}_T$. We consider the case when the distance information d_{ik} has no noise. For all $i \in \mathcal{V}_R$, the opinion $\boldsymbol{o}_i(0) = [o_i^1(0), \cdots, o_i^{N_T}(0)]^\intercal$ is initialized as

$$o_i^k(0) = \frac{1}{\alpha_i \log(d_{ik} + 1 + \epsilon)},\ \forall k = 1, \cdots, N_T, \tag{6}$$

where $\alpha_i = \sqrt{\sum_{k=1}^{N_T}\left(\frac{1}{\log(d_{ik}+1+\epsilon)}\right)^2} > 0$ ensures $\|\boldsymbol{o}_i(0)\|_2 = 1$, and $\epsilon \in \mathbb{R}_+$ is a small tuning parameter to ensure $\log(d_{ik} + 1 + \epsilon) > 0$. According to (6), robots prefer closer tasks. This is reasonable in the real world since robots can detect more accurate information in a smaller region.

Besides, if robot i coincides with task k at the same location, then $d_{ik} = 0$, leading to $o_i^k(0) \to 1, o_i^{k'}(0) \to 0$ for all $k' \neq k$, and $\boldsymbol{o}_i(0) \to \boldsymbol{e}_k$ in this special scenario.

The distances between the robots and tasks satisfy that $d_{ik} \geq 0$ for all $i \in \mathcal{V}_R, k \in \mathcal{V}_T$. This further leads to $o_i^k(0) > 0$ for all $k = 1, \cdots, N_T$ according to the initialization described in (6). Thus, the initial opinion states satisfy $\boldsymbol{o}_i(0) \in \mathbb{S}_+^{N_T-1}$ for all $i \in \mathcal{V}_R$.

B. Design of Interaction Control

The overall design goal is to induce convergence behaviors of $\boldsymbol{o}_i \to \boldsymbol{e}_k$ for some k and for all $i \in \mathcal{V}_R$. This requires when $\dot{\boldsymbol{o}}_i = \boldsymbol{0}$ for all $i \in \mathcal{V}$ at equilibrium, opinion states satisfy $\boldsymbol{o}_i = \boldsymbol{o}_j$ or $\boldsymbol{o}_i \perp \boldsymbol{o}_j$ for all $(i, j) \in \mathcal{E}$. Based on the unit-length property, this also means $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j \in \{0, 1\}\ \forall (i, j) \in \mathcal{E}$.

To obtain the desired convergence behaviors, we design a scalar function $f(\cdot) : [-1, 1] \mapsto \mathbb{R}$ in (2) as $f(x) = x(\beta x^2 - 1)$, where $\beta \in \mathbb{R}_+, \beta > 1$. The polynomial function $f(x)$ has roots of $0, \pm\frac{1}{\sqrt{\beta}}$ within $[-1, 1]$ as shown in Fig. 1. When the inner product values satisfy $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j \in \{0, \pm 1\}$ for all $(i, j) \in \mathcal{E}$, then the equilibria are stable (denoted by green closed dots). When $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j = \pm\frac{1}{\sqrt{\beta}}$ for some $(i, j) \in \mathcal{E}$, then the equilibria are unstable (represented by green open dots). More details will be provided in Section V.

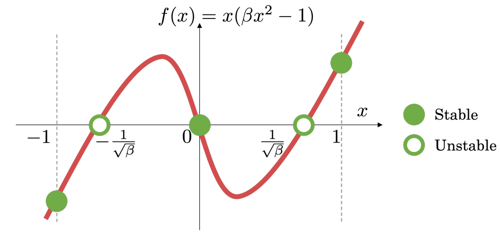


Fig. 1: Scalar function $f(x)$ (red curve) within $[-1, 1]$. Green closed (open) dots represent stable (unstable) equilibria.

Utilizing the polynomial function $f(x)$, we can obtain

$$\boldsymbol{u}_i = \sum_{j \in \mathcal{N}_i} (\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)[\beta(\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)^2 - 1]\boldsymbol{o}_j,\ \forall i \in \mathcal{V}_R. \tag{7}$$

Since $N_R \gg N_T$ by Assumption III.2, the influence from neighboring tasks is much weaker than that from neighboring robots, which might lead to undesired convergence of $\boldsymbol{o}_i \not\to \boldsymbol{e}_k$ for all $i \in \mathcal{V}_R, k \in \mathcal{V}_T$. Thus, we add a small modification to the control in (7) as follows

$$\boldsymbol{u}_i = \sum_{j \in \mathcal{N}_i} (\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)[\beta(\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)^2 - 1] w_{ij} \boldsymbol{o}_j,\ \forall i \in \mathcal{V}_R, \tag{8}$$

where $w_{ij} \triangleq \begin{cases} 1,\ j \in \mathcal{N}_i \cap \mathcal{V}_R, \\ \left\lfloor \dfrac{N_R}{N_T} \right\rfloor,\ j \in \mathcal{N}_i \cap \mathcal{V}_T. \end{cases}$ The influence of adding w_{ij} can be interpreted as putting a number of $\lfloor \frac{N_R}{N_T} \rfloor$ same tasks at the location of task k for any $k \in \mathcal{V}_T$ to increase the influence of task k on the neighboring robots.

Thus, the opinion dynamics (1) can be rewritten as

$$\dot{\boldsymbol{o}}_i = \begin{cases} \boldsymbol{0},\ \forall i \in \mathcal{V}_T, \\ (\boldsymbol{I} - \boldsymbol{o}_i \boldsymbol{o}_i^\intercal) \sum_{j \in \mathcal{N}_i} (\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)[\beta(\boldsymbol{o}_i^\intercal \boldsymbol{o}_j)^2 - 1] w_{ij} \boldsymbol{o}_j, \forall i \in \mathcal{V}_R. \end{cases} \tag{9}$$

The opinion-based strategy for distributed multi-robot task allocation is presented in Algorithm 1, where the convergence behaviors of $\boldsymbol{o}_i \to \boldsymbol{e}_k$ is utilized for task selection.

Algorithm 1: Opinion-based Strategy

Initialize opinion states $\{\boldsymbol{o}_i(0)\}_{i \in \mathcal{V}}$ using (4), (5), or (6)

while $\dot{\boldsymbol{o}}_i \neq \boldsymbol{0}\ \forall i \in \mathcal{V}$ **do**

  Run opinion dynamics (9)

end while

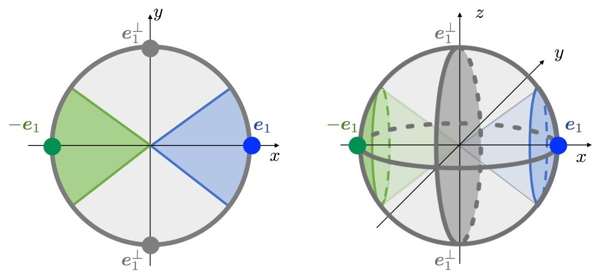
Select $k \in \mathcal{V}_T$ such that $\boldsymbol{o}_i \to \boldsymbol{e}_k$ when $\dot{\boldsymbol{o}}_i = \boldsymbol{0}$ for all $i \in \mathcal{V}$

V. Convergence Behaviors

According to the design of $\boldsymbol{u}_i$ in (8) and the explicit opinion dynamics in (9), it is true that $\dot{\boldsymbol{o}}_i = \boldsymbol{0}$ for all $i \in \mathcal{V}$ when $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j \in \{0, \pm\frac{1}{\sqrt{\beta}}, \pm 1\}$ for any $(i, j) \in \mathcal{E}$.

Theorem V.1. *The equilibrium $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j \in \{0, \pm 1\}$ for any $(i, j) \in \mathcal{E}$ is asymptotically stable while the equilibrium associated with $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j = \pm\frac{1}{\sqrt{\beta}}$ for some $(i, j) \in \mathcal{E}$ is unstable.*

The stability regarding different inner product values is also illustrated in Fig. 1. Based on Theorem V.1 and Fig. 1, the unstable inner products $\pm\frac{1}{\sqrt{\beta}}$ separate the stable values $\{0, \pm 1\}$. Detailed stability proof is presented in [5].



(a) Example in $\mathbb{S}^1$ (b) Example in $\mathbb{S}^2$

Fig. 2: Examples on regions of attraction for fixed opinion $\boldsymbol{e}_1$ in $\mathbb{S}^1, \mathbb{S}^2$. Blue and green dots represent $\boldsymbol{e}_1, -\boldsymbol{e}_1$, respectively. Blue and green shaded areas show the corresponding regions of attraction. Grey dots in (a) and dark grey circle $\{(x, y, z) | x = 0, y^2 + z^2 = 1\}$ in (b) represent $\boldsymbol{e}_1^{\perp}$, and light grey shaded areas denote the regions of attraction for $\boldsymbol{e}_1^{\perp}$.

Example V.1. *We consider two illustrative examples regarding the regions of attraction concerning fixed opinion $\boldsymbol{e}_1$ in $\mathbb{S}^1, \mathbb{S}^2$, respectively, as shown in Fig. 2. We only analyze the robots directly connected to the fixed opinion $\boldsymbol{e}_1$. Vector $\boldsymbol{e}_1$ satisfies $\boldsymbol{e}_1 = [1, 0]^\intercal \in \mathbb{S}^1$ and $\boldsymbol{e}_1 = [1, 0, 0]^\intercal \in \mathbb{S}^2$. The convergence of opinion state towards $\boldsymbol{e}_1$, $-\boldsymbol{e}_1$ and $\boldsymbol{e}_1^{\perp}$ can be summarized as follows*

- $\boldsymbol{e}_1$*: For robot $i \in \mathcal{N}_1$ satisfying $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i(0) > \frac{1}{\sqrt{\beta}}$, opinion state $\boldsymbol{o}_i$ will converge to the equilibrium $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i = 1$, i.e. $\boldsymbol{o}_1 \rightarrow \boldsymbol{e}_1$, with region of attraction $\{\boldsymbol{o}_i | \boldsymbol{e}_1^\intercal \boldsymbol{o}_i > \frac{1}{\sqrt{\beta}}, i \in \mathcal{N}_1\}$ (blue shaded areas).*
- $-\boldsymbol{e}_1$*: For robot $i \in \mathcal{N}_1$ satisfying $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i(0) < -\frac{1}{\sqrt{\beta}}$, opinion state $\boldsymbol{o}_i$ will converge to the equilibrium $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i = -1$, i.e. $\boldsymbol{o}_1 \rightarrow -\boldsymbol{e}_1$, with region of attraction $\{\boldsymbol{o}_i | \boldsymbol{e}_1^\intercal \boldsymbol{o}_i < -\frac{1}{\sqrt{\beta}}, i \in \mathcal{N}_1\}$ (green shaded areas).*
- $\boldsymbol{e}_1^{\perp}$*: For robot $i \in \mathcal{N}_1$ satisfying $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i(0) \in (-\frac{1}{\sqrt{\beta}}, \frac{1}{\sqrt{\beta}})$, opinion state $\boldsymbol{o}_i$ will converge to the equilibrium $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i = 0$, i.e. $\boldsymbol{o}_1 \rightarrow \boldsymbol{e}_1^{\perp}$, with region of attraction $\{\boldsymbol{o}_i | \boldsymbol{e}_1^\intercal \boldsymbol{o}_i \in (-\frac{1}{\sqrt{\beta}}, \frac{1}{\sqrt{\beta}}), i \in \mathcal{N}_1\}$ (light grey shaded areas).*

Based on opinion initializations in (3), (4), (5) and (6), all opinion states satisfy $\boldsymbol{o}_i \in \mathbb{S}_{\geq 0}^{N_T - 1}$ for any $i \in \mathcal{V}$ and $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j \geq 0$ for any $(i, j) \in \mathcal{E}$ at $t = 0$, implying that $\boldsymbol{o}_i^\intercal \boldsymbol{o}_j \rightarrow -1$ cannot be achieved and achievable equilibria satisfy

$$\boldsymbol{o}_i^\intercal \boldsymbol{o}_j = 0, 1, \ \forall (i, j) \in \mathcal{E}. \tag{10}$$

We want to avoid the convergence scenario when robots' opinions are not converging to a neighboring task's opinion, but instead to another task's opinion. Example V.1 indicates that the value $\frac{1}{\sqrt{\beta}}$ separates different regions of attractions. For example when robots' opinions are initialized as (4), $\boldsymbol{e}_1^\intercal \boldsymbol{o}_i(0) = \frac{1}{\sqrt{N_T}}$ for all $i \in \mathcal{N}_1$. To ensure $\boldsymbol{o}_i \rightarrow \boldsymbol{e}_1$, the parameter β needs to satisfy $\frac{1}{\sqrt{\beta}} < \frac{1}{\sqrt{N_T}}$, i.e. $\beta > N_T$.

By the definition of basis vectors $\{\boldsymbol{e}_k\}_{k=1}^{N_T}$, they always satisfy $\boldsymbol{e}_{k_1}^\intercal \boldsymbol{e}_{k_2} = 0$ for any $k_1 \neq k_2$. Since tasks have been incorporated into opinion systems as virtual robots with fixed opinions of $\{\boldsymbol{e}_k\}_{k=1}^{N_T}$ and the graph is connected by Assumption III.3, the equilibrium satisfying (10) can also be described as: for any $i \in \mathcal{V}_R$, there exists $k \in \mathcal{V}_T$ such that $\boldsymbol{o}_i \rightarrow \boldsymbol{e}_k$ as $t \rightarrow \infty$. This means that robot i chooses task k.

VI. Simulation Experiments and Discussions

In this section, we validate our opinion-based task allocation strategy through simulations. A 100-meter by 100-meter environment hosts 200 homogeneous robots and up to 8 tasks to satisfy Assumption III.2, with robots having a 15-meter communication range. Each run varies robot positions, task numbers, and task locations randomly. We generate four cases with 2, 4, 6, and 8 tasks, conducting 500 runs for each case. If Assumption III.3 is violated, we reinitialize until it is valid, discarding less than 10% of the total runs.

We compare our proposed algorithm with the **Local Voting Strategy** described in Algorithm 2 in simulation. In the local voting strategy, we adopt the majority voting rule on task selection. Each robot i maintains a task selection vector $\mathbf{a}_i = [a_{i1}, \cdots, a_{iN_T}]^\intercal \in \{0, 1\}^{N_T}$ with $\sum_{k=1}^{N_T} a_{ik} \leq 1$, and $a_{ik} = 1$ indicates robot i selects task k. At each timestep in the voting process, robot i receives the task selection $\mathbf{a}_j$ of its neighbors $j \in \mathcal{N}_i$ and updates its task selection based on which task has the most votes from its neighbors and itself.

Algorithm 2: Local Voting Strategy

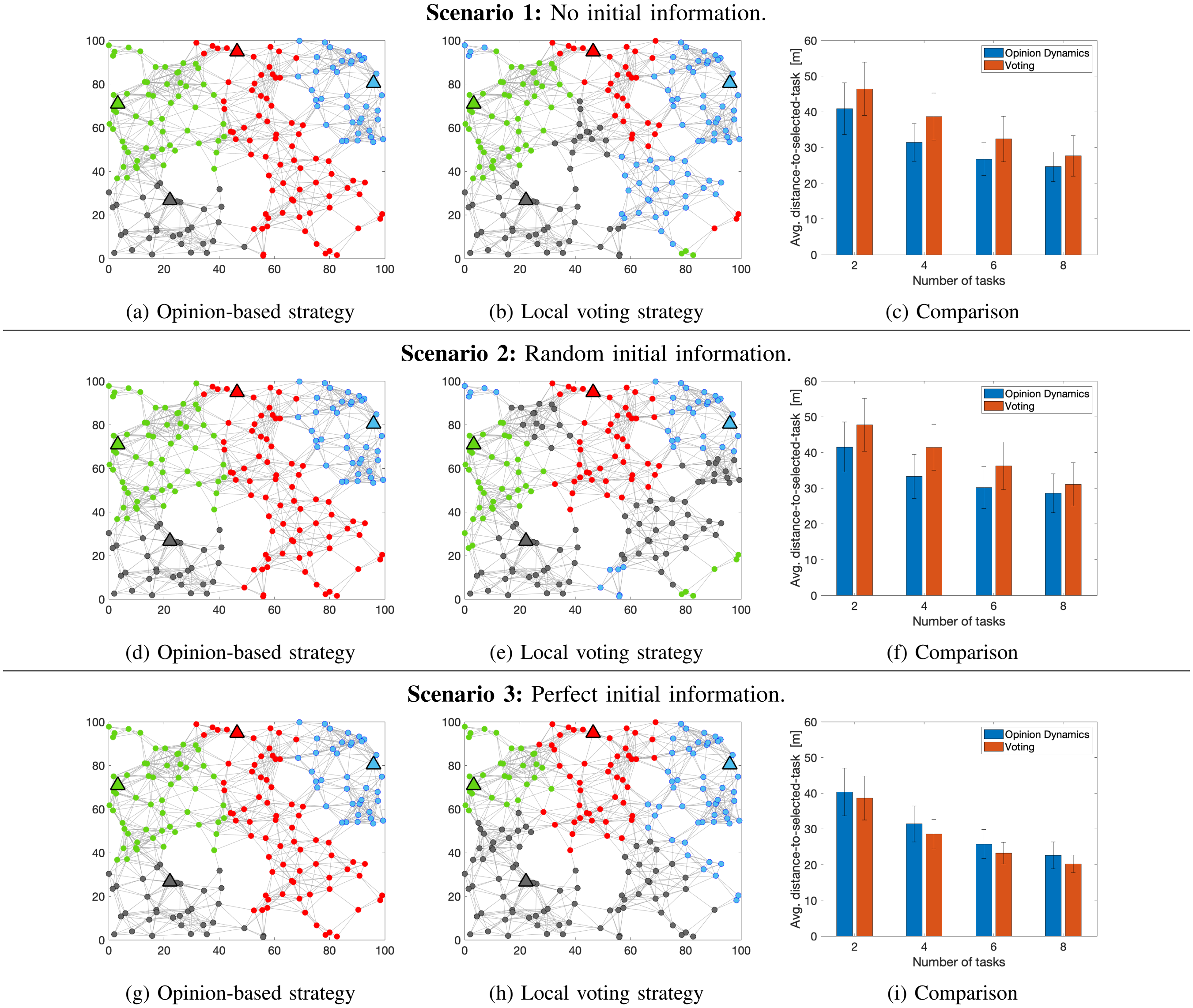
Input: $\mathbf{a}_j \ \forall j \in \mathcal{N}_i$
Output: $\mathbf{a}_i$
$\mathbf{v}_i = \sum_{j \in \mathcal{N}_i \cup i} \mathbf{a}_j = [v_{i1}, \cdots, v_{iN_T}]^\intercal$
if $\|v_i\| > 0$ **then**
  $k \leftarrow \arg\max_l v_{il} \in \mathbf{v}_i$
else
  Select $k \in \mathcal{V}_T$ at random
$\mathbf{a}_i \leftarrow \boldsymbol{e}_k$

A. No Initial Information

For the opinion-based strategy, robots' initial opinions are set to (4). In the local voting strategy, each robot i's initial task selection is set to $\mathbf{a}_i = \mathbf{0}$. The opinion-based strategy forms clusters including the selected tasks (Fig. 3a), while the local voting strategy results in clusters not including their selected tasks (Fig. 3b). A comparison of both strategies (Fig. 3c) shows the opinion-based strategy consistently achieves a lower average distance-to-selected-task, with improvement of 12.96%, 20.44%, 18.59%, and 10.73% for the 2-task, 4-task, 6-task, and 8-task cases, respectively.

B. Random Initial Information

For the local voting strategy, each robot i selects a task k at random with $\mathbf{a}_i = \boldsymbol{e}_k$. Due to random initial selection, these robots behave similarly to those with no initial information, forming disconnected clusters from their tasks (Fig. 3e). Using the opinion-based strategy, robots develop random opinions about tasks initially. As interactions between neighbors continue, robots perceive "strong" opinions originally from tasks and converge toward a stable equilibrium $\boldsymbol{e}_k$ where k is the index of a connected task. Therefore, these



(a) Opinion-based strategy (b) Local voting strategy (c) Comparison

(d) Opinion-based strategy (e) Local voting strategy (f) Comparison

(g) Opinion-based strategy (h) Local voting strategy (i) Comparison

Fig. 3: Simulation results for three initialization scenarios (no information, random information, and perfect information). First two columns show task assignments after convergence using the same robot positions, task number, and task positions, but different initializations. The colors of the 200 robots (dots) indicate task (triangles) assignments. The last column shows the comparison between opinion-based and local voting strategy on the average distance-to-selected-task over 500 runs.

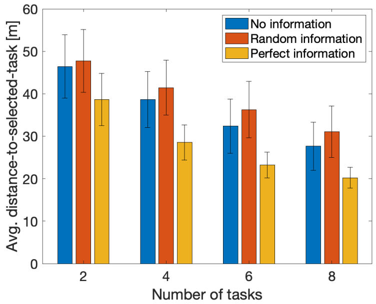
robots form connected clusters with their assigned tasks as shown in Fig. 3d. Comparing the opinion-based strategy with the local voting regarding average distance-to-selected-task, the results (Fig. 3f) show the opinion-based strategy has significantly lower travel distances than the local voting strategy (13.99%, 21.93%,18.29% and 8.25% differences for the 2-task, 4-task, 6-task and 8-task cases, respectively).

C. Perfect Initial Information

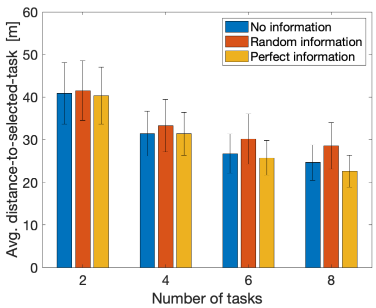
With perfect information, robots are aware of all task locations and use this for initialization. For the opinion-based strategy, opinions are initialized as (6). In the local voting strategy, each robot i sets its initial task selection $\mathbf{a}_i = \boldsymbol{e}_k$, where task k is the closest in terms of travel distance. This resembles k-means clustering with task locations as means.

During the voting process, robots within their clusters maintain their initial task selections. Robots at the cluster boundaries update task selections based on neighbors' choices, similar to the k-nearest neighbor algorithm. For the opinion-based strategy, a robot's task selection is influenced not only by the number of neighbors choosing a task but also by their opinions' strength toward that task. These opinions propagate through the network from task vertices to robots, depending on both distances and network connectivities.

In comparison, the opinion-based strategy, as depicted in Fig. 3g, tends to assign more robots to tasks with denser connectivities, such as the green task. Consequently, the local voting strategy (Fig. 3h), generally results in shorter travel



(a) Local voting with different types of initial information



(b) Opinion-based strategy with different types of initial information

Fig. 4: Intra-strategy comparisons. a) Local voting strategy's performance varies with initializations. b) Opinion-based strategy is less sensitive to initializations.

distances for robots to assigned tasks as shown in Fig. 3i. The distance reductions for the 2-task, 4-task, 6-task, and 8-task cases are 3.95%, 8.68%, 9.20%, and 9.73%, respectively.

D. Intra-Strategy Comparisons in Different Initializations

Local voting strategy depends heavily on the robots' initial information. For the same seed (robots and tasks in the same position) across different types of initial information, the outcomes of the local voting strategy were vastly different (Fig. 4a). The strategy in the perfect information scenario provides significantly better results than the other two scenarios. On the other hand, initial information from robots has less effect on opinion-based strategy (Fig. 4b). It has more comparable results from the three different initialization scenarios.

VII. Conclusion and Future Work

This paper introduces a scalable task allocation algorithm for robotic swarms, leveraging opinion dynamics for computational efficiency and stability, and avoiding complex combinatorial optimization. Using an interaction control with opinion dynamics, robots select tasks and form coalitions. In the cases of no initial information available and random initial information, our algorithm outperforms local voting strategies in terms of average distance-to-selected-task. Moreover, our algorithm is not only computationally efficient and scalable but also with stability guarantee, making it a promising solution for large-scale multi-robot task allocation. Future research will investigate the algorithm's performance in noisy environments and the integration of heterogeneous robots. Additionally, we aim to assess the robustness of the opinion-based approach against adversarial attacks.

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