

# Heading Control for Obstacle Avoidance using Dynamic Posture Manipulation during Tumbling Locomotion

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**Abstract**—Passive tumbling structures are energy efficient, but often sacrifice control authority due to their under actuated nature. Unlike many passive tumbling robots, Northeastern University’s COBRA is a snake robot with eleven articulated joints that transforms into a wheel-like structure with a high degree of posture control during tumbling, and using this posture manipulation, COBRA can control its forward velocity and heading angle while tumbling. This paper presents a mathematical framework that describes the dynamics of posture manipulation during tumbling and identifies two types of control actions that allow it to control its movement. This is validated in hardware testing to demonstrate obstacle avoidance during passive tumbling using only posture manipulation.

## I. INTRODUCTION

Navigating challenging terrain poses a hurdle for mobile robots. Legged robots that dynamically interact with their surroundings to adjust their center of mass show great promise [1]–[3] in this regard. Unlike traditional wheeled systems with fixed points of contact, legged robots can select their contact locations to navigate diverse environments. However, steep inclines present unique obstacles that demand further investigation [4].

Terrestrial and extraterrestrial rugged landscapes have scientific significance [5], [6], however traversing them is a challenge. The lunar terrain, for example, is marked by craters stretching over tens of kilometers in diameter, with slopes as steep as tens of degrees covered in porous regolith (lunar topsoil) that makes locomotion challenging [7].

In 2022, NASA’s Innovative Advanced Concepts (NIAC) Program invited academic institutions in the United States to participate in the Breakthrough, Innovative, and Game-changing (BIG) Idea competition, seeking novel mobility systems capable of traversing lunar craters [8], [9]. Northeastern University secured NASA’s prestigious Artemis Award with their proposal of COBRA (Crater Observing Bio-inspired Rolling Articulater) [10], an articulated robot tumbler showcased in Fig. 1. Open loop tumbling locomotion was successfully demonstrated on the steep slopes of hills near Pasadena, where the competition was held. This accomplishment led to the research question at hand, centered on active posture manipulation and heading angle steering for closed-loop control.

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Fig. 1. Illustrates a frontal view of COBRA [11]–[13] engaged in tumbling while navigating around an obstacle using posture manipulation.

This study develops a mathematical framework for tumbling locomotion and explores control methodologies for tracking. We first present rigorous mathematical models of tumbling, followed by experimental validation. Finally, we employ an optimization framework to derive solutions for heading angle control and demonstrate path tracking for obstacle avoidance in experiments.

Tumbling robots have gained attention in the robotics community for their energy-efficient design and adaptability in challenging environments. Passive systems like NASA/JPL’s Mars Tumbleweed Rover [14] are valued for their minimal energy consumption, making them ideal for remote exploration, while active rolling robots such as MIT’s Kickbot [15] offer robust, omnidirectional mobility, benefiting from a low center of gravity and the ability to traverse uneven terrain. The spherical nature of these robots also allows for excellent maneuverability in confined spaces.

Despite these advantages, tumbling locomotion presents challenges, particularly in control design. Passive rolling systems, while efficient, often sacrifice controllability, relying on morphology or posture to maneuver. Additionally, the use of the entire body for movement complicates sensor placement and localization.

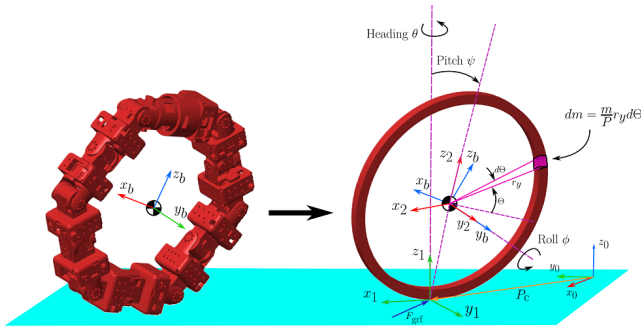


Fig. 2. Illustrates model parameters, coordinate frames, etc., used to derive Eqs. 1, 7, 22, and 23.

Early designs like Rollo [16] and SMR [17] used a spherical shell with a spring-mass system for motion, but faced issues with maintaining contact between the driving wheel and sphere. Later systems, such as Sphericle [18] and Festo's SRR [19], improved upon these designs with car-driven mechanisms, though they remained unstable on rough terrain.

Weight-shifting mechanisms, like those in Spherobot [20] and August Robot [21], offer better control but require additional weights, reducing energy efficiency. More recent approaches using deformable structures, such as Ritsumeikan's Deformable Robot [22] and Ourobot [23], provide a more efficient way to control the center of mass.

Northeastern University's COBRA builds on this concept, featuring a morphing body for multi-modal locomotion, including tumbling, slithering, and sidewinding. COBRA's unique head-tail locking mechanism enhances its ability to tumble dynamically across rough surfaces, while its active posture control allows for two-dimensional steering. This paper focuses on COBRA's tumbling locomotion and presents an overview of the head-tail locking mechanism.

### Brief Overview of COBRA Hardware

As shown in Fig. 1, COBRA has eleven actuated joints. The head module houses the computing system, radio antenna for lunar communication, and an IMU for navigation, while the tail contains an interchangeable payload. The ten identical joint modules between them each include a joint actuator and battery.

A latch mechanism in the head connects with a passive element in the tail, forming a closed-loop tumbling structure. COBRA can adjust its posture within this loop using its actuated joints to control tumbling motion, which is tested here for heading regulation during tumbling.

## II. REDUCED-ORDER MODEL (ROM) DERIVATIONS

In the tumbling configuration, COBRA forms an elliptical ring, as shown in Fig. 2. Our analysis is based on the following assumptions: (1) The ring has a negligible cross-sectional area (Fig. 2); (2) The mass is uniformly distributed; (3) The shape is defined by the principal axes,  $u_1$  and  $u_2$ , controlled by the robot's joints (Fig. 2); and (4) The postures

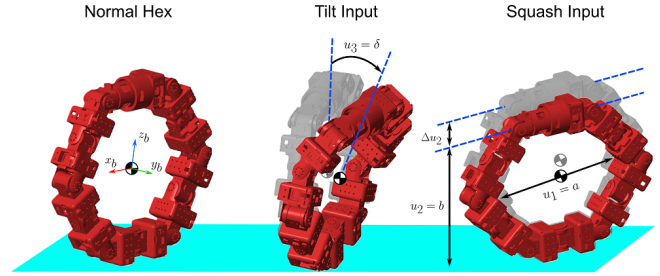


Fig. 3. Illustrates posture manipulation by considering two imaginary actuators, denoted as  $u_1$  and  $u_2$ , which act along the principal axes of the ring to induce planar deformations and one imaginary actuator denoted  $u_3$  to induce off-plane deformation.

are symmetric, ensuring the ellipse's center aligns with the center of mass (CoM).

Using these assumptions, we derive the ring's inertia tensor,  $\mathcal{I}(u_i)$ , where  $u_i$  adjusts the ring's shape. This tensor is then used to derive the nonholonomic equations of motion for the rolling ring, capturing how control actions affect its posture.

### A. Cascade Model

In our approach to mathematically formalizing tumbling servoing through shape manipulations in the ring, we introduce a cascade nonlinear model structured as follows:

$$\begin{aligned} \Sigma_{tbl} : \dot{x} &= f(x, y) \\ \Sigma_{pos} : \begin{cases} \dot{\xi} &= f_{\xi}(\xi, u) \\ y &= h_{\xi}(\xi) \end{cases} \end{aligned} \quad (1)$$

Here,  $x$  and  $y$  represent the state vector and output function, which include the ring's orientation (via Euler angles), CoM position, and the mass moment of inertia about its body axes ( $x$ - $y$ - $z$ ). The function  $f(\cdot)$  governs the state dynamics.

The terms  $f_{\xi}$  and  $h_{\xi}$  represent the dynamics of the internal states  $\xi$  (see Section II). The control input  $u$  applies actuation along the ring's principal axes, as shown in Figs. 3 and 2. The cascade model separates the dynamics into tumbling ( $\Sigma_{tbl}$ ) and posture manipulation ( $\Sigma_{pos}$ ), where tumbling is controlled by internal posture adjustments to maintain the robot's shape in the inertial frame. The following subsections derive the equations for these models.

### B. Governing Dynamics for Posture Manipulation $\Sigma_{pos}$

As shown in Fig. 2, the control actions  $u = [u_1, u_2]^T$  adjust the principal axes for the ring. This section derives the governing equations for posture dynamics.

While no closed-form equation exists for the perimeter of an ellipse, several approximations exist that are sufficiently accurate. One such approximation is employed in this work to maintain the perimeter at a fixed value as the length of the axes changes. To calculate the inertia tensor of the line mass, we use a simpler integral-based method, computable through numerical integration.

Consider the general equation of the center line in the ring depicted in Fig. 3 in the  $x$ - $z$  plane of the body frame with

principal axes of length  $a$  and  $b$ ,

$$\frac{p_{i,x}^2}{a^2} + \frac{p_{i,z}^2}{b^2} = 1 \quad (2)$$

where  $p_i = [p_{i,x}, 0, p_{i,z}]^\top$  denotes the body-frame coordinates of a point on the ring. Consider the following change of variables:

$$\begin{aligned} \xi_1 &= r_y C_\theta \\ \xi_2 &= r_y S_\theta \end{aligned} \quad (3)$$

where  $r_y$  and  $\theta$  are polar coordinates and are shown in Fig. 3.  $S_\theta$  and  $C_\theta$  denote  $\sin \theta$  and  $\cos \theta$ . We take the time-derivative of the equation above and Eq. 2, which yields

$$\begin{aligned} \dot{\xi}_1 &= \dot{r}_y C_\theta - \xi_2 \dot{\theta} \\ \dot{\xi}_2 &= \dot{r}_y S_\theta + \xi_1 \dot{\theta} \\ \frac{\xi_1 \dot{\xi}_1}{a^2} - \frac{2\xi_1^2 u_1}{a^3} + \frac{\xi_2 \dot{\xi}_2}{b^2} - \frac{2\xi_2^2 u_2}{b^3} &= 0 \end{aligned} \quad (4)$$

where  $\dot{a} = u_1$  and  $\dot{b} = u_2$ . The perimeter of the ring is fixed and given by the following equation

$$P = \int_0^{2\pi} \sqrt{a^2 C_\theta^2 + b^2 S_\theta^2} d\theta \quad (5)$$

therefore, we can write the following relationship between  $\dot{P}$  and  $\dot{\theta}$

$$\dot{P} = \left( \sqrt{a^2 C_\theta^2 + b^2 S_\theta^2} \right) \dot{\theta} = 0 \quad (6)$$

This equation constitutes the remaining ordinary differential equations necessary to establish the state-space model for the posture dynamics. By defining  $\xi_3 = r_y$ ,  $\xi_4 = \theta$ ,  $\xi_5 = a$ , and  $\xi_6 = b$ , and considering Eqs. 2, 3, 4, 6, the state-space model governing the state vector  $\xi = [\xi_1, \dots, \xi_6]^\top$  is given by

$$\begin{bmatrix} 1 & 0 & -C_{\xi_4} & \xi_2 & 0 & 0 \\ 1 & 0 & -S_{\xi_4} & \xi_1 & 0 & 0 \\ \xi_1 & \xi_2 & 0 & 0 & 0 & 0 \\ \xi_5^2 & \xi_6^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma(\xi) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \\ \dot{\xi}_5 \\ \dot{\xi}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{2\xi_1^2}{\xi_5^3} & \frac{2\xi_2^2}{\xi_6^3} \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (7)$$

where  $\gamma(\xi) = \sqrt{\xi_5^2 C_{\xi_4}^2 + \xi_6^2 S_{\xi_4}^2}$ . The matrix in the left-hand side of Eq. 7 is invertible, and therefore, the normal form  $\dot{\xi} = f_\xi(\xi, u)$  can be obtained, which is skipped here. Now, it is possible to show that the mass moments of inertia about the body-frame  $x$ ,  $y$ , and  $z$  axes, denoted by  $\mathcal{I}_{xx}$ ,  $\mathcal{I}_{yy}$ , and  $\mathcal{I}_{zz}$ , are functions of the hidden state vector  $\xi$ .

The mass of the differential element on the ring can be calculated assuming uniform distribution as follows:

$$dm = \frac{m}{P} dP = \frac{m}{P} \gamma(\xi) d\xi_4 \quad (8)$$

where  $m$  is the total mass of the elliptical ring. Thus, the mass moment of inertia around each body frame axis can be obtained by:

$$\begin{aligned} \mathcal{I}_{kk} &= \frac{m}{P} \int_{\xi_4} r_k^2 \gamma(\xi) d\xi_4, \quad k \in \{x, y, z\} \\ r_x &= \xi_3 C_{\xi_4} \\ r_y &= \xi_3 \\ r_z &= \xi_3 S_{\xi_4} \end{aligned} \quad (9)$$

In the equation above, the output function  $y = h_\xi(\xi) = [\mathcal{I}_{xx}, \mathcal{I}_{yy}, \mathcal{I}_{zz}]^\top$  encapsulates the mass moments of inertia. Next, we will derive the equations of motion for the tumbling ring using these posture dynamics as follows.

### C. Governing Dynamics for Tumbling $\Sigma_{tbl}$

Consider the ring in Fig. 2 equipped with virtual actuators  $u_i$  along its principal axes for posture control. We define the following frames of reference: (1) the world frame  $x_0-y_0-z_0$ ; (2) the contact frame  $x_1-y_1-z_1$  at the contact point  $p_c$ , with the  $z$ -axis perpendicular to the ground; (3) the gimbal frame  $x_2-y_2-z_2$  at the CoM  $p_{cm}$ , inert to the body's motion; and (4) the body frame  $x_b-y_b-z_b$  at the CoM, rotating with the ring.

An inclination is introduced between the contact and world frames, forming an inclined plane at an angle  $\alpha$ . The ring's orientation  $R_b^0$ , described in roll, pitch, and yaw angles  $\theta$ ,  $\psi$ , and  $\phi$ , is parameterized as follows:

$$R_b^0 = R_z(\theta) R_y(\phi) R_x(\psi) \quad (10)$$

The angular velocity vector in the body frame  $\omega_b = [\omega_{b,x}, \omega_{b,y}, \omega_{b,z}]^\top$ , in terms of  $\dot{\theta}$ ,  $\dot{\psi}$ , and  $\dot{\phi}$ , is expressed as:

$$\begin{aligned} \omega_{b,x} &= \dot{\psi} \sin(\theta) \sin(\phi) + \dot{\theta} \cos(\phi) \\ \omega_{b,y} &= \dot{\psi} \sin(\theta) \cos(\phi) - \dot{\theta} \sin(\phi) \\ \omega_{b,z} &= \dot{\psi} \cos(\theta) + \dot{\phi} \end{aligned} \quad (11)$$

From the  $\Sigma_{pos}$  model, the ring's principal moments of inertia are denoted as  $y_1$ ,  $y_2$ , and  $y_3$ . The angular momentum of the ring about  $p_{cm}$  is represented by

$$\mathbf{H}_b = \begin{bmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{bmatrix} \omega_b \quad (12)$$

We define the radius of rotation as the vector extending from  $p_{cm}$  to  $p_c$ .

Since the ring is in pure rolling at the contact point  $p_c$ , three constraints must be considered, including one holonomic constraint ( $v_{c,z} = 0$ ) and two nonholonomic constraints ( $v_{c,x} = 0$ ) and ( $v_{c,y} = 0$ ), where  $v_c = [v_{c,x}, v_{c,y}, v_{c,z}]^\top$  denotes the contact velocity.

We formulate the equations of motion by resolving the linear and angular momentum balances concerning the ring's

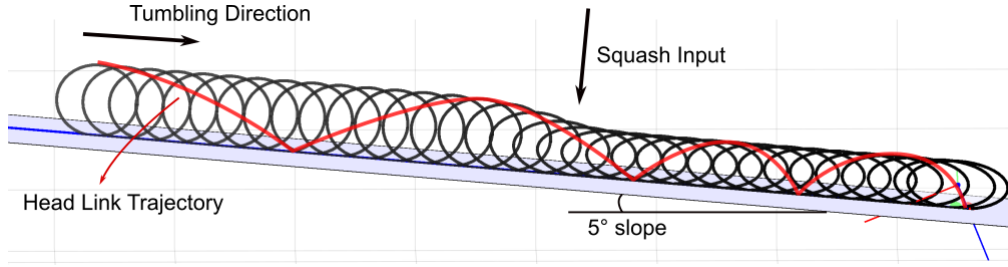


Fig. 4. Shows the overlaid snapshots of the tumbling ring squashed along its body axes.

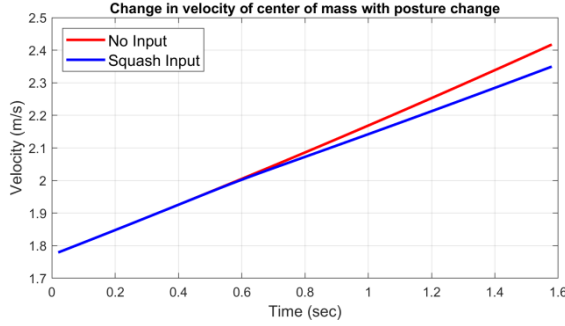


Fig. 5. Depicts the change in the tumbling velocity due to the posture manipulation in the cascade model given by Eq. 1.

CoM. The resulting equations, derived from applying the balance laws alongside the non-integrable constraints, constitute a set of differential equations describing the ring's orientation and the lateral translation of its CoM over time.

This system of equations is expressed in first-order form

$$\dot{x} = f(x, y) = M^{-1}(x, y)N(x, y) \quad (13)$$

for numerical integration in MATLAB where the nonlinear terms  $M(\cdot)$  and  $N(\cdot)$  are given in the Appendix Section. The state vector is represented as  $x = [\theta, \psi, \phi, \dot{\theta}, \dot{\psi}, \dot{\phi}, p_{c,x}, p_{c,y}]^T$ . Subsequently, we propose a controller based on the collocation method to utilize  $u$  for steering the heading angle of the tumbling ring.

### III. CONTROLS

To tackle this control problem, involving the regulation of the ring's heading angle within the tumbling dynamics  $\Sigma_{tbl}$  using the ring's posture dynamics  $\Sigma_{pos}$ , we introduce the following cost function:

$$J = \|\theta - \theta_d\|_2 \quad (14)$$

where  $\theta_d$  denotes the desired heading angle. This cost function  $J$  is evaluated within the framework provided by the cascade model specified in Eq. 1 to find open-loop trajectories.

We discretize Eq. 1 temporally, dividing it into  $n$  discrete time intervals ( $t_i, i = 1, \dots, n, 0 \leq t_i \leq t_f$ ) to yield the following system of equations:

$$\dot{x}_i = f_i(x_i, y_i), \quad i = 1, \dots, n, \quad 0 \leq t_i \leq t_f \quad (15)$$

Here,  $x_i$  encapsulates the values of the state vector  $x$  at the  $i$ -th discrete time, while  $y_i$  represents the principal mass moment of inertia of the ring at the  $i$ -th discrete time.  $f_i$  denotes the governing dynamics of tumbling at the  $i$ -th discrete time.

We organize all discrete values from  $x_i$  and  $y_i$  into vectors  $X = [x_1^T(t_1), \dots, x_n^T(t_n)]^T$  and  $Y = [y_1^T(t_1), \dots, y_n^T(t_n)]^T$ , respectively.

To ensure continuity, we impose  $2n$  boundary conditions at the boundaries of  $n$  discrete periods, given by:

$$r_i(x(0), x(t_f), t_f) = 0, \quad i = 1, \dots, 2n \quad (16)$$

Considering the three entries in  $y$ , we establish three inequality constraints to maintain the principal mass moment of inertia within bounds. These constraints, denoted as  $g_i$ , are defined as follows:

$$g_i(x(t_i), y(t_i), t_i) \geq 0, \quad i = 1, 2, 3, \quad 0 \leq t_i \leq t_f \quad (17)$$

To approximate the nonlinear dynamics of the tumbling ring, we employ a method based on polynomial interpolations, greatly simplifying computational efforts. Let's consider the  $n$  time intervals, defined as follows:

$$0 = t_1 < t_2 < \dots < t_n = t_f \quad (18)$$

We stack the states  $x_i$  and the principal mass moment of inertia terms  $y_i$  from the ring at these discrete times into a single vector denoted as  $\mathcal{Y}$ . Additionally, we append the final discrete time  $t_f$  as the last entry of  $\mathcal{Y}$ , allowing the optimizer to determine the tumbling speed.

$$\mathcal{Y} = [x_1^T, \dots, x_n^T, y_1^T, \dots, y_n^T, t_f]^T \quad (19)$$

We approximate the output function  $y_i(t_i)$  at time  $t_i \leq t < t_{i+1}$  using the linear interpolation function  $\tilde{y}$  between  $y_i(t_i)$  and  $y_{i+1}(t_{i+1})$ , given by

$$\tilde{y} = y_i(t_i) + \frac{t - t_i}{t_{i+1} - t_i} (y_{i+1}(t_{i+1}) - y_i(t_i)) \quad (20)$$

Similarly, we interpolate the states  $x_i(t_i)$  and  $x_{i+1}(t_{i+1})$  using a nonlinear cubic interpolation, ensuring continuity. This interpolation function, denoted as  $\tilde{x}$ , is determined by

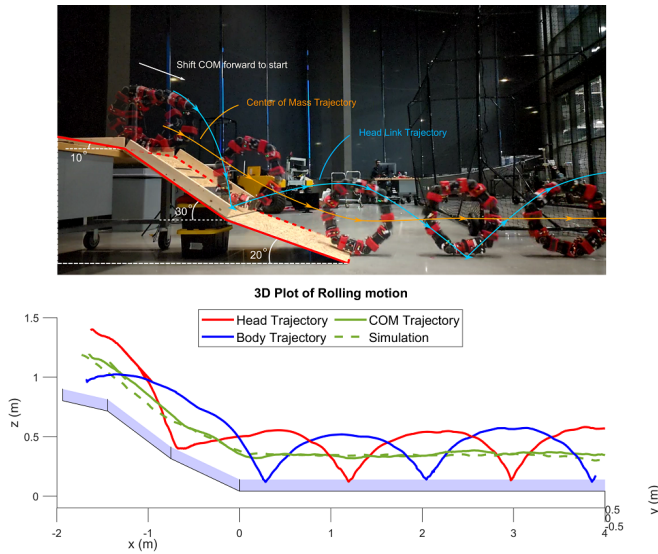


Fig. 6. Top figure Shows the experimental setup. Bottom figure depicts high fidelity model predictions of the tumbling system behavior.

the following system of equations:

$$\begin{aligned} \tilde{x}(t) &= \sum_{k=0}^3 c_k^j \left( \frac{t-t_j}{h_j} \right)^k, \quad t_j \leq t < t_{j+1}, \\ c_0^j &= x(t_j), \\ c_1^j &= h_j f_j, \\ c_2^j &= -3x(t_j) - 2h_j f_j + 3x(t_{j+1}) - h_j f_{j+1}, \\ c_3^j &= 2x(t_j) + h_j f_j - 2x(t_{j+1}) + h_j f_{j+1}, \\ \text{where } f_j &:= f(x(t_j), y(t_j)), \quad h_j := t_{j+1} - t_j. \end{aligned} \quad (21)$$

This cubic interpolation  $\tilde{x}$  for  $x$  ensures continuity at discrete points and at the midpoint of sample times. Thus, the only remaining constraints in the nonlinear programming problem are the collocation constraints at the midpoint of  $t_i - t_{i+1}$  time intervals, the inequality constraints at  $t_i$ , and the constraints at  $t_1$  and  $t_f$ , all of which are handled within the optimization process. We address this optimization problem using MATLAB's `fmincon` function.

#### IV. RESULTS

The dynamics framework was implemented in MATLAB for planar posture manipulation. Figure 4 shows simulation snapshots where an initial angular velocity of 5 rad/s is applied, followed by a reduction of the ellipse's axis from 0.6 m to 0.5 m at 0.5 seconds, adjusting  $u_1$  to maintain a 2 m perimeter. The corresponding velocity change due to inertia variation is plotted in Fig. 5. Hardware implementation of this actuation mode, requiring a rotation-invariant shape, necessitates closed-loop control and precise contact location estimation.

Steering control via the tilt input  $u_3$  was tested in both high-fidelity Simulink simulations and hardware experiments. Figure 6 shows the experimental setup, where the robot is released down a ramp and tracked with motion

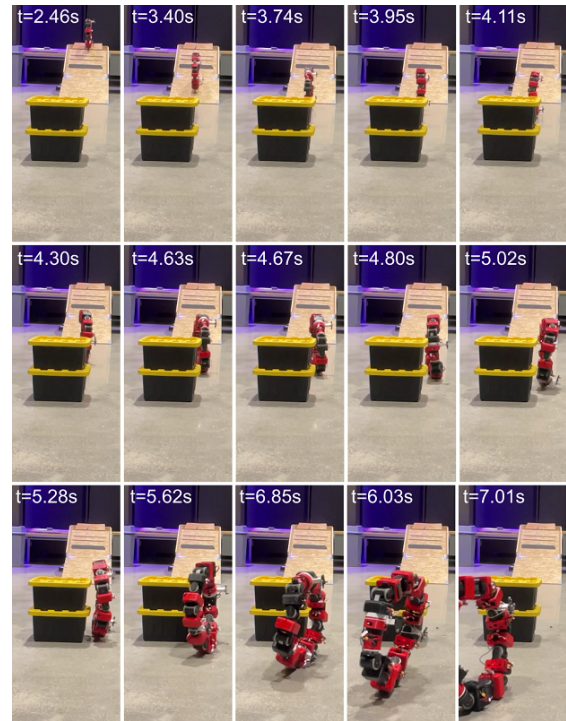


Fig. 7. Snapshots demonstrating obstacle (box) avoidance using posture manipulation in COBRA.

capture cameras. The resulting trajectory is compared with the simulation (Fig. 6).

To test steering control, we place an obstacle in the path of the robot and use posture manipulation with a fixed tilt input of  $20^\circ$  to change the heading and avoid the obstacle. Figure 7 shows snapshots of the robot avoiding the obstacle. Figure 8 shows the trajectory, with the robot successfully avoiding the obstacle and returning to its path after successive tilt actuation to  $-20^\circ$  and back to  $+20^\circ$ .

#### V. CONCLUDING REMARKS

This study presents a mathematical framework for controlling COBRA's tumbling velocity and heading. Using a cascade model in a collocation framework, we determine how to compress or tilt the robot to achieve desired heading angles. Our experiments show that compression modulates speed, allowing acceleration or deceleration, while off-plane tilts enable active steering. These results are validated with successful open-loop demonstrations. Future work will refine this framework with closed-loop control for posture manipulation, incorporating position and orientation estimation.

#### APPENDIX

The nonlinear term  $M$  is given by

$$M(x, y) = \left[ \begin{array}{ccc|c} y_1 S_\theta & 0 & 0 & \mathbf{0} \\ 0 & y_2 + mr^2 & 0 & \\ (y_3 + mr^2) C_\theta & 0 & y_3 + mr^2 & \\ \hline & \mathbf{0} & & \mathbf{I} \end{array} \right] \quad (22)$$

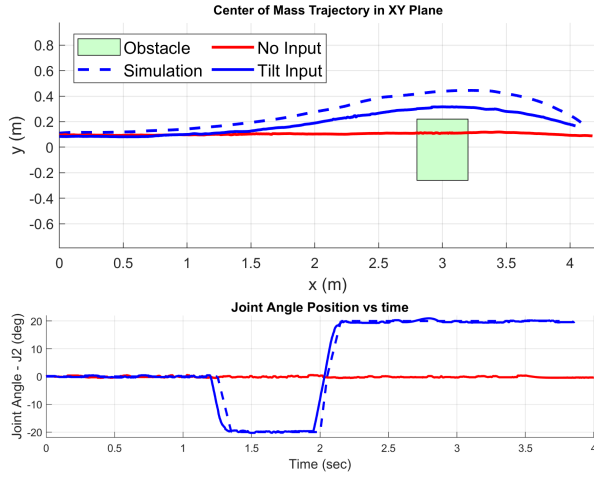


Fig. 8. Top figure shows the top view of the center of mass trajectory of the tumbling COBRA and obstacle avoidance. Bottom figure depicts the desired joint trajectory applied to the robot from the obstacle avoidance experiment.

where  $\mathbf{0}$  and  $\mathbf{I}$  denote zero and identity block matrices, respectively.  $r$  signifies the distance to the contact point. The nonlinear vector  $N$  is defined as follows:

$$N(x, y) = \begin{bmatrix} (y_3 - 2y_1) \dot{\psi} \dot{\theta} C_\theta + y_1 \dot{\theta} \dot{\phi} \\ \beta_1 \\ (y_3 + 2mr^2) \dot{\psi} \dot{\theta} S_\theta \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad (23)$$

where  $\beta_i$  are given by:

$$\beta_1 = (y_1 - y_3 - mr^2) \dot{\psi}^2 S_\theta C_\theta - (y_3 + mr^2) \dot{\psi} \dot{\phi} S_\theta - mgr C_\theta \quad [22]$$

$$\beta_2 = -r \dot{\psi} C_\psi C_\theta + r \dot{\theta} S_\psi S_\theta - r \dot{\phi} C_\psi$$

$$\beta_3 = -r \dot{\psi} S_\psi C_\theta - r \dot{\theta} C_\psi S_\theta - r \dot{\phi} S_\psi$$

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