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Differential-Algebraic Equation Control Barrier Function for Flexible Link Manipulator

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Abstract— **This paper presents a control barrier function (CBF) for systems described by differential-algebraic equations and applies the method to guarantee the safety of a two-link flexible-link manipulator. The two main contributions of the paper are: a) an extension of CBFs to systems governed by differential-algebraic equations; b) a framework for simulation of flexible-link robots in a floating frame of reference formulation (FFRF) finite element method (FEM). Numerical simulations demonstrate the minimally invasive safety control of a flexible two-link manipulator with position constraints through CBF quadratic programming without converting the differential-algebraic equations to a control-affine system.**

I. INTRODUCTION

The dynamics of flexible-link manipulators can be approximated by differential algebraic equations that describe joint motion and link deformation. The deflection of several robotic systems can be approximated by simple flexible-link robots, e.g., [1] concludes that the most important deflections of the manipulator used for remote maintenance of the fusion power plant DEMO occur in only two links; similarly, [2] concludes that only two links have a significant deflection of standard collaborative robots. For implementing a safety-critical system, the control barrier function [3] is a well-known technique that can be used for ensuring collision avoidance by forward invariance. Differential-algebraic systems also called descriptor systems include an algebraic constraint equation. Because the descriptor matrix of the system does not have full rank, the descriptor system cannot be converted to a control-affine system.

In practice, an iterative equilibrium model can control a flexible robot. In [4], [5], the authors address theoretical and practical problems associated with controlling a flexible-link system whose dynamic model is based on the finite element method with joint constraint or contact. The first step toward developing a controller is calculating the instantaneous optimal control equation. The discrete-time model can be used as an optimal control problem in combination with algebraic constraints. The mapping function preserves the algebraic condition during control.

Differential-algebraic equations can be converted to ordinary differential equations when a problem-specific mapping is found. The minimal coordinate set formulation cancels out the dependent term of the differential-algebraic equation. This can be achieved by finding a projection onto the tangent space of the constraint manifold. This method can be used as a robot controller [6], [7] by a pseudo-inverse operator. Other methods can be used to find null space projectors, such as QR factorization, known as the Maggi-Kane equations [8]. Using a control barrier function in an ordinary differential equation originating from a finite element, the proposed method was shown to be capable of guaranteeing safety [9]. The limitation of the control barrier function for a finite element is considered a monolithic system.

This work extends a control barrier in two ways, enabling its application to systems with elasticity and algebraic conditions. Our differential-algebraic equation control barrier function for flexible link robots can be used for guaranteeing collision avoidance over a dynamic model. First, we present an extension of the control barrier function with consistency space. Second, a framework is proposed for simulating flexible-link robots in a floating frame of reference formulation (FFRF) finite element method (FEM).

The paper is organized as follows. Section II introduces the problem statement. Section III introduces prerequisites on dynamics analysis of a floating frame-of-reference formulation (FFRF) finite element method (FEM) and control barrier function (CBF). In Section IV, we present our main results in the form of a definition and implementation of control barrier function. Section V presents the simulation results of a controller used to move a manipulator with two flexible links. Finally, we provide our conclusion in Section VI.

II. PROBLEM STATEMENT

This section presents the problem addressed in this paper. We consider a flexible-link robot as shown in Fig. 1. We describe the model of the system based on a floating frame of reference formulation (FFRF), which can describe joint motion and link deformation separately. It includes axial and lateral deformation that has a large rigid body motion. The FFRF method divides the coordinates into rigid body motion of the reference frame and flexible material frame. The advantage over absolute coordinate formulation is the structure of the equation that relates to each flexible beam equation. The schematic and coordinate frames of the FFRF model are shown in Fig. 1. The differential-algebraic equation constitutes such an FFRF system because link connections are represented by constraint algebraic equations.

Current control barrier methods address only ordinary differential equations; hence, the safety of flexible-link robots cannot be guaranteed using current methods. Therefore, we propose control barrier functions for systems governed by differential-algebraic equations.

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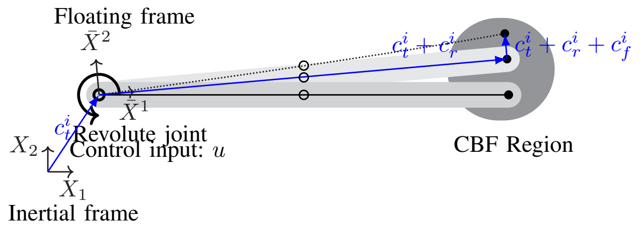

Fig. 1. Flexible system modeling with large joint displacement and small deformation. Rotation and deformation have a map from inertial frame X to floating frame $\bar{X}$. The displacement with respect to X is a sum of nodal displacement of the floating frame of reference c_t^i, rotational nodal displacement c_r^i, and flexible nodal displacement c_f^i. A CBF regulates the displacement of the tip.

III. Preliminaries

This section presents preliminaries of differential-algebraic equations in Section III-A and control barrier function in Section III-B. A modeling framework for systems with large rigid motion and small elastic deformation is presented in Section III-C. Finally, the numerical scheme to address this problem is described in Section III-D.

A. Differential-Algebraic Equation

We consider quasi-linear differential algebraic equations (DAE) as:

$$E(x)\dot{x} = f(x) + g(x)u. \tag{1}$$

where $x \in \mathfrak{C}_{E,f,g} \subseteq \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $E : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $f : \mathbb{R}^n \to \mathbb{R}^n$, and $g : \mathbb{R}^n \to \mathbb{R}^m$. The set $\mathfrak{C}_{E,f,g}$ is called the consistency space and is defined as follows.

Definition 3.1 (Consistency Space [10]): The consistency space of (1), denoted by $\mathfrak{C}_{E,f,g}$, is the space that satisfies the algebraic condition and the differential equation simultaneously:

$$\mathfrak{C}_{E,f,g} := \left\{ x_0 \in \mathbb{R}^n \;\middle|\; \begin{array}{c} \exists\,\text{solution}\, x : [0,T) \to \mathbb{R}^n \\ \text{such that}\, E(x)\dot{x} = f(x) + g(x)u \\ \text{with}\, x(0) = x_0 \end{array} \right\} \tag{2}$$

B. Control Barrier Function

In [3], [11], the authors proposed a Lyapunov-like method for enforcing safety:

Definition 3.2 (Safety [3]): A set $\mathcal{C}$ is considered to be safe if it is forward invariant.
This set $\mathcal{C}$ is defined as the zero super level set of a continuously differentiable function $B : \mathbb{R}^n \to \mathbb{R}$, i.e.

$$\begin{aligned} \mathcal{C} &= \{\; x \in \mathbb{R}^n \;|\; B(x) \geq 0 \;\}, \\ \partial\mathcal{C} &= \{\; x \in \mathbb{R}^n \;|\; B(x) = 0 \;\}. \end{aligned} \tag{3}$$

The safety of a system with respect to set $\mathcal{C}$ can be ensured if a control barrier function exists.

Definition 3.3 (Control Barrier Function [12]): Consider a dynamic system with state vector $x \in \mathbb{R}^n$ and control input $u \in \mathbb{R}^m$:

$$\dot{x} = \bar{f}(x) + \bar{g}(x)u. \tag{4}$$

Let $B : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that $\nabla B(x) \neq 0$ for all $x \in \partial B$. Then, $B(x)$ is a control barrier function if:

$$L_{\bar{f}}B(x) + L_{\bar{g}}B(x)u \geq 0, \forall x \in \mathbb{R}^n \tag{5}$$

with L denoting the Lie derivative.

Remark 3.1: Differential algebraic equation (DAE) of (1) can be converted into (4), if E^{-1} exists to be $\bar{f} = E^{-1}f$, $\bar{g} = E^{-1}g$. If we use pseudo-inverse E^+ instead of E^{-1}, the control law would require additional constraint to abide states $x \in \mathfrak{C}_{E,f,g}$, the tangent space of $z \in T_x\mathfrak{C}_{E,f,g}$.

C. Flexible System Modeling

The architecture of the FFRF FEM is shown in Fig. 1. The derivation in this section is a summary of [13] and [14]. The displacement of the beam is approximated in the i-th position. The global position of discretized i-th nodes is denote $c^i \in \mathbb{R}^2$. The global position is the sum of translational nodal displacement of the floating frame of reference c_t^i, rotational nodal displacement c_r^i, and flexible nodal displacement c_f^i. $\bar{c}_r^i$ and $\bar{c}_f^i$ are rotated by rotation matrix A, parameterized by rotation parameter θ_t^i, from c_t^i. The nodal displacement given in the inertial frame is:

$$\begin{aligned} c^i &= c_t^i + c_r^i + c_f^i, \\ c_r^i + c_f^i &= A(\theta_t^i)(\bar{c}_r^i + \bar{c}_f^i). \end{aligned} \tag{6}$$

Reference point displacement $\bar{c}_f^i$ mapping S to an isoparametric element has $\bar{q}$ where l is the length of the element and $\zeta = \bar{x}_1/l$ where $\bar{x}_1$ is the position at floating frame [14]:

$$\begin{aligned} \bar{c}_f^i &= S\bar{q}, \\ S &= \begin{bmatrix} 1-\zeta & 0 \\ 0 & 1-3\zeta^2+2\zeta^3 \\ 0 & l\left(\zeta-2\zeta^2+\zeta^3\right) \\ \zeta & 0 \\ 0 & 3\zeta^2-2\zeta^3 \\ 0 & l\left(\zeta^3-\zeta^2\right) \end{bmatrix}^T. \end{aligned} \tag{7}$$

There is a map from global nodal displacement to a generalized coordinate denoted by q:

$$q := [\theta_t^1, \bar{c}_f^1, \ldots, \theta_t^i, \bar{c}_f^i]. \tag{8}$$

The spatially discretized nodal-based FFRF equation of motion is derived via Lagrange's equation with Langrangian L for a general mechanical system:

$$\begin{aligned} \frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) - \frac{dL}{dq} &= 0, \\ L = T - V + W &- \lambda^T\phi_q. \end{aligned} \tag{9}$$

where T is kinetic energy, V is elastic strain energy, W is nodal based work, λ is a Langrange multiplier, and $\phi(q) = 0$ stands for the holonomic constraints equation. The velocity constraint equation is $\dot{\phi}(q) = \dot{q}^T\phi_q = 0$ where ϕ_q is Jacobian with respect to q.

The equation of motion, which incorporates Coriolis force Q_v, elastic energy Q_{el}, and generalized external control input $u \in \mathbb{R}^m$ with actuation direction $Q_e \in \mathbb{R}^{n\times m}$ [13], [15]:

$$\begin{aligned} M\ddot{q} + \phi_q^T\lambda &= Q_v + Q_{el} + Q_e u, \\ M &= \frac{\partial}{\partial \dot{q}^T}\left(\frac{\partial T}{\partial \dot{q}^T}\right), \\ Q_v &= \frac{\partial T}{\partial q^T} - \frac{\partial}{\partial q^T}\left(\frac{dT}{d\dot{q}^T}\right), \\ Q_{el} &= \frac{\partial V}{\partial q^T}, \\ \phi_q\ddot{q} &= -(\phi_q\dot{q})_q\dot{q} - 2\phi_{qt}\dot{q} - \dot{\phi}_{tt}. \end{aligned} \tag{10}$$

Therefore, the equation of motion with velocity $v := \dot{q}$ constraint is:

$$\begin{bmatrix} M & \phi_q^T \\ \phi_q & 0 \end{bmatrix}\begin{bmatrix} \dot{v} \\ \lambda \end{bmatrix} = \begin{bmatrix} Q_v + Q_{el} + Q_e u \\ -(\phi_q\dot{q})_q\dot{q} - 2\phi_{qt}\dot{q} - \dot{\phi}_{tt} \end{bmatrix}. \tag{11}$$

D. *Differential Algebraic Equation of a Flexible System*

We aim to use control barrier functions for bounding the configuration $q(t)$ which is not included in (11). This method incorporates configuration $q(t)$ which augments the system and abides by safety constraints:

$$q(t) = \int_0^t v(\tau)d\tau + q(0). \tag{12}$$

As a result, the control system of concern is:

$$\underbrace{\begin{bmatrix} I & 0 & 0 \\ 0 & M & \phi_q^T \\ 0 & \phi_q & 0 \end{bmatrix}}_{E(x)}\underbrace{\begin{bmatrix} \dot{q} \\ \dot{v} \\ \lambda \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} v \\ Q_v + Q_{el} \\ \gamma \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} 0 \\ Q_e \\ 0 \end{bmatrix}}_{g(x)} u \tag{13}$$

$$\gamma := -(\phi_q\dot{q})_q\dot{q} - 2\phi_{qt}\dot{q} - \dot{\phi}_{tt} \tag{14}$$

Fig. 2 shows a diagram of a numerical solution scheme based on [13]. The differential-algebraic equation of motion of (1) is index-3 Hessenberg form [16], which can be solved by the scheme. The scheme consists of regularization, linearization, Moore-Penrose pseudo-inverse, and implicit ordinary differential equation solving. At each time step, the constraint is regularized and γ is passed to the solver. The solver retrieves the numerical value of matrices M, Q, ϕ_q, and γ from the analytical formula (13). Then, it is augmented to have auxiliary states. The augmented system is converted by Moore-Penrose pseudo-inverse denoted by A^+. Finally, the system was integrated using the MATLAB `ODE15s`, which employs the backward differentiation formula for ordinary differential equations.

IV. Main Results

This section presents the two main results of this paper, which enable the use of control barrier functions for systems described by differential-algebraic equations.

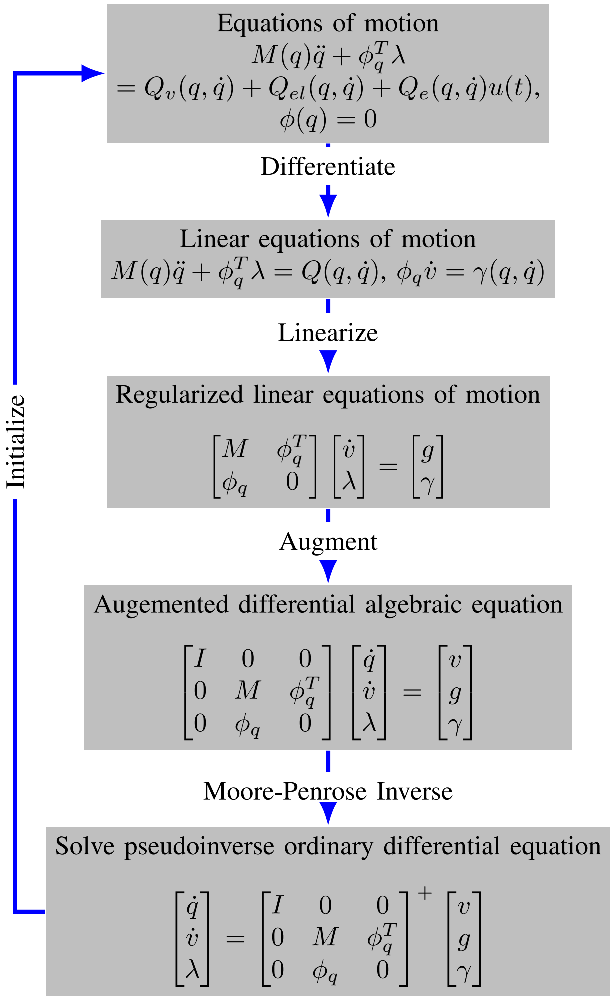

Fig. 2. The solution process for the differential-algebraic equation in the framework of a constrained floating frame of reference formulation for multibody dynamics. The equations of motion are differentiated, linearized, augmented, and inverted. The solution was obtained from the stiff ordinary differential solver. The sum of generalized forces Q_v, Q_{el}, and $Q_e u$ is denoted by Q.

A. *Control Barrier Function of Differential-algebraic Function*

We propose a control barrier function for differential-algebraic equations:

Definition 4.1 (Control Barrier Function of Differential-algebraic Equation): Consider a dynamic system with state vector $x \in \mathfrak{C}_{E,f,g} \subset \mathbb{R}^n$ and control input $u \in \mathbb{R}^m$:

$$E(x)\dot{x} = f(x) + g(x)u \tag{15}$$

where $f(x)$ and $g(x)$ are Lipschitz continuous and $E(x) \in \mathbb{R}^{n\times n}$. Assume that there is a function F that satisfies the following:

1) There exists a continuous $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ such that $\nabla B(x)z = F(x, E(x)z), \forall x \in \mathfrak{C}_{E,f,g}, \forall z \in T_x\mathfrak{C}_{E,f,g}$.
2) There exists $F(x)$ such that $\dot{B}(x) = F(x, f(x) + g(x)u)$.

Then, $B(x)$ is a control barrier function if:

$$L_f B(x) + L_g B(x)u \geq 0, \forall x \in \mathfrak{C}_{E,f,g}. \tag{16}$$

Theorem 4.1 (reformulated from [12]): Let $\mathcal{C} \subset \mathfrak{C}_{E,f,g}$ be defined as a continuously differentiable function $B : \mathcal{C} \subset \mathfrak{C}_{E,f,g} \to \mathbb{R}$. If B is a control barrier function of definition 4.1 on $\mathcal{C}$ and $\frac{\partial B}{\partial x} \neq 0$ for all x in the boundary of $\mathcal{C}$, then any Lipschitz continuous controller $u \in U \subset \mathbb{R}^m$ for the system (1) renders the set $\mathcal{C}$ safe.

Proof: The existence of B is derived from the control barrier function in [3] with smooth and compact consistency space $\mathfrak{C}_{E,f,g}$ and existence of map F:

$$\begin{aligned} \frac{d}{dt}B(x(t)) &= \nabla B(x(t))\dot{x}(t) = F(x(t), E(x(t))\dot{x}(t)) \\ &= F(x(t), f(x) + g(x)u) = \dot{B}(x(t)) \geq 0 \end{aligned} \tag{17}$$

Assume that $F(x,z) := \frac{\partial B}{\partial x}Ez$ and $\dot{B}(x) := L_f B(x) + L_g B(x)u(x)$ on $\forall x \in \mathfrak{C}_{E,f,g}, \forall z \in T_x\mathfrak{C}_{E,f,g}$. L_f, L_g, and $u \in U \subset \mathbb{R}^m$ is Lipschitz continuous. The time derivative of B is:

$$\dot{B}(x) = L_f B(x) + L_g B(x)u \geq 0, \quad \forall x \in \mathcal{C} \tag{18}$$

By Nagumo's theorem, the invariance of $\mathbb{C}$ is equivalent to

$$B(x) = 0 \Rightarrow L_f B(x) + L_g B(x)u \geq 0 \tag{19}$$

Thus, the Lipschitz continuous feedback law will ensure that $B(x) \geq 0$ on $\mathfrak{C}_{E,f,g}$. ■

The difference between Theorem 4.1 and the ordinary control barrier function of [12] is the states in the consistency space $x \in \mathfrak{C}_{E,f,g} \subset \mathbb{R}^n$ instead of $x \in \mathbb{R}^n$. The essence of the proof is the existence of map F. Futhermore, the control law does not require the inversion of the matrix of the descriptor system.

B. Extending Control Barrier Function to Operate for an Under-Actuated Flexible System

The presented control barrier function can only ensure forward invariance of systems with relative degree one with respect to the barrier function B. The higher order zeroing control barrier functions of [3], [17] extend the notion of safety to systems with higher relative degree, e.g. flexible-link robots with position constraints. In this context, a set $\mathcal{D}$ satisfies $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$.

Theorem 4.2 (Higher-order Zeroing Control Barrier Function of Differential Algebraic Equation, reformulated from [3], [17]): Consider a dynamic system with the state vector $x \in \mathfrak{C}_{E,f,g}$ and control input $u \in U \subset \mathbb{R}^m$ that satisfies (15) and has the relative degree r. A function $B : \mathbb{R}^n \to \mathbb{R}$ is a high-order zeroing control barrier function on $\mathcal{D}$ with $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$, if there exists an extend class $\mathcal{K}_\infty$ function α_i for each $i \leq r$ such that:

$$\begin{aligned} &\sup_{u \in U}[L_f\psi_{r-1}(x) + L_g\psi_{r-1}(x)u + \alpha_{r-1}(\psi_{r-1}(x))] \geq 0 \\ &\text{with } \psi_0 = B(x), \\ &\quad \psi_i = \frac{\partial \psi_{i-1}(x)}{\partial x}E(x)z + \alpha_i(\psi_{i-1}(x)). \end{aligned} \tag{20}$$

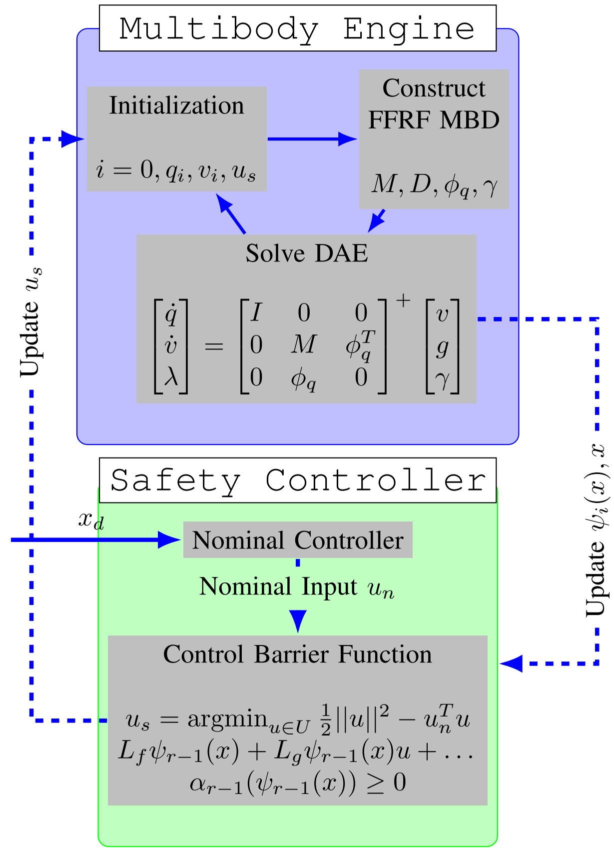

Fig. 3. Schematic of differential-algebraic equation control barrier function for flexible link robots. The system consists of a multibody engine and a safety controller. The multibody engine calculates matrices and calculates the solution of the equation with the input of the controller. The safety controller takes nominal input u_n by the nominal controller and enforces safety with $\psi(x)$. Nominal controller takes unsafe trajectory x_d.

Positive invariance holds open set $\mathcal{D} \subset \mathbb{R}^n$. We tailor the quadratic programming controller which imposes a zeroing control barrier function and input constraints to (15), where u_s denotes safe input, u_n denotes nominal control:

$$\begin{aligned} u_s &= \operatorname*{argmin}_{u \in U} \frac{1}{2}||u||^2 - u_n^T u \\ \text{s.t.} \quad & L_f\psi_{r-1}(x) + L_g\psi_{r-1}(x)u + \alpha(\psi_{r-1}(x)) \geq 0 \end{aligned} \tag{21}$$

Fig. 3 illustrates the elevated view of the main components of the differential algebraic equation control barrier function for flexible link robots. Notice that the multibody engine calculates the augmented states x on the consistency space:

$$x = [q, v]^T \in \mathfrak{C}_{E,f,g}. \tag{22}$$

As a result, the control barrier function in the differential-algebraic equation always exists in the safety controller. The proposed technique that implements DAE-CBF establishes a novel control framework that is capable of regulating the class of the descriptor control system.

V. Application

We showcase the presented method by designing a safety controller for a two-link flexible manipulator. We address a

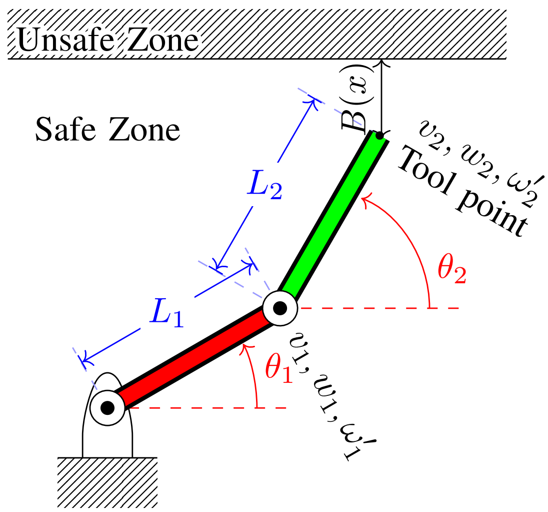

Fig. 4. Two flexible link manipulators. Safety $B(x)$ is defined as a collision with the upper wall. Each joint has an angle θ_i; axial displacement v_i; lateral displacement w_i; and angle of lateral displacement ω_i:

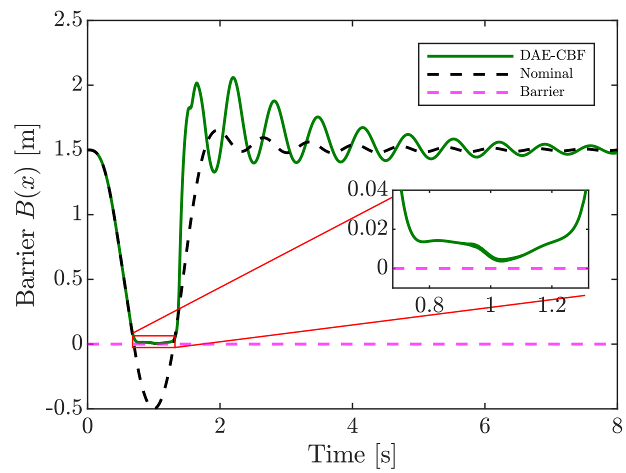

Fig. 5. Simulation result of differential-algebraic equation control barrier function compared with the nominal situation. The barrier function $B(x)$ ensures safety during the run time.

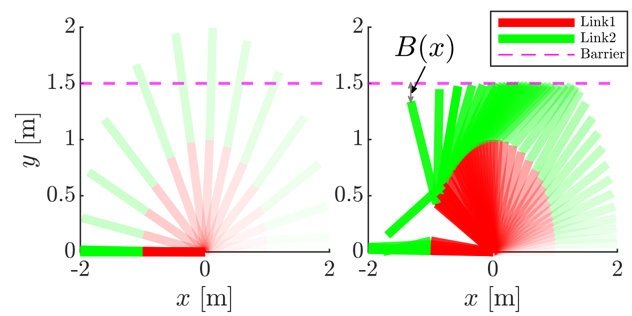

Fig. 6. (Left) Nominal PD control of 2-link manipulator with elasticity. Safety is violated. (Right) The safety control is incorporated with PD control of a 2-link pendulum with elasticity. Safety is enforced. The black line represents a limit of 1.5 m on the robot's tip.

specific maneuver that poses a safety hazard when executed with the nominal input [1].

A. Dynamic Model of Flexible-link Manipulator

Fig. 4 shows a diagram of a two-link flexible manipulator. The system dynamics has relative degree two with respect to the barrier function. The model has the following parameters. The lengths are $L_1 = L_2 = 1$ m. Elasticity is $E_1 = E_2 = 1$ GPa. The inertia of moments are $I_1 = I_2 = 6.75 \times 10^{-4}$ m^4. The densities are $\rho_1 = \rho_2 = 2.77 \times 10^3$ kg/m^2. The model has 16 states which are denoted as x:

$$x = [\theta_1, v_1, w_1, \omega_1, \theta_2, v_2, w_2, \omega_2, \dots \\ \dots \theta_1', v_1', w_1', \omega_1', \theta_2', v_2', w_2', \omega_2'] \tag{23}$$

The barrier function $B(x)$ is defined as:

$$B(x) = 1.5 - L_1 \sin\theta_1 - v_1 - L_2 \sin\theta_2 - v_2 \, [\mathrm{m}] \tag{24}$$

and represents a limit of 1.5 m on the tip of the robot in the y-direction.

B. Performance Evaluation of Safety Controller

The manipulator is initialized to be horizontal with zero velocity. In the following, we simulate a PD controller with and without the addition of a CBF-based controller. The PD control exerts a force on each joint as a nominal controller. Fig. 5 illustrates the barrier function, which represents the distance between the tool point and the upper wall, by the horizontal line (black) and safety controller (green). Fig. 6 illustrates a trace of the position of the links. The initial position is horizontal to the right. The set position of the PD control is horizontal to the left.

Fig. 7 shows the states of the multibody engine, which is integrated with the safety controller. The angle of each joint θ_i is illustrated in the first figure. The desired value for θ_i is 180°. The axial deformation v_i is illustrated in the second figure. The third figure represents the lateral displacements w_i. The fourth figure represents the control force u_i of the links, which is compared with nominal controller force u_i^n. Note that DAE-CBF control is minimally invasive, as it only deviates the nominal joint trajectory θ_i^n near the safety barrier from 0.5 s to 4 s. The deflections of the axial and lateral directions are also shown in Fig. 7. Note that the deflection exhibits a small correlation with the presence of safety.

VI. Conclusion

A control barrier function for differential-algebraic equations was proposed for a floating frame of reference formulation (FFRF) finite element method (FEM), which forms a two-link flexible-link manipulator, which facilitates its implementation in real-world conditions. The descriptor system with the control barrier function can enforce safety instead of the control-affine system with the control barrier function.

The first contribution is an extension of control barrier functions to differential-algebraic equations. The second contribution is an amalgamation of a safety controller based on a higher-order zeroing control barrier function and a differential-algebraic solver based on the Moore-Penrose inverse. As a result, we can enforce safety by solving a

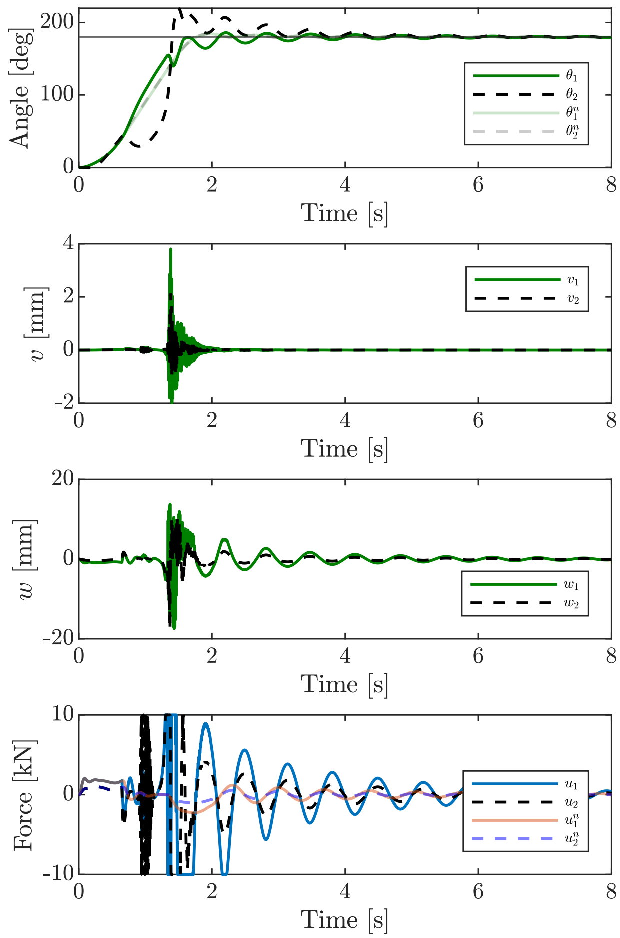

Fig. 7. States during multibody simulation with integrated safety controller. Each plot represents the angle of each joint θ_i, the axial deformation v_i, and lateral displacements w_i. θ_i^n denotes the trajectory of nominal PD-control. u_i denotes the forces of our framework. u_i^n denotes the forces of the nominal controller.

quadratic programming problem online. Numerical simulation addresses safety-critical system which demonstrates the benefits of a linear regularized differential-algebraic equation solver-based system control strategy.

To generalize the strategy, future work includes: 1) combining our framework with 3-dimensional space remote handling. 2) Investigating the collision barrier that represents complicated geometry. 3) Exploring the Lyapunov stability for differential-algebraic systems to extend methodologies like those presented in [10] or incorporating input shaping methods to ensure intrinsically stable and safe inputs.

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References

[1] G. Burroughes, N. Hamilton, and R. Skilton, "Towards controlling flexible deforming manipulators/structures with limited available sensing in DEMO," in *30th Symposium on Fusion Technology (SOFT)*, 2018.

[2] D. K. Thomsen, R. Søe-Knudsen, O. Balling, and X. Zhang, "Vibration control of industrial robot arms by multi-mode time-varying input shaping," *Mechanism and Machine Theory*, vol. 155, p. 104072, 2021.

[3] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, 2017.

[4] H. Peng, F. Li, J. Liu, and Z. Ju, "A symplectic instantaneous optimal control for robot trajectory tracking with differential-algebraic equation models," *IEEE Transactions on Industrial Electronics*, vol. 67, no. 5, pp. 3819–3829, 2019.

[5] C. Duriez, "Control of elastic soft robots based on real-time finite element method," in *2013 IEEE international conference on robotics and automation*. IEEE, 2013, pp. 3982–3987.

[6] A. Müller and T. Hufnagel, "Model-based control of redundantly actuated parallel manipulators in redundant coordinates," *Robotics and Autonomous Systems*, vol. 60, no. 4, pp. 563–571, 2012.

[7] J. Reher, C. Kann, and A. D. Ames, "An inverse dynamics approach to control Lyapunov functions," in *2020 American Control Conference (ACC)*. IEEE, 2020, pp. 2444–2451.

[8] P. Zhou, A. Zanoni, and P. Masarati, "A projection continuation approach for minimal coordinate set constrained dynamics," *Multibody System Dynamics*, vol. 57, no. 3, pp. 237–257, 2023.

[9] Y. Park and C. Sloth, "Discretization-robust safety barrier of partial differential equation," in *2023 11th International Conference on Control, Mechatronics and Automation (ICCMA)*, 2023, pp. 49–54.

[10] D. Liberzon and S. Trenn, "Switched nonlinear differential algebraic equations: Solution theory, Lyapunov functions, and stability," *Automatica*, vol. 48, no. 5, pp. 954–963, 2012.

[11] H. Durand and A. D. Ames, "A control barrier function perspective on Lyapunov-based economic model predictive control," in *2022 American Control Conference (ACC)*, 2022, pp. 2823–2828.

[12] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath, and P. Tabuada, "Control barrier functions: Theory and applications," in *2019 18th European control conference (ECC)*. IEEE, 2019, pp. 3420–3431.

[13] R. N. Hollari, A. Taghvaeipour, M. M. Aghdam, and F. González, "Novel co-simulation technique in the flexible analysis of a parallel robot," in *2022 10th RSI International Conference on Robotics and Mechatronics (ICRoM)*, 2022, pp. 224–230.

[14] A. Shabana, *Computational Continuum Mechanics*. Wiley, 2018, ch. 6.

[15] P. M. E. J. Wijckmans, "Conditioning of differential algebraic equations and numerical solution of multibody dynamics," 1996.

[16] P. Kunkel and V. Mehrmann, *Differential-algebraic Equations: Analysis and Numerical Solution*, ser. EMS textbooks in mathematics. European Mathematical Society, 2006, pp. 151–201.

[17] W. Xiao and C. Belta, "Control barrier functions for systems with high relative degree," in *2019 IEEE 58th conference on decision and control (CDC)*. IEEE, 2019, pp. 474–479.