

On a Magnetically Driven Array System with Autonomous Motion and Object Delivery for Biomedical Microrobots

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Abstract—The application of microrobots in the biomedical field has attracted great interest, among which drug transportation is one of the application scenarios. Traditional studies used the global magnetic field to control single microrobot, therefore it is impossible to control multiple microrobots. To address this problem, this paper develops a local magnetic field generation system to realize the independent control of multiple microrobots. The proposed multi-microrobot motion system integrates perception, planning, and actuation, enabling autonomous multi-task drug delivery. In our system, we first develop a printed circuit board (PCB) array magnetic driven microrobot system based on a micro coil array, then the Yolov8 framework is employed for the target/environment recognition, accurately identifying microrobots and magnetic fluids, while the Rapidly-exploring Random Trees (RRT) algorithm is used for path planning. We have conducted experiments on obstacle avoidance, droplet transport, and drug fusion. The results clearly demonstrate the significant potential of magnetic field-driven microcoil array devices in transportation and drug fusion engineering.

Index Terms—Magnetic microrobots, Object delivery, Yolov8, Rapidly-exploring Random Trees (RRT).

I. INTRODUCTION

The remote navigation of microrobots using magnetic fields has garnered considerable interest owing to its potential biomedical applications [1]–[3]. These applications encompass a broad spectrum of therapeutic procedures, such as cancer treatment, vascular interventions, tumor-targeted therapy, targeted drug delivery, hyperthermia therapy, and microsurgery [4]–[7].

In the past several years, using a global magnetic field to drive and control a single microrobot was a common method. For example, in [8], an adaptive neural learning based control method was proposed to achieve single microrobot tracking with unknown dynamics and disturbances in 2D space. In [9], this work achieved the motion of a single micro robot in 3D space by constructing an electromagnetic drive system. Such works are all aimed at controlling a single micro robot with Maxwell coils and Helmholtz coils. However, in biomedical scenarios, it is often necessary to sequentially fuse multiple drugs, making the coordinated movement of multiple magnetic-driven microrobots crucial. In addition,

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This work was supported in part by the National Natural Science Foundation of China under Grant 62203186 and 62373168, and in part by the 2022 open research project of Guangxi Key Laboratory of Automobile Components and Vehicle Technology under Grant 2022GKLACVTKF06. (Corresponding author: Qigao Fan.)

the complex environment in the medical application is another obstacle for the autonomous task navigation. How to control multiple microrobots moving in an unconstructed environment is a challenging issue.

Therefore, controlling multiple microrobots independently has attracted much attention. One method to control multiple microrobots is to use a global magnetic field to manipulate microrobots with different shapes or magnetic properties. The work of [10] demonstrates the use of magnetic gradients as a control method for the motion and independent movement of two different microrobots. In [11], the independent motion of dual-tailed microrobots with different tail length ratios in a jointly driven magnetic field is developed. In [12], heterogeneous nanorobots with the same geometric shape but different magnetization directions were used to achieve independent control, resulting in the same velocity response curve to the oscillating magnetic field but with different phases.

However, in the independent control of multiple microrobots using a global magnetic field, it is necessary to distinguish the structure or magnetism of the micro robots to ensure that independent control can be achieved, and it is still not possible to completely decouple the control of multiple micro robots. Therefore, when faced with a large number of micro robots, these methods are no longer applicable. To overcome these problems that occur in a single global magnetic field, some discrete local magnetic fields generated by planar microcoil array offer a new train of thought. In [13], a 64-coil array electromagnetic microrobot system was designed to independently control multiple microrobots on the platform. In [14], a novel local magnetic field generation system is proposed, consisting of micro-wire strips positioned in different layers within a printed circuit board (PCB). Each group of micro-wire strips is orthogonal, creating local magnetic field gradients in the orthogonal direction within the plane, allowing for independent control of multiple microrobots. The aforementioned systems primarily use micro coils or micro-wire strips to generate local magnetic fields, which effectively enable the independent control of multiple microrobots. These driving structures eliminate the need to manufacture separate microrobots for independent tasks.

Although the aforementioned work introduces the concept of array-based magnetic field design for independently controlled microrobots, most research has primarily focused on system design rather than on the execution of multiple tasks by independently controlled robots. For example, achieving coordinated movement of multiple microrobots for tasks such

as target transportation and fusion is a significant challenge in the field of magnetic-driven microrobots. Among them, two key points should be addressed, i.e., task sensing and planning [15]–[18].

First, it is crucial for robots to obtain real-time information about their current position and orientation. Additionally, autonomous navigation becomes challenging due to complex obstacles. Therefore, real-time tracking and obstacle recognition for microrobots are key issues that need to be addressed. In practical applications such as transportation, it is essential to distinguish and tracking between different categories of objects being transported by microrobots to obtain accurate information. Although traditional machine learning methods can accomplish certain recognition tasks [19], they generally fall short in terms of generalization, accuracy, and robustness compared to deep learning approaches. Deep learning methods, with their complex network architectures and extensive data training, are better at capturing intricate features in the data, thereby enhancing recognition accuracy and system robustness [20], [21].

Among them, YOLOv8, the latest iteration of the You Only Look Once (YOLO) series, represents a significant advancement in real-time object detection. YOLO models are known for their ability to simultaneously predict object locations and classes with high efficiency, making them well-suited for applications requiring rapid and accurate detection [22]. Therefore, due to the accuracy and real-time performance of the recognition algorithm, we employed YOLOv8 for the identification of microrobots in this study. Besides, to achieve multi-robot and multi-task motion planning, we employed the Rapidly-exploring Random Tree (RRT) algorithm. RRT efficiently explores high-dimensional spaces and finds feasible paths while avoiding obstacles. Its ability to quickly navigate complex environments makes it well-suited for coordinating multiple robots in dynamic scenarios [23], [24].

This paper introduces a novel microrobot system with two primary contributions. First, we present the design of a PCB-based coil array system capable of controlling microrobots using independently generated local magnetic fields. Second, it integrates YOLOv8 for real-time object recognition and RRT for efficient path planning, enabling the microrobots to autonomously perform multi-task delivery, transportation, and target fusion. These advancements underscore the system's potential for versatile applications in complex environments. The organization of the paper is as follows: Section II introduces the magnetic field driving and sensing system based on PCB array coils. Section III presents the autonomous navigation functions and algorithms. Section VI is dedicated to the experimental validation of the proposed system. The final section provides a summary and conclusion of the paper.

II. LOCAL MAGNETIC FIELD ACTUATION SYSTEM

This section introduces a local magnetic field platform for independent control of multiple micro robots. We primarily introduce the modeling of the magnetic drive system, the

target recognition and tracking algorithms for microrobots based on YOLOv8, and the structural design of the driving system.

A. Local Magnetic Field Platform Design

To achieve the independent control of multiple microrobots for droplet transportation, we utilized a local magnetic field generation platform comprising 144 planar micro coils, each with a diameter of 3mm.

By independently powering the micro coils, the system can achieve the independent control of multiple magnetic microrobots. The magnetic force \vec{F} on a microrobot with magnetization \vec{M} , generated by the external magnetic field \vec{B} from the microcoils, can be described as:

$$\vec{F} = V(\vec{M} \cdot \nabla)\vec{B} \quad (1)$$

where V (m^3) represents the volume of the microrobot. For a disc-shaped magnetic microrobot magnetized along the thickness direction, the magnetization \vec{M} can be approximated as \vec{M}_z . Therefore, the magnetic force \vec{F} can be simplified to:

$$\vec{F} = V \left(\vec{M}_z \left(\frac{\partial \vec{B}_x}{\partial z} \vec{a}_x + \frac{\partial \vec{B}_y}{\partial z} \vec{a}_y + \frac{\partial \vec{B}_z}{\partial z} \vec{a}_z \right) \right) \quad (2)$$

B. YOLOv8-Based Detection and Tracking

For the effective control of microrobots, real-time and accurate positioning is crucial to ensure that the microcoils near the microrobots can generate precise external magnetic fields. However, commonly used visual algorithms are often inadequate for tracking multiple microrobots. In this study, we employ YOLOv8 to identify and analyze the position information of multiple microrobots and the droplets they transport. The YOLOv8 network model, depicted in Fig. 1, consists of three main components: the Backbone, Neck, and Head.

YOLOv8's Backbone comprises modules such as CBS, C2F, and SPPF, which are designed to extract feature maps from the input data. The Neck part of the network further processes these feature maps, employing a design concept that includes a Path Aggregation Network (PAN) and a Feature Pyramid Network (FPN) [25], [26]. This structure facilitates a bottom-up path aggregation and a top-down feature pyramid, enhancing the network's ability to detect and classify objects at different scales.

YOLOv8 uses a decoupled Head structure, separating the classification head from the detection head to improve performance. By assigning different colors to the microrobots, they acquire distinct features, enabling YOLOv8 to accurately recognize their positions. The recognition results are shown in Fig. 2.

C. Multiple Microrobots Actuation and Control

A point-to-point discrete control method is used to actuate the microrobots. When a micro coil is internally energized, it functions like an electromagnet. If the magnetic moment of the microrobot aligns with the local magnetic field generated by the micro coil, the microrobot will be attracted towards

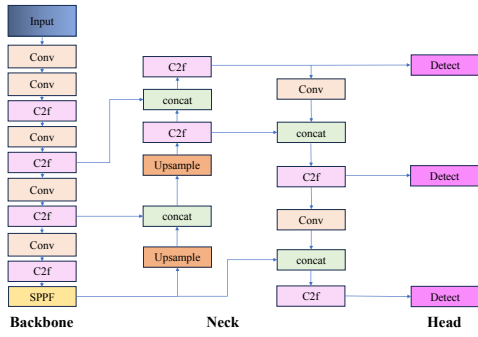


Fig. 1. YOLOv8 network model.

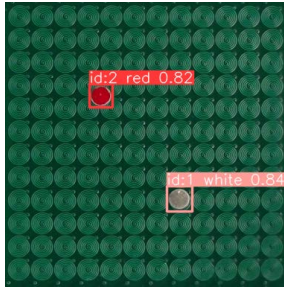


Fig. 2. The recognition effect of two micro robots with different features.

it. Conversely, if the directions are opposite, the force on the micro robot will repel it. The control strategy for the micro coils is based on the error between the current position and the target position of the micro robot.

Due to the very low inductance of the micro coils, it is possible to quickly switch the current, allowing for relatively easy control of the micro robots using a switch-type controller to output the required current for actuation. Fig. 3 illustrates the steps for the feedback control of the micro robot based on visual input, and the current output module is shown in Fig. 4. And the motor driver (TB6612FNG) is controlled for current direction through an I/O expander (PCF8575), with each set of four motor drivers connectable to one I/O port (PCF8575), so each current control board has the capability to manage up to 16 microcoils.

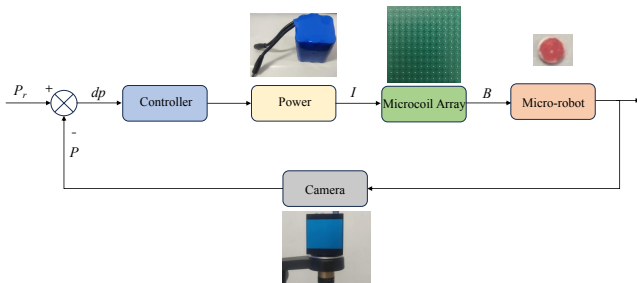


Fig. 3. Visual feedback control of micro robot.

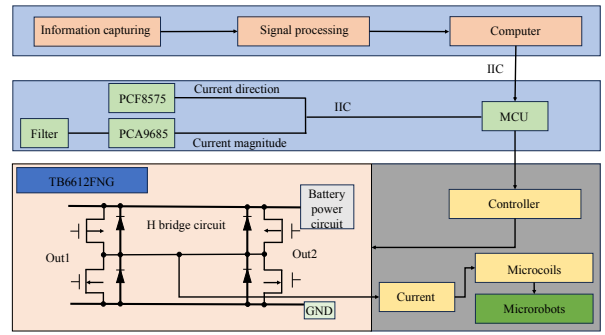


Fig. 4. Schematic diagram of the driver board.

III. PATH NAVIGATION FOR DROPLET TRANSPORT

A. RRT

The Rapidly-exploring Random Tree (RRT) algorithm, first introduced by LaValle in 1998 [27], is an effective path planning approach crucial for the multi-task planning and collaborative navigation of micro robots. RRT is a sampling-based algorithm that initiates from a starting point, samples points through random functions, and employs a node expansion strategy to generate new nodes. Collision detection determines whether a new node can be incorporated into the random tree. The algorithm continues expanding the tree until the target point is reached, at which point the expansion halts.

The RRT algorithm provides several benefits:

- 1) Effective exploration of unexplored spaces, facilitating comprehensive path planning.
- 2) Probabilistic completeness, ensuring reachability of the target with sufficient iterations.
- 3) High efficiency in obstacle avoidance due to its reliance on collision detection rather than precise obstacle modeling.

For multi-task planning and collaborative navigation of micro robots, the RRT algorithm is utilized to efficiently generate paths for tasks such as object delivery and coordination. The process involves initializing a random tree from the starting point, sampling new points, expanding towards these points while performing collision detection, and iteratively building the tree until reaching the goal. This method, while not our primary innovation, provides a robust solution for path planning in complex environments. The path planning process is outlined in Algorithm 1.

Note that in our designed system, each micro robot within the array of magnetic coils is limited to four possible movement directions: forward, backward, left, and right, as shown in Fig. 5. The step size for each movement is $\sqrt{2}L$, where L represents the distance between the centers of two adjacent microcoil arrays. One example of the generated path under these conditions is illustrated in Fig. 6.

Remark 1: To optimize path planning for the array-based magnetic micro robot system, we introduce a probability parameter p during the expansion step of the RRT algorithm.

Algorithm 1 RRT

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1: function RRT( $T_{init}, T_{goal}, maxiters, p$ )
2:    $tree \leftarrow CreateTree(T_{init})$ 
3:    $iters \leftarrow 0$ 
4:   while  $iters \leq maxiters$  do
5:      $T_{rand} \leftarrow RandSample(T_{rand}, T_{goal}, p)$ 
6:      $T_{nearest} \leftarrow NearestNeighbor(tree, T_{rand})$ 
7:      $T_{new} \leftarrow Extend(T_{nearest}, T_{rand})$ 
8:     if  $ObstacleNo(T_{nearest}, T_{new})$  then
9:        $newNode \leftarrow AddNode(tree, T_{nearest}, T_{new})$ 
10:      if  $Distance(T_{new}, T_{goal}) < threshold$  then
11:        if  $ObstacleNo(T_{new}, T_{goal})$  then
12:           $path \leftarrow GeneratePath(T_{new}, T_{goal})$ 
13:          return  $path$ 
14:      $iters \leftarrow iters + 1$ 
15:   return  $path_{RRT}$ 
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Depending on the value of p , the tree either grows randomly or towards the target position. If the randomly generated probability is less than p , the tree expands randomly; otherwise, it grows towards the target point. This method enhances the path planning efficiency for microrobots controlled by the magnetic field array.

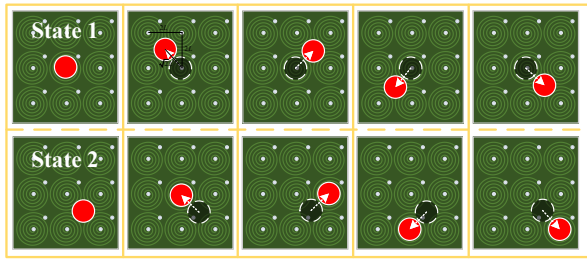


Fig. 5. Motion diagram of microrobot

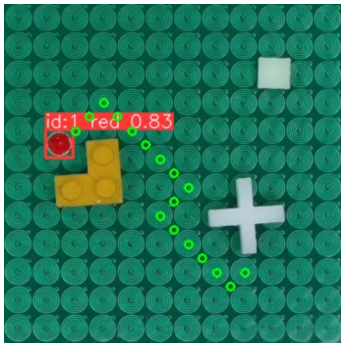


Fig. 6. Example of RRT algorithm path planning

IV. EXPERIMENTS VERIFICATION

The array PCB microrobot drive system, as shown in Fig. 7, comprises a microcoil array platform, nine current drive boards, and a camera. The microcoil array platform generates local magnetic fields for microrobot actuation. The nine

current drive boards supply power through 144 channels, each powering an individual microcoil. The camera is used to capture the real-time location of the microrobots.

A. Experimental Setup and Objectives

The primary goal of the experiment is to validate the effectiveness of the proposed array PCB microrobot drive system in microfluidic transport and mixing applications. To achieve this, the experiment utilizes microrobots with distinct colors, enabling the YOLOv8 algorithm to accurately track and identify each microrobot based on its unique features.

The experimental setup involves a series of steps:

- **Microrobot Identification and Tracking:** Different colored microrobots are used to facilitate accurate tracking by the YOLOv8 algorithm. This step is crucial for monitoring the position and movement of the microrobots in real-time.
- **RRT Path Planning for Single Microrobot:** Initially, the experiment focuses on the path planning of a single microrobot using the RRT algorithm. This stage helps to establish a baseline for the control and navigation capabilities of the microrobot system.
- **Microfluidic Transport and Mixing:** The core objective is to demonstrate the system's potential in microfluidic transport. Two microrobots are tasked with transporting magnetic droplets to a designated reaction zone for mixing. The magnetic droplets, which are water-based magnetic fluids, are situated in a culture dish filled with silicone oil. The droplets and microrobots are positioned in separate layers, ensuring clear separation and control during the experiment (as shown in Fig. 9).
- **Design and Implementation of Droplet-Carrying Structure:** A specially designed concave structure is integrated at the bottom of the culture dish. This structure serves to separate the droplets from the microrobots, facilitating smooth transport and preventing unintended interactions.
- **Path Point Acquisition and Navigation:** The RRT algorithm is employed to determine the path points for the microrobots. Upon reaching the mixing zone, node direction is carefully controlled to ensure that the microrobots can quickly exit the area, allowing for a streamlined and efficient process.

The experiment aims to showcase the system's capability in precisely transporting and mixing magnetic droplets, with successful mixing observed in the reaction zone. This setup not only demonstrates the feasibility of the microrobot system for practical applications but also highlights the potential for further advancements in microfluidic transport technologies.

B. Results and Analysis

In our experiments, we used a magnet with a diameter of 3mm and a thickness of 0.5mm as the microrobot. The RRT algorithm was employed to determine the final path for the microrobot by pre-inputting obstacle information, enabling precise navigation, as depicted in Fig. 8.

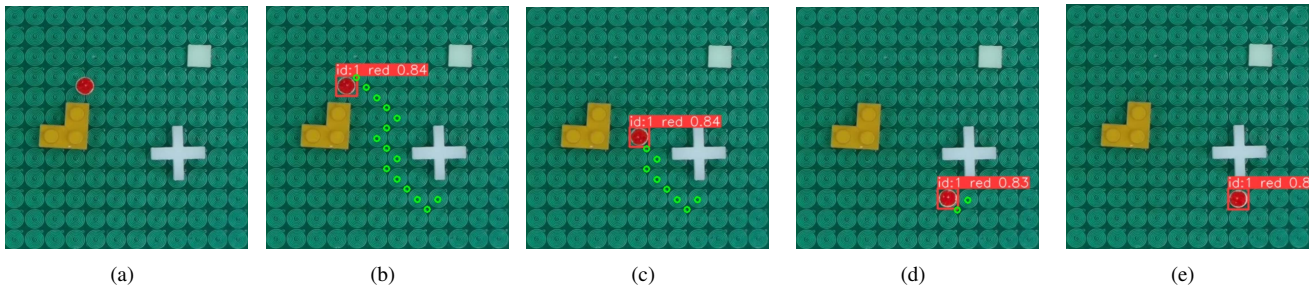


Fig. 8. Obstacle avoidance of a single micro robot based on RRT

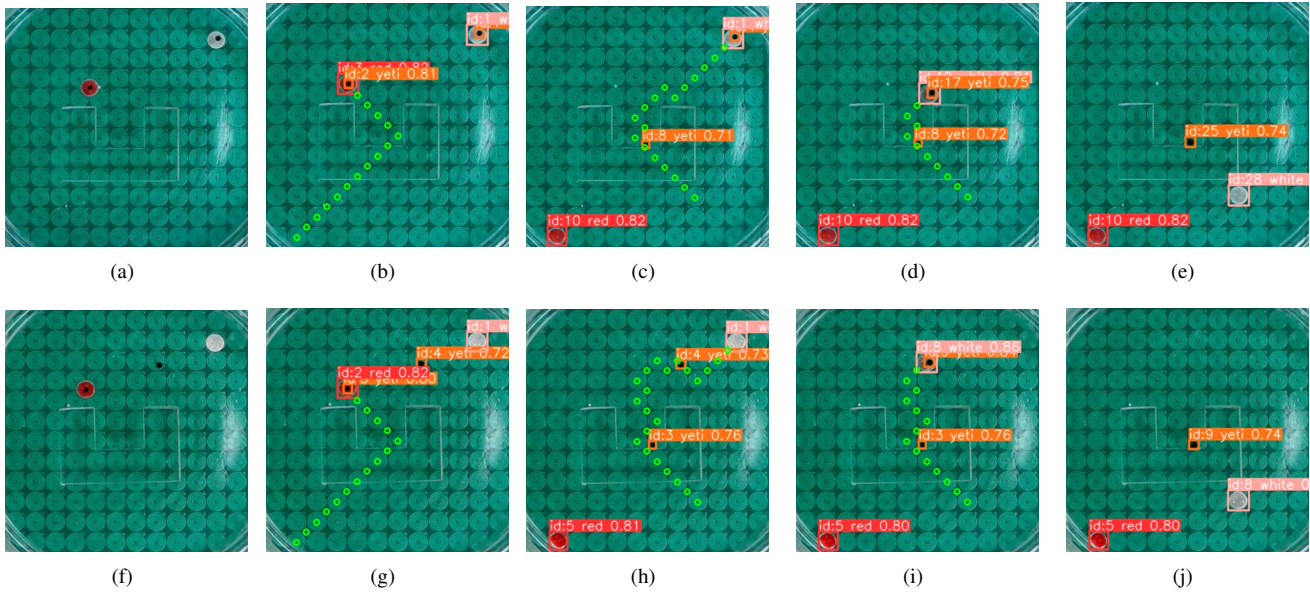


Fig. 11. Snapshots of the delivery experiments of Case1 and Case 2

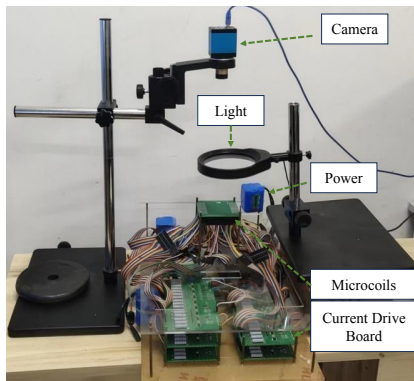


Fig. 7. Microrobot system with the 12×12 planar circular microcoil array.

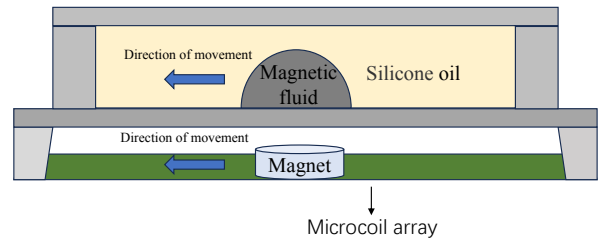


Fig. 9. Schematic diagram of magnetic fluid transportation. The micro robot is driven by a coil array and carries the magnetic fluid movement in the culture dish.

The process of transporting magnetic fluid by the microrobots is shown in Fig. 9. The microrobots transported the magnetic fluid using the magnetic forces between them. During the transport process, concave structures were used as obstacles to prevent collisions between the magnetic fluids, while the microrobots also acted as obstacles to each other to avoid interference. After the magnetic fluid was separated

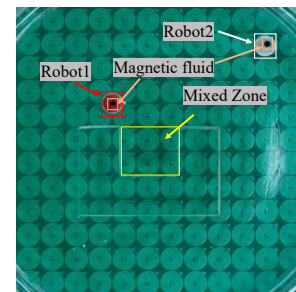


Fig. 10. Schematic diagram of experimental scenario.

from the microrobot, the concave structure was no longer used as an obstacle, and the direction between the RRT nodes was constrained to allow the microrobot to quickly exit the mixing zone.

The schematic diagram of the experimental scenario is shown in Fig. 10. In this setup, the map was rasterized, and only four diagonal directions were selected as possible motion directions. The distance between the two microrobots was kept greater than $12L$, with an RRT algorithm probability p set at 0.95.

In Case 1, all the magnetic fluids were transported by the microrobot. In Case 2, one of the microrobots did not carry magnetic fluid. In this case, the RRT algorithm first generated a path from the microrobot to the magnetic fluid and then a path to the mixing zone.

The use of the RRT algorithm in these experiments allowed for accurate path planning and navigation, enabling the microrobots to successfully complete the transportation and mixing tasks. The results, as illustrated in Fig. 11, showed that the magnetic droplets were effectively transported to the reaction zone and mixed in a controlled manner. This demonstrates the system's capability for precise navigation and handling of complex tasks, validating the effectiveness of the proposed approach for microrobot control.

V. CONCLUSIONS

In this paper, we drive micro robots using a local magnetic field drive device composed of 144 micro coils. And it was found that using Yolov8 can accurately track micro robots and obtain corresponding position information. By using mature RRT algorithms to input physical constraints and obstacle information, a feasible path can be obtained. After combining Yolov8 with RRT, we successfully achieved obstacle avoidance and magnetic fluid transportation and fusion functions, proving its feasibility. On this basis, large-scale automated operation of magnetic droplets will be studied. In addition, encapsulating droplets with reagents for drug synthesis, sample analysis, or chemical synthesis is also research work.

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