2025 IEEE/SICE International Symposium on System Integration (SII)
January 21-24, 2025, Munich, Germany

Vibrotactile Feedback for Training Tempo and Stroke Length in Golf Putting

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***Abstract*— Traditionally, to learn a sport effectively, people receive feedback from an expert, such as an instructor. However, personal instructors are not only expensive, but the demand for training is also larger than the number of instructors available. As an alternative to traditional instruction, we are developing a sports training system to provide multimodal feedback to users based on their performance and environment information. The feedback is based on both the user's performance and the motion of expert athletes, enabling users to train effectively. Initially, we chose Golf as a case study to implement this system. In this paper, we introduce an approach to train putting tempo and stroke length using vibrotactile feedback in order to enable users to train their putting ability consistently. Using the proposed vibrotactile cues, users were able to perform putts with tempos close to the ones provided by the vibrotactile cues. Furthermore, they were able to control the length of the backswing stroke following the proposed vibrotactile cues. We believe the proposed framework can be used to train the basic skills of putting effectively.**

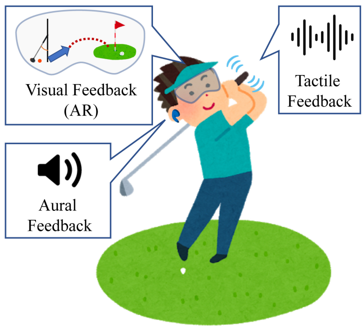

Fig. 1: Concept of Augmented Golf Training

I. INTRODUCTION

Whenever a person wants to learn a sport, they usually receive some instruction from an expert, such as an instructor or coach. However, this has some limitations: first, the instruction's quality depends on the coach or instructor's proficiency; their ability to teach directly impacts the trainee's progress; second, cost and availability: personal instructors are limited in numbers and can be quite pricey. Furthermore, it's impossible to receive such instruction in situations where the trainee is home-ridden.

To address this, we propose a system that enables users to train independently by providing motion feedback through multiple perception channels (auditory, tactile, visual). This approach enhances the trainee's awareness and helps them improve without an instructor. We selected golf as the initial target sport due to its turn-based nature, allowing feedback before, during, and after performance. As shown in Fig. 1, the system uses augmented reality for visual guidance, tactile feedback for real-time motion correction, and auditory feedback to convey semantic insights informed by traditional coaching.

In the past, we have proposed methods to use vibrotactile feedback to guide the motion of the user's elbow [1] and wrist [2], [3]. In these cases, we only considered the spatial characteristics of the motion and not the temporal characteristics. However, temporal information is equally important in some motions. In the past, we've used multimodal feedback to train ballroom dancing [4], [5]. In golf, the temporal characteristics of the swing and putt play a key role in its effectiveness. Professional golfers exhibit a tempo, measured as the ratio between the backswing and downswing times, of 3 to 1 when swinging and a tempo close to 2 to 1 when putting [6].

In this paper, we focus on using vibrotactile feedback to train the user's putt shots. Putting is an essential part of the game, as it comprises more than half of all the strokes in a round, and the putter is generally used twice more than the driver [7]. Putting is a task that requires precision and consistency, and its characteristics have been widely studied. In putting, controlling the distance traveled by the ball, also known as "touch", is achieved through a combination of a consistent tempo (i.e., the ratio between the backswing and downswing motion) and the length of the stroke. In this paper, we propose using vibrotactile cues to convey the temporal characteristics of the putting motion to users using vibrotactile illusions to train the skill of distance control in putting.

Through experiments, we evaluate using vibrotactile feedback based on the Phantom Sensation illusion in order to determine whether the temporal characteristics of the putting motion can be conveyed effectively. Additionally, we propose a real-time vibrotactile cue to indicate the end of the backswing to the users, which in turn controls the distance traveled by the ball. We believe that continued training using this framework could help users understand and improve the characteristics of their putts in a shorter time, even without an instructor.

*This work was supported by JSPS KAKENHI Grant Number 20K19838.
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II. Feedback Systems for Golf

As a complement to conventional golf instruction, several digital systems have been developed to aid the process of learning golf. Most proposed systems focus on the practice of the golf swing, although some focus on analyzing and providing feedback about the player's putting skills.

A. Post-hoc Feedback

The majority of the proposed systems to date focus on providing players with Knowledge of Performance (KP) about their swing or putt. The most common approach is tracking the golf club using optical motion tracking systems or Inertial Measurement Units (IMU), processing the data, and displaying it to players so they can make informed improvements to their skills.

There are several IMU based systems to evaluate the swing performance of the golfer based on the club motion [8], [9], [10], [11], [12], [13], including commercial products [14], [15]. Other approaches track and evaluate the golfer's motion while swinging by attaching sensors to the golfer's body [16], [17].

There are less systems that focus on measuring and evaluating the putting motion. The most common approach is using an IMU attached either to the shaft [18], [19], [20] or to the head of the putter [21]. Jensen et al. [21] additionally propose the use of machine learning to detect a putting stroke automatically. Commercial solutions using optical motion tracking [22] and IMUs [23] are also available.

B. Real-time Feedback

Some approaches have studied providing instantaneous feedback when playing golf. As the visual channel is used when putting or swinging, most of these systems use auditory cues to convey information about their performance to the golfer in real-time.

For people practicing the swing, one approach consists in converting the acceleration at the club's head into an audio signal where the magnitude varies the volume of the auditory cue [24]. A similar approach varying the pitch of the sound based on the speed of the swing was made commercially available in the past [25]. Other systems evaluate different parameters, such as the foot pressure or the head rotation of the golfer, and use auditory signals such as beeps or buzzers to alert the golfer [26], [27].

Haptic feedback is also a suitable alternative to provide real-time feedback, as the haptic channel can elicit faster reaction times than the auditory channel [28], [29]. Some systems provide force feedback to correct the motion, including robotic systems that can be attached to the club, to help users train their swing motion [30], [31].

Huang [32] proposed a cable-driven device to guide the putter's head angle. Some of the drawbacks about the system stated by the authors include that the freedom to move the putt was reduced due to the constrained nature of the system, making their putt unnatural. Another haptic system commercialized under the name "The Can't-Miss" used gyroscopic precession to create torque around the shaft to keep the putter's face straight during its motion.

Vibrotactile cues are a form of haptic feedback that can guide human motion [2]. One of its main advantages is that it can be felt anywhere on the skin, and due to the reduced size of vibrotactile actuators, they can be easily integrated into wearable devices or sports gear (e.g., the grip of the putter), which allows users to perform the task naturally, by themselves, while receiving feedback. However, we couldn't find any related work where vibrotactile feedback was used for golf.

III. Mechanics of Putting

Many people affirm that touch "cannot be taught" and it's an unconscious competence that players develop with practice [7]. However, new models have been proposed to understand the decisive factors when putting and provide quantitative information to allow players to improve their shots [33], [34].

Most golfers and researchers agree that the motion of the putter can be modeled as a pendulum driven at twice its resonant frequency, which can be evidenced in professional golfers having a consistent tempo (i.e., the ratio between the backswing time τ_b and the downswing time τ_d).

Based on this observation, Grober [34] proposed a thorough model of the ideal putter's motion. From his results, the author indicates that the most important parameters to achieve a consistent putt, observed in all world-class golfers, are (1) a constant head velocity when the putter hits the ball (2) a tempo close to $\tau_b/\tau_d \approx 2$ in all their putts (in contrast, non-expert putters usually exhibit a tempo close to 1:1 [35]) and (3) a stroke duration independent of the final velocity of the putter club.

A. Model of Putter Motion

The model proposed by Grober is based on the equation of motion of a pendulum, and it satisfies the three conditions above. The model proposes that the velocity of the pendulum $\dot{x}_s(t)$ can be calculated as:

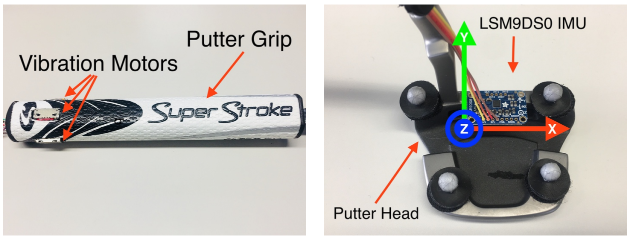
$$\dot{x}_s(t) = -A(cos\omega_0 t - cos2\omega_0 t) \tag{1}$$

where $x_s(t)$ is the position of the pendulum mass (the putter's head, in this case), $\omega_0 = \frac{2\pi}{3\tau_b}$ is the resonant frequency of the system, τ_b is the duration of the backswing and the amplitude A is a constant used to normalize the velocity. Using this model, we can calculate the position of the putter $x_s(t)$ by integrating the expression above, under initial conditions $x_s(t_0) = 0, \dot{x}_s(t_0) = 0$.

B. Resonance and Distance Control in Putting

Grober [36] also estates that in cases when the golfer has a putt that follows the previously introduced model, the distance traveled by the ball x_b is related to the length of the stroke, which he defines as $x_l = x_s(\tau_b)$ through the following relation:

$$x_b \propto {x_l}^2 \tag{2}$$



(a) Putter grip embedded with vibration motors (b) IMU attached to putter head

Fig. 2: Vibrotactile feedback enabled golf putter

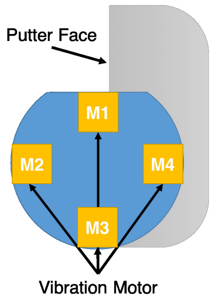


Fig. 3: Distribution of motors in the grip (top view)

Under the assumption that the proposed model is valid, controlling the length of the backstroke would allow players to control the distance traveled by the ball, in other words, mastering the skill of "touch". Due to its unobtrusive nature and its variable parameters, vibrotactile feedback can be used to convey tempo to users, and it can also be used in real-time to control the stroke length. We believe that the proposed models can be used to generate the characteristics of an ideal putt stroke and create vibrotactile cues based on them in order to convey the motion characteristics.

IV. Vibrotactile Device Implementation

A. Hardware

Given the nature of golf, where users are required to hold the putter, we chose to embed the vibrotactile actuators on the grip, as the tactile sensitivity of the hand is high, so they will provide a clear vibration to the user and would not impede the motion.

We used a Mizuno T-Zoid RV104 34" putter, in which we attached a Super Stroke Fatso 5.0 putter grip, characterized by it's 1.67" diameter, which allowed us to embed the motors inside while preserving the shape of the grip, as can be seen in Fig. 2a. Additionally, an IMU was attached to the putter's head, in order to measure its motion. The orientation in which the IMU was attached can be seen in Fig. 2b. The device is controlled using a Raspberry Pi 3 running ROS, connected to the motors and the IMU.

1) Characteristics of the Vibration: To produce the vibrotactile cues, we used 4 HAPTIC™ Reactor Linear Resonant Actuators (LRA) from ALPS Electric distributed 90° apart from each other, as can be seen in Fig. 3. We selected these LRAs as they provide a strong vibration (up to 3G) and their actuation time is much faster than conventional eccentric mass motors (∼10 ms after voltage applied).

The motors are driven using a 160 Hz PWM pulse, and oriented so that the vibration direction is perpendicular with the contact surface of the hand. The amplitude of the vibration produced by the LRA can be controlled by changing the duty cycle of the driving PWM signal. Additionally, by changing the Inter Stimuli Interval, or ISI (i.e., the time between each burst of vibration and the next), we can produce vibrations that feel "slower" for longer time intervals, or "faster" for shorter time intervals. This is a parameter that, along with the amplitude of the vibration, can be used to convey different information to the user.

2) Putter Motion Estimation: To measure the characteristics of the putter's motion we used a LSM9DS0 9 DoF IMU from Adafruit, which includes a 3-axis accelerometer, a 3-axis gyroscope and a 3-axis magnetometer. The pose was estimated using a Madgwick filter. Gyroscope bias are calculated and fed to the system before using it to avoid the effects of gyro drift. We carried out a preliminary experiment to verify the measurements using a Motion Analysis Motion Capture System (MCS). We measured the putt of a proficient golf player using both MCS and IMU. Additionally, we calculated the ideal position change of the putt by integrating Eq. 1 assuming initial conditions $x_s(t_0) = 0, \dot{x}_s(t_0) = 0$. The results can be seen in Fig. 4. There is a difference between the position at the end of the follow-through phase between the model based estimations and the actual putt measurements, which we don't consider relevant as the putter is not in contact with the ball after the impact, and the measurements of tempo are based on the characteristics of the backswing and downswing phase.

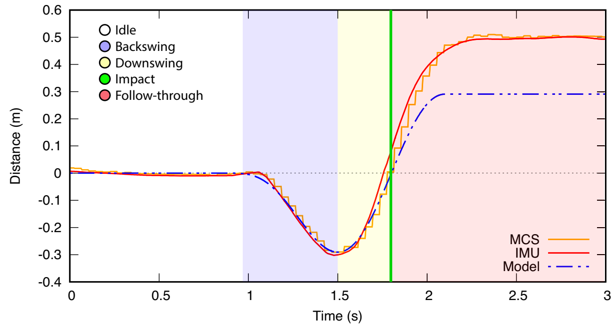


Fig. 4: Comparison between shaft position measurements using IMU, MCS and model generated data. The colored zones represent the corresponding phases of the Putt motion

V. Vibrotactile Feedback for Putting

When designing effective vibrotactile cues, they must be intuitive and easy to follow with minimal explanation. Timing and frequency are also important; continuous cues can distract users from the task and harm performance [37][38]. Our framework combines online (during motion) and offline (before/after motion) feedback for golf putting. Offline cues help users internalize the ideal tempo before putting, while a

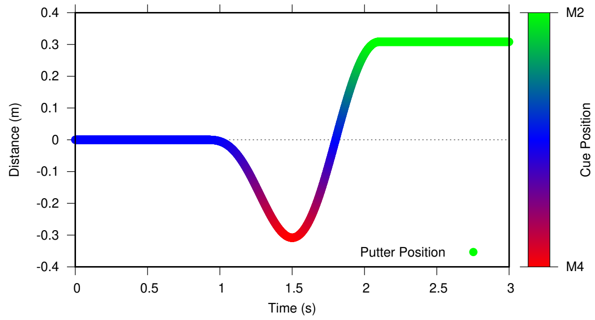


Fig. 5: Mapping of x_s to virtual cue's position inside the grip

simple online cue signals when to transition from backswing to downswing, ensuring correct stroke length.

A. *Vibrotactile Feedback to Convey Tempo*

To generate vibrotactile cues for conveying tempo, we used the model from Eq. 1 to recreate an ideal stroke based on commercial reports. For a tempo of 2, with $\tau_b = 600ms$, the model gave $\tau_d = 300ms$, allowing us to calculate the putter's position $x_s(t)$, velocity $\dot{x}_s(t)$, and acceleration $\ddot{x}_s(t)$. We found $x_s(t)$ to be the most intuitive for users to follow, as it represents a tangible measurement (displacement).

By controlling three parameters—cue location, intensity, and ISI—we produce vibrotactile feedback that emulates the ideal change of x_s. We leverage the Phantom Tactile Sensation (PTS) illusion [39], where a virtual vibration moves between two actuators (M2 and M4) on the putter's grip, simulating the putter head's motion. As the user swings, the virtual cue moves in sync with the ideal putter position, creating a tactile sensation of motion from the backswing to the follow-through.

We map x_s to the virtual cue's position inside the grip, as shown in Fig. 5. The cue starts in the middle, shifts towards M4 during the backswing, and returns to the center and then M2 during the downswing and follow-through. The ISI is modulated based on $|x_s|$, ranging from 30ms near zero to 10ms at maximum values, providing more detailed feedback as the stroke progresses.

B. *Vibrotactile Feedback to Control Stroke Length*

Grober's model suggests that proficient golfers can consistently control putt distance by managing the backswing length x_l and maintaining a tempo of 2, as described in Eq. 2. The pendulum model allows us to calculate the putter's displacement $x_s(t) = r \sin \theta(t)$, where r is the distance from the golfer's shoulder to the putter head, and $\theta(t)$ is the angle change of the putter's shaft.

To provide subtle guidance without distracting from the putt, we use a short vibrotactile cue, called the Transition Timing Cue (TTC), which activates all four motors briefly to create a tap-like sensation. This cue signals the user to transition from the backswing to the downswing. The TTC is emitted once during the stroke for about 10 ms, minimizing disruption.

To account for human reaction times of 200-300 ms [40], [41], we use a Linear Time Invariant Kalman Filter to estimate $\theta(t)$ 200 ms into the future and trigger the TTC when the estimated angle $\hat{\theta}$ reaches the desired value θ_d.

VI. Experimental Study

A. *Tempo Control Experiment*

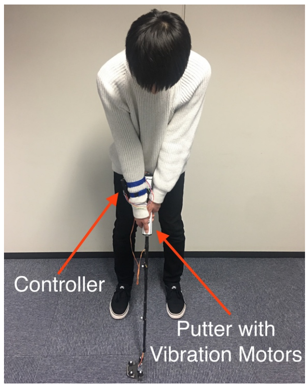


Fig. 6: Experimental setup

We evaluated the proposed conceptual mapping to convey tempo based on the PTS illusion through a user study, in order to determine whether users can understand the notion of tempo from the cues, and elicit the desired motion. The participants were 5 males (aged 22-31) with no history of physical or neurological disorders, and no previous golf experience. Subjects received a basic introduction about putting (how to hold grip, putt phases).

In order to determine whether the cues actually conveyed tempo information to the users in order to reproduce the desired motion, we had users perform under two conditions:

- Condition 1: Using cues where the change in $x_s(t)$ is represented using a simple sinusoidal function. The desired backswing time is $\tau_b = 1000\ ms$, and the downswing time $\tau_d = 1000\ ms$, which yields a tempo of $\tau_b/\tau_d = 1$.
- Condition 2: Using cues based on the change of $x_s(t)$ according to the ideal putt motion proposed in section V-A. The desired backswing time is $\tau_b = 600\ ms$, and the downswing time $\tau_d = 300\ ms$ conveying a tempo of $\tau_b/\tau_d = 2$.

A comparison between the functions to produce the vibrotactile feedback for each experimental condition can be seen in Fig. 7.

Users received an explanation about how to interpret the vibrotactile cues, and then practiced performing the putt while receiving the feedback 5 times before performing the motion. Next, users wore the controller (Raspberry Pi) on the wrist while holding the putter, as can be seen in Fig. 6.

Users were requested to perform the putting motion guided by the vibrotactile cue 20 times, resulting in 100 samples per condition. The 20 repetitions were performed sequentially, in intervals of 5 s for the first condition and 3 s for the second condition, with 2 s of separation between one

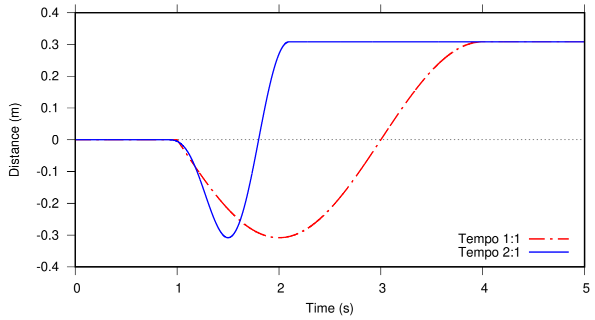
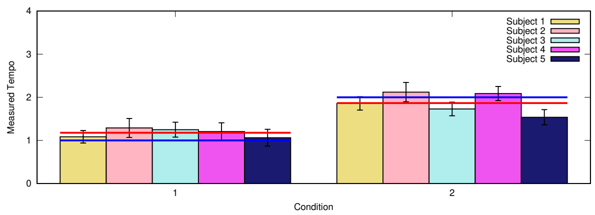
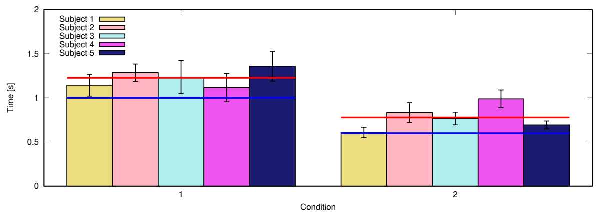


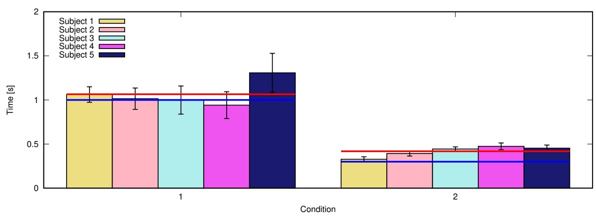
Fig. 7: Change of $x_s(t)$ for conditions 1 and 2



(a) Tempo

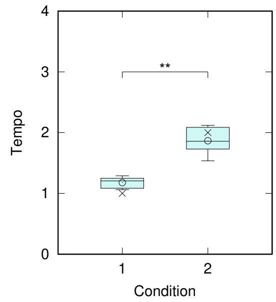


(b) Backswing Time

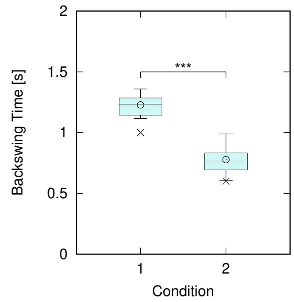


(c) Downswing Time

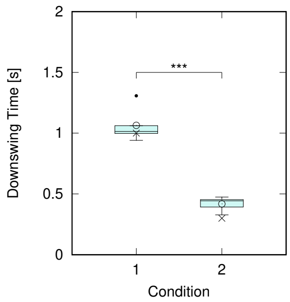
Fig. 8: Individual Subject (1-5) mean results for (a) tempo (b) backswing time and (c) downswing time under both conditions ($\tau_b/\tau_d = 1$ and $\tau_b/\tau_d = 2$). Error bars illustrate the standard deviation, the blue line indicates the desired value and the red line indicates the mean value from all users under each condition



(a) Tempo



(b) Backswing Time



(c) Downswing Time

Fig. 9: Distribution of (a) tempo (b) backswing time and (c) downswing time for each condition ($\tau_b/\tau_d = 1$ and $\tau_b/\tau_d = 2$). Blue boxes represent the interquartile range (IQR), which contains 25%-75% of the data (P_{25} and P_{75}). The line inside the boxes represents the sample median, and the cross and circle marks represent the desired and mean values, respectively. Points represent samples outside $\pm$1.5IQR. Asterisks represent the statistical significance

cue and the following. Each condition was performed on a different day, with more than one week of separation between performances, to avoid task-learning bias. For each measured putt stroke, we determined the backswing time τ_b, downswing time τ_d and tempo using a Finite State Machine approach with a heuristic based on the characteristics of the putting stroke.

1) Results: The average tempo, backswing time and downswing time exhibited by each user while following the vibrotactile cues are presented in Fig. 9 for each condition. Users averaged tempos of 1.178 using the cues conveying a tempo of 1, and 1.866 for the cues conveying a tempo of 2. This seems to indicate that users are able to follow the cues and reproduce the desired motion appropriately. The backswing and downswing times were also close to the values used to produce the cues, averaging $\tau_b = 1.228\ s, \tau_d = 1.064\ s$ for the first condition, and $\tau_b = 0.778\ s, \tau_d = 0.418\ s$ for the second condition.

The distribution of the tempo, backswing time and downswing time for both tasks is shown in Fig. 9. We performed Shapiro-Wilk tests, and none of the measurements deviate from normality ($p > 0.05$). We did a paired samples t-test for each characteristic (tempo, backswing time and downswing time) between the two conditions and found that the difference is statistically significant for all characteristics (tempo $p < 0.05$, backswing time $p < 0.001$, downswing time $p < 0.001$). This seems to indicate that the users are able to understand the vibrotactile cues and reproduce the putting motion with characteristics similar to those conveyed by the cues.

B. Angle Control

Next, we carried out a second experiment with the same five subjects to determine the effectiveness of the proposed vibrotactile cues to control the putter stroke length. Two

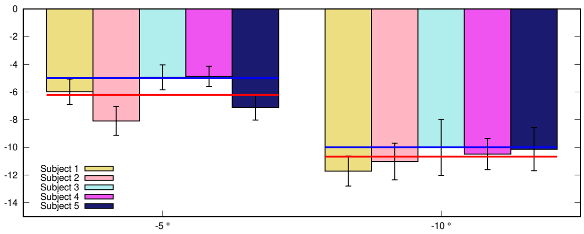


Fig. 10: Individual subject (1-5) mean results for backswing stroke length for $\theta_d = -5°, -10°$ respectively. Error bars illustrate the standard deviation, the blue line indicates the desired value and the red lines indicate the mean for all users for a given pattern

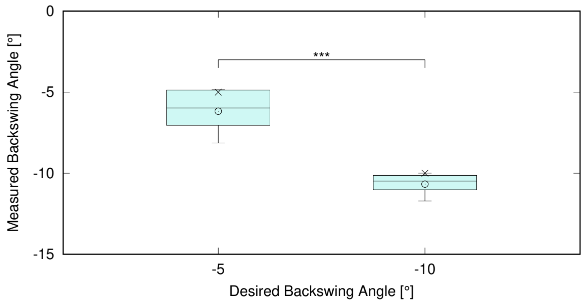


Fig. 11: Distribution of backswing angle for for $\theta_d = -5°, -10°$ respectively. Blue boxes represent the interquartile range (IQR), which contains 25%-75% of the data (P_{25} and P_{75}). The line inside the boxes represents the sample median, and the cross and circle marks represent the desired and mean values, respectively. Points represent samples outside ± 1.5IQR. Asterisks represent the statistical significance

different desired backswing angles $\theta_d = -5°, -10°$ were set, and users were instructed to try to elicit a tempo of 2, remembering its characteristics from the previous experiment. For the experiments, we controlled the angle $\theta(t)$ instead of controlling the stroke distance $x_s(t)$, as controlling the latter would imply a different θ_d depending on the golfer's characteristics (i.e., the length of the pendulum) for a desired backstroke length x_l.

Users performed 20 consecutive putting strokes for each θ_d, and a TTC was applied when the estimated angle $\hat{\theta}$ exceeded the desired angle θ_d. The results are shown in Fig. 10.

In average, users were able to produce stroke angles close to the desired angle θ_d, with averages of $-6.205°$ and $-10.669°$ respectively. The measurements didn't deviate from normality (Shapiro-Wilk test, $p > 0.05$), and a paired samples t-test showed statistical significance for θ_d ($p < 0.001$). This seems to indicate that users are able to use the produced TTC to determine the moment to transition from the backswing phase to the downswing phase, which allows them to control the stroke angle of the putting motion.

C. Discussion and Contributions

In this paper, we introduced vibrotactile cues to control the tempo and the stroke angle of the putt motion. The existing systems to train the putt motion provide either post-task performance metrics and KP or forcefully guide users through the motion by applying forces to the user/putter. However, the cues proposed in this paper just convey the information, and users are in charge of performing the motion, which in turn allows them to evaluate the proprioception and use these indicators to achieve motor learning. Moreover, we can adjust the parameters such as the stroke angle based on the environment information, which in turn, along a consistent tempo of 2, will enable users to control the distance traveled by the ball and thus master the skill of "touch".

In the first experiment, we could observe that the vibrotactile cues allowed novice golfers to elicit the desired tempo characteristics conveyed by the cue. Although non-expert golfers tend to naturally exhibit a tempo of 1 [35], users were able to putts with tempos close to both 1 and 2 by following the respective cues. We can observe that when performing under the second condition, Subject 5 presents a comparatively lower tempo. This might be caused by the predominance of the user's preexisting concept of the putting motion over the interpretation of the vibrotactile cues. A long-term experiment with multiple training sessions using the vibrotactile cues could give more insight into whether this personal preference can effectively be overcome by the cues.

Additionally, the results from the second experiment seem to indicate that the estimation made for the angle gave the users sufficient time to react to the TTC cues, as the measured stroke angles are quite close to the desired values. For $\theta_d = -5°$, we can see the average error is about $1.20°$, almost double as the average error for $\theta_d = -10°$ of $0.669°$. This is understandable as the desired stroke angle is quite small, so some users (e.g., Subject 2) overshot past the desired angle when performing the putt. This might be caused by slow reaction times to the vibrotactile cue, or a large initial backswing acceleration that caused the user to require more time to decelerate and end the backswing phase.

The experiments introduced in this section were measurements of the first time the users trained with the proposed approach. Nonetheless, results suggest that training the putting motion using cues based on the ideal motion allows players to control the temporal characteristics of their putt shots in the short-term. Long-term experimentation is required to evaluate the retention of the characteristics conveyed by the cues, and the impact in the putting performance of the users.

VII. CONCLUSIONS

In this paper, vibrotactile cues based on tactile illusions to convey motion tempo and timing were introduced as the first step of a framework to train golf by augmenting the user's senses using different types of feedback. Using these vibrotactile cues, subjects were able to perform putting motion under arbitrary tempos (1 and 2), and they also were able to control the angle of the backswing stroke following the produced vibrotactile cues.

We believe that training the putt motion using the proposed vibrotactile cues might help users achieve a consistent putt and master the skill of touch, but there are skills required in putting that we have not addressed. This framework needs to be complemented to train other important aspects of the putt, such as face aim, face impact angle and putt reading. Furthermore, we need to test whether using the proposed cues actually enable users to have a better distance control, and whether it's necessary to control additional parameters, such as the velocity of the user when driving the putt.

Finally, we want to extend the concept to utilize multimodal feedback (haptic, auditory and visual) in order to provide users feedback that goes beyond traditional methods, by using real world information such as the surface characteristics or the speed of the green.

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