

Non-Monotonic Lyapunov Function-Based Design of Decentralized Triggering Conditions in Output Feedback Event-Triggered Control

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Abstract—Event-triggered control is a method in which the measured values are sent from sensors to controllers only when an event-triggering condition is satisfied. In the case where the controller is given in advance, it is necessary to design event-triggering mechanisms (ETMs). In this paper, a design method of decentralized event-triggering mechanisms (DETMs) is proposed based on non-monotonic Lyapunov functions. Decentralized ETMs efficiently work for a sensor network in which multiple sensors are located in a distributed way. Using non-monotonic Lyapunov functions, it is expected that the number of times that the event occurs is decreased. The design problem of DETMs is reduced to an LMI (linear matrix inequality) feasibility problem. The effectiveness of the proposed method is presented by a numerical example.

I. INTRODUCTION

In networked control systems in which feedback loops are closed through a communication network [1], it is important to decrease the communication cost while maintaining the control performance. To achieve this purpose, various control methods such as time delay compensation have been widely studied. In particular, event-triggered control and self-triggered control are well known. In event-triggered control, the measured values are sent from sensors to the controller only when an event-triggering condition is satisfied [2]. A mechanism that judges an event-triggering condition, which is implemented in a sensor, is called an event-triggering mechanism (ETM). Self-triggered control is a method in which the next measurement time is calculated based on the current measured value [2], [3].

Roughly, there are three types of problems for event-triggered control, that is, (i) event-triggering conditions are given and the controller is designed (see, e.g., [4], [5]), (ii) the controller is given and event-triggering conditions are designed (see, e.g., [6], [7]), and (iii) both the controller and event-triggering conditions are designed (see, e.g., [8], [9]). In the type (i), event-triggering conditions are given based on the estimation/assumption of the communication cost. Under the assumption on errors that occur by event-triggering conditions, the controller is designed based on some control specification. In the type (ii), first, the controller is designed according to a conventional control method in which event-triggering conditions are ignored. After that, event-triggering conditions are designed. In the type (iii),

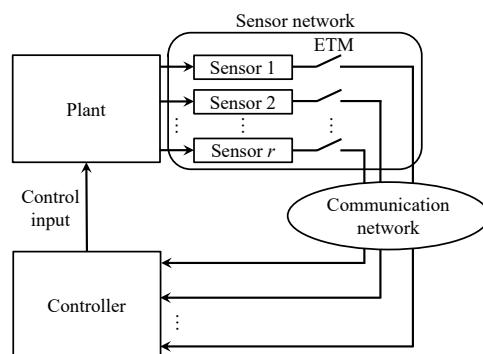


Fig. 1: Closed-loop system with a communication network.

both control and communication performances are considered. From the viewpoint of the control performance, it is desirable that the output is measured at each time. Then, the type (ii) is relatively easier than the other two types, because we may evaluate only the degradation of the control performance.

In many stabilization methods, the controller is designed such that the Lyapunov function is decreased monotonically. As more relaxed methods, a non-monotonic Lyapunov function approach has been developed (see, e.g., [10]). In this approach, stability of the closed-loop system is guaranteed if the Lyapunov function is decreased averagely. In event-triggered control, a non-monotonic Lyapunov function approach has been used in [11], [12]. However, in these methods, simple ETMs with only a scalar parameter are handled. To the best of our knowledge, this approach has not been used to design more general ETMs with various design parameters.

In this paper, based on a non-monotonic Lyapunov function approach, a design method of decentralized event-triggering mechanisms (DETMs) is proposed. Decentralized ETMs [13] are well known as an ETM in control systems over a sensor network in which multiple sensors are dispersively located (see Fig. 1). For example, in building control, various sensors such as thermometers, luxmeters, and human presence sensors are dispersively put in many places. In the conventional centralized ETMs, one sensor or the controller must aggregate all measured values at each time. By decentralized ETMs, this aggregation can be avoided. Moreover, using non-monotonic Lyapunov functions, it is expected that the number of times that the event occurs is decreased. We transform the design problem of DETMs into an LMI (linear matrix inequality) feasibility problem.

Notation: Represent by \mathcal{R} the set of real numbers. For

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the matrix M , denote by M^\top the transpose matrix of M . Represent by I_n and $0_{m \times n}$ the $n \times n$ identity matrix and the $m \times n$ zero matrix, respectively. Instead of $0_{m \times n}$ and I_n , the symbols 0 and I may be used, respectively. Represent by $M \succ 0$ that the matrix M is positive-definite. For the two matrices X and Y , represent by $X \otimes Y$ the Kronecker product of X and Y . Represent by $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ the symmetric matrix $\begin{bmatrix} A & B^\top \\ B & C \end{bmatrix}$.

II. PRELIMINARIES

First, we briefly review the outline of non-monotonic Lyapunov functions for discrete-time linear systems. For the discrete-time autonomous linear system $x_a(k+1) = A_a x_a(k)$, we consider the quadratic form $V(k) = x_a^\top(k) P_a x_a(k)$, where x_a is the state vector, A_a is a given matrix, $k \in \{0, 1, 2, \dots\}$ is the discrete time, and $P_a = P_a^\top \succ 0$. In [10], a non-monotonic Lyapunov function has been defined as follows.

Definition 1 ([10]): The quadratic form $V(k)$ is called a non-monotonic Lyapunov function if the following condition holds for any $k \in \{0, 1, 2, \dots\}$:

$$\sum_{i=1}^M \tau_i (V(k+i) - V(k)) < 0, \quad (1)$$

where $\tau_1, \tau_2, \dots, \tau_M$ are given non-negative scalars. The positive integer M is also given in advance.

The following result has been obtained in [10].

Lemma 1 ([10]): The system $x_a(k+1) = A_a x_a(k)$ is asymptotically stable if (1) holds for any $k \in \{0, 1, 2, \dots\}$.

As an example, consider the case of $M = 2$ and $\tau_i = 1$. Then, (1) is given as

$$(V(k+2) - V(k)) + (V(k+1) - V(k)) < 0,$$

that is

$$\frac{V(k+2) + V(k+1)}{2} < V(k),$$

which implies that V decreases averagely, and allows that V increases temporarily. See, e.g., [10] for further details.

Next, we summarize the \mathcal{S} -procedure technique. The following result is known as the \mathcal{S} -procedure [14].

Lemma 2 ([14]): For any real vector $\zeta \neq 0$, $\zeta^\top \Theta_0 \zeta > 0$ holds under $\zeta^\top \Theta_i \zeta \geq 0$, $i = 1, 2, \dots, r$ if there exist $\kappa_i > 0$ satisfying $\Theta_0 + \sum_{i=1}^r \kappa_i \Theta_i \succ 0$.

III. PROBLEM FORMULATION

In this section, first, a plant and a controller considered in this paper are explained. Next, a DETM is introduced. Finally, the design problem of this mechanism is formulated based on a non-monotonic Lyapunov function approach.

A. Mathematical Model of the Closed-Loop System

In this paper, we suppose that the mathematical model of the plant is given by the following discrete-time linear system:

$$\begin{cases} x_p(k+1) = A_p x_p(k) + B_p u(k), \\ y(k) = C_p x_p(k), \end{cases} \quad (2)$$

where $x_p(k) \in \mathcal{R}^{n_p}$ is the state of a given plant, $u(k) \in \mathcal{R}^m$ is the control input, $y(k) \in \mathcal{R}^r$ is the measured output, $A_p \in \mathcal{R}^{n_p \times n_p}$, $B_p \in \mathcal{R}^{n_p \times m}$, and $C_p \in \mathcal{R}^{r \times n_p}$ are given coefficient matrices/vectors, and $k \in \{0, 1, 2, \dots\}$ is the discrete time. Suppose that the sensor i measures the i -th element of $y(k)$, i.e., the number of sensors is given by r . Suppose also that each sensor is connected to the plant and the controller through a (wireless) communication network (see Fig. 1). In other words, we consider a sensor network composed of multiple sensors. Each sensor has an ETM. See the next subsection for details of ETMs. Since we focus on sensor networks, it is supposed that the controller is directly connected to the plant.

Let us consider the following output-feedback controller:

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c \hat{y}(k), \\ u(k) = C_c x_c(k) + D_c \hat{y}(k), \end{cases} \quad (3)$$

where $x_c(k) \in \mathcal{R}^{n_c}$ is the state of the controller. We suppose that $A_c \in \mathcal{R}^{n_c \times n_c}$, $B_c \in \mathcal{R}^{n_c \times r}$, $C_c \in \mathcal{R}^{m \times n_c}$, and $D_c \in \mathcal{R}^{m \times r}$ are given in advance. The vector $\hat{y}(k) \in \mathcal{R}^r$ is the input to the controller from sensors, and is defined as follows:

$$\hat{y}(k) := \begin{cases} y(k) & \text{if an event-triggering condition} \\ & \text{is satisfied for at least one sensor,} \\ \hat{y}(k-1) & \text{otherwise.} \end{cases} \quad (4)$$

In other words, $\hat{y}(k)$ implies the measured value that the controller has and manages. Defining

$$e(k) := \hat{y}(k) - y(k),$$

the closed-loop system composed of the plant (2) and the controller (3) is given by

$$x(k+1) = Ax(k) + Be(k), \quad (5)$$

where $x(k) = [x_p^\top(k), x_c^\top(k)]^\top \in \mathcal{R}^n$ ($n = n_p + n_c$) and

$$A = \begin{bmatrix} A_p + B_p D_c C_p & B_p C_c \\ B_c C_p & A_c \end{bmatrix}, \quad B = \begin{bmatrix} B_p D_c \\ B_c \end{bmatrix}.$$

B. DETMs

In this subsection, we introduce DETMs. To utilize a non-monotonic Lyapunov function, for the sensor $i \in \{1, 2, \dots, r\}$, we suppose that the following event-triggering condition, which is checked at time $k+M-1$ ($k = 0, 1, 2, \dots$), is given:

$$\begin{bmatrix} y_i(k) \\ \bar{e}_i'(k) \end{bmatrix}^\top \begin{bmatrix} Q_{11}^i & * \\ Q_{21}^i & Q_{22}^i \end{bmatrix} \begin{bmatrix} y_i(k) \\ \bar{e}_i'(k) \end{bmatrix} < 0, \quad (6)$$

where

$$\begin{aligned} \bar{e}'_i(k) &:= [e_i(k), e_i(k+1), \dots, e_i(k+M-2), \\ &\quad \hat{y}_i(k+M-2) - y_i(k+M-1)]^\top, \end{aligned}$$

and y_i and e_i are the i -th elements of y and e , respectively. The positive integer M is given in advance. If the event-triggering condition (6) is satisfied, then $\bar{e}'_i(k)$ is reset to a zero vector. The scalar Q_{11}^i , the vector $Q_{21}^i \in \mathcal{R}^M$, and the matrix $Q_{22}^i \in \mathcal{R}^{M \times M}$ are design parameters. We impose the constraint $Q_{11}^i \geq 0$. Hereafter, we may call (6) a DETM.

C. Design Problem of DETMs

Based on the concept of non-monotonic Lyapunov functions, we consider the design problem of the matrices Q_{11}^i, Q_{21}^i , and Q_{22}^i . As a candidate of non-monotonic Lyapunov functions, we consider

$$V(k) = x^\top(k)Px(k), \quad (7)$$

where $P = P^\top \succ 0$. For the inequality (1), we add the term to adjust the convergence rate of the Lyapunov function. We consider the following inequality:

$$\sum_{j=1}^M \tau_j (V(k+j) - V(k)) < -\beta V(k), \quad (8)$$

where β is a given parameter, and its range depends on M . For example, in the case of $M = 2$ and $\tau_j = 1$, we can obtain $(V(k+2) + V(k+1))/2 < (1 - \beta/2)V(k)$, where β satisfies $0 < 1 - \beta/2 \leq 1$. From Lemma 1, it is directly shown that the closed-loop system is asymptotically stable if (8) holds for any $k \in \{0, 1, 2, \dots\}$. Hereafter, (8) is called a non-monotonic Lyapunov inequality.

Finally, we formulate the following design problem of the DETM (6).

Problem 1: For the system (5), suppose that coefficient matrices A and B are given. For the non-monotonic Lyapunov inequality (8), suppose that the parameter M , τ_j , and β are given. Consider $V(k)$ of (7) as a candidate of non-monotonic Lyapunov functions. Then, find the matrices $P \succ 0$, $Q_{11}^i \geq 0$, Q_{21}^i , and Q_{22}^i satisfying (8).

In this problem, we design both the non-monotonic Lyapunov function $V(k)$ and the event-triggering condition (6) under the condition where the controller (3) is given. When (8) is satisfied, the closed-loop system (5) with the DETM (6) is asymptotically stable.

IV. LMI-BASED SOLUTION METHOD FOR PROBLEM 1

In this section, a solution for Problem 1 is derived. First, we consider rewriting the event-triggering condition (6). Next, we focus on the non-monotonic Lyapunov inequality (8). Finally, we reduce Problem 1 to an LMI feasibility problem.

A. Conversion of Event-Triggering Condition

First, when the event-triggering condition is given by (6), from (4) and $Q_{11}^i \geq 0$, the following condition is always satisfied:

$$\begin{bmatrix} y_i(k) \\ \bar{e}_i(k) \end{bmatrix}^\top \begin{bmatrix} Q_{11}^i & * \\ Q_{21}^i & Q_{22}^i \end{bmatrix} \begin{bmatrix} y_i(k) \\ \bar{e}_i(k) \end{bmatrix} \geq 0, \quad (9)$$

where $\bar{e}_i(k) := [e_i(k), e_i(k+1), \dots, e_i(k+M-1)]^\top$.

Defining $\bar{e}(k) := [\bar{e}_1^\top(k), \bar{e}_2^\top(k), \dots, \bar{e}_r^\top(k)]^\top$, we can obtain

$$\begin{aligned} \begin{bmatrix} y_i(k) \\ \bar{e}_i(k) \end{bmatrix} &= G_i \begin{bmatrix} x(k) \\ \bar{e}(k) \end{bmatrix}, \\ G_i &= \begin{bmatrix} G_i^1 & 0 \\ 0 & G_i^2 \end{bmatrix}, \\ G_i^1 &= [\text{Row}_i(C_p), 0_{1 \times n_c}], \\ G_i^2 &= [0_{M \times M(i-1)}, I_M, 0_{M \times M(r-i)}]. \end{aligned}$$

Then, (9) can be rewritten as

$$\begin{bmatrix} x(k) \\ \bar{e}(k) \end{bmatrix}^\top G_i^\top \begin{bmatrix} Q_{11}^i & * \\ Q_{21}^i & Q_{22}^i \end{bmatrix} G_i \begin{bmatrix} x(k) \\ \bar{e}(k) \end{bmatrix} \geq 0. \quad (10)$$

B. Conversion of Non-Monotonic Lyapunov Inequality

We consider the non-monotonic Lyapunov inequality (8). First, from (5), we can obtain

$$x(k+j) = A^j x(k) + \sum_{l=0}^{j-1} A^{j-1-l} B e(k+l).$$

Using this expression, we can obtain

$$\bar{x}(k) = \bar{A}x(k) + \bar{B}\bar{e}(k), \quad (11)$$

where

$$\begin{aligned} \bar{x}(k) &= [x^\top(k), x^\top(k+1), \dots, x^\top(k+M)]^\top, \\ \bar{A} &= [I \ A^\top \ (A^2)^\top \ \dots \ (A^M)^\top]^\top, \\ \bar{B} &= \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ B & 0 & & & \vdots \\ AB & B & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ A^{M-1}B & \dots & AB & B & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} I_r \otimes [1, 0, \dots, 0] \\ I_r \otimes [0, 1, 0, \dots, 0] \\ \vdots \\ I_r \otimes [0, \dots, 0, 1] \end{bmatrix}. \end{aligned}$$

Next, the non-monotonic Lyapunov inequality (8) can be rewritten as

$$\bar{\beta} x^\top(k)Px(k) - \sum_{j=1}^M \tau_j x^\top(k+j)Px(k+j) > 0, \quad (12)$$

where $\bar{\beta} = \sum_{j=1}^M \tau_j - \beta$. Using (11), the non-monotonic Lyapunov inequality (12) can be rewritten as

$$\begin{bmatrix} x(k) \\ \bar{e}(k) \end{bmatrix}^\top [\bar{A} \ \bar{B}]^\top \bar{P} [\bar{A} \ \bar{B}] \begin{bmatrix} x(k) \\ \bar{e}(k) \end{bmatrix} > 0, \quad (13)$$

where

$$\bar{P} = \text{block-diag}(\bar{\beta}P, -\tau_1 P, -\tau_2 P, \dots, -\tau_M P).$$

C. Reduction of Problem 1 to an LMI feasibility Problem

Based on the above result, we arrive at the following theorem as a main result.

Theorem 1: For the closed-loop system (5) with the DETM (6), suppose that $M, \tau_j \geq 0, \beta$ in (8) are given. The non-monotonic Lyapunov inequality (8) holds under the condition (9) if there exist the matrices $P = P^\top \succ 0, Q_{11}^i \geq 0, Q_{21}^i$, and Q_{22}^i satisfying the following LMI with respect to P, Q_{11}^i, Q_{21}^i , and Q_{22}^i :

$$[\bar{A} \quad \bar{B}]^\top \bar{P} [\bar{A} \quad \bar{B}] - \sum_{i=1}^r G_i^\top \begin{bmatrix} Q_{11}^i & * \\ Q_{21}^i & Q_{22}^i \end{bmatrix} G_i \succ 0. \quad (14)$$

Proof: Applying the \mathcal{S} -procedure (Lemma 2) to (10) and (13) yields the following condition:

$$[\bar{A} \quad \bar{B}]^\top \bar{P} [\bar{A} \quad \bar{B}] - \sum_{i=1}^r G_i^\top \begin{bmatrix} \kappa_i Q_{11}^i & * \\ \kappa_i Q_{21}^i & \kappa_i Q_{22}^i \end{bmatrix} G_i \succ 0,$$

where $\kappa_i > 0$ is a design parameter. Noting that Q_{11}^i, Q_{21}^i , and Q_{22}^i are design parameters, without loss of generality, $\kappa_i Q_{11}^i, \kappa_i Q_{21}^i$, and $\kappa_i Q_{22}^i$ are replaced with Q_{11}^i, Q_{21}^i , and Q_{22}^i . Thus, we can obtain the LMI (14). ■

Finally, we summarize the procedure of the proposed control method.

Procedure of event-triggered control using non-monotonic Lyapunov functions ($M \geq 2$):

Step 1: Set the current time t as $t = 0$. Generate the control input using initial information, and apply it to the plant. Update $t := t + 1$ and go to Step 2.

Step 2: If $t \geq M - 1$ holds, then go to Step 3, otherwise go to Step 4.

Step 3: Check whether the event-triggering condition (6) is satisfied or not. If (6) is satisfied, then go to Step 4, otherwise go to Step 5.

Step 4: Send the measured output from each sensor to the controller. Go to Step 5.

Step 5: Update the control input using (3). Update $t := t + 1$ and go to Step 2.

V. NUMERICAL EXAMPLE

We present a numerical example. First, we describe a problem formulation. Consider the coefficient matrices/vectors in the unstable plant that are given by

$$A_p = \begin{bmatrix} 0.95 & 0.2 & 0.6 \\ -0.5 & 0.5 & 0.3 \\ 0.6 & -0.2 & 0.5 \end{bmatrix}, \quad B_p = \begin{bmatrix} 0.17 \\ 0.15 \\ -0.18 \end{bmatrix},$$

$C_p = [I_2, 0_{2 \times 1}]$. The coefficient matrices/vectors in the stabilizing controller for the plant are given by

$$A_c = \begin{bmatrix} -2.91 & -0.31 & -1.29 \\ -2.42 & -0.24 & -1.36 \\ 2.60 & 0.20 & 2.48 \end{bmatrix}, \quad B_c = \begin{bmatrix} -1.20 & -0.27 \\ 0.41 & -0.53 \\ -0.78 & 0.14 \end{bmatrix},$$

$C_c = [15.54, 1.43, 11.06]$, and $D_c = 0_{1 \times 2}$.

In the non-monotonic Lyapunov inequality (8), the parameters M, τ_j , and β are set as $M = 2, \tau_1 = 0, \tau_2 = 1$, and

$\beta = 0.35$, respectively. That is, we consider the following non-monotonic Lyapunov inequality:

$$V(k+2) - V(k) < -0.35V(k).$$

Next, we present computation results. By solving the LMI (14) in Theorem 1, we can obtain

$$P = \begin{bmatrix} 1.39 & * & * & * & * & * \\ 0.04 & 0.10 & * & * & * & * \\ 0.90 & 0.06 & 0.99 & * & * & * \\ 1.20 & 0.18 & 0.91 & 1.43 & * & * \\ 0.18 & 0.04 & 0.09 & 0.21 & 0.05 & * \\ 0.80 & 0.12 & 0.97 & 1.04 & 0.12 & 1.11 \end{bmatrix},$$

and

$$\begin{bmatrix} Q_{11}^1 & * \\ Q_{21}^1 & Q_{22}^1 \end{bmatrix} = \begin{bmatrix} 0.38 & * & * \\ 0.28 & -0.21 & * \\ 0.01 & -0.06 & -0.20 \end{bmatrix},$$

$$\begin{bmatrix} Q_{11}^2 & * \\ Q_{21}^2 & Q_{22}^2 \end{bmatrix} = \begin{bmatrix} 0.06 & * & * \\ 0 & -4.40 & * \\ 0 & -0.53 & -0.10 \end{bmatrix},$$

where we used MATLAB with YALMIP [15] and MOSEK.

In the simulation, the initial states of the plant and the controller are given by $x_p(0) = [0.3, -0.3, 0.3]^\top$ and $x_c(0) = [0.1, 0.1, 0.1]^\top$, respectively. Figs. 2–5 show the time response of the plant state, the control input, and the event occurrence, respectively. From Fig. 2, we see that the state converges to zero. From Fig. 3 and Fig. 4, we see that the control input is changed not depending on the event occurrence (see also Fig. 1). From Fig. 4, we see that the update of the measured output is sometimes skipped. Fig. 5 shows the time response of the non-monotonic Lyapunov function. From Fig. 5, we see that although the non-monotonic Lyapunov function sometimes increases, it eventually converges to zero.

Thus, through this numerical example, we see that introducing the non-monotonic Lyapunov function can reduce the number of event occurrences while ensuring system stability.

VI. CONCLUSION

In this paper, a design method of DETMs was proposed based on non-monotonic Lyapunov functions. By solving an LMI feasibility problem, design parameters in DETMs can be derived. A numerical simulation validated that the proposed method is effective in inhibiting the event occurrence. The proposed method will expand the scope of applications of event-triggered control.

In future work, it is important to extend the proposed method to sensor and actuator networks in which multiple sensors and actuators are dispersively located. It is also important to apply the proposed method to practical systems such as networked robotic systems.

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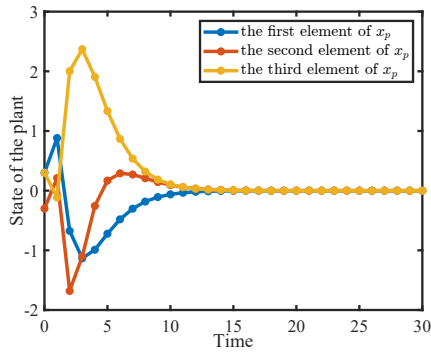


Fig. 2: Time response of the state of the plant.

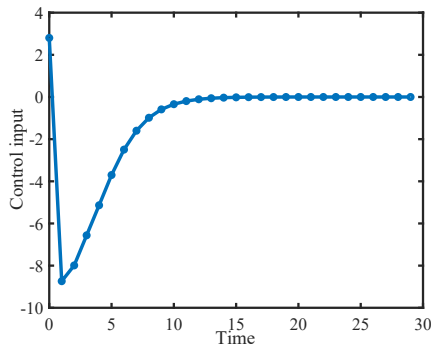


Fig. 3: Control input.

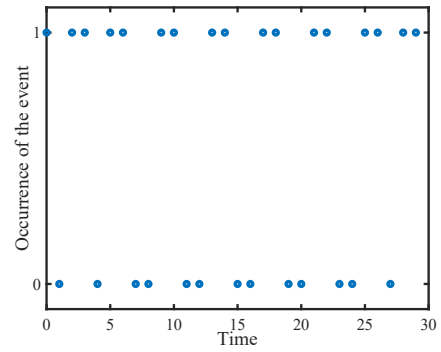


Fig. 4: Occurrence of the event. 1: the event occurs. 0: the event does not occur. The number of event occurrences is 13.

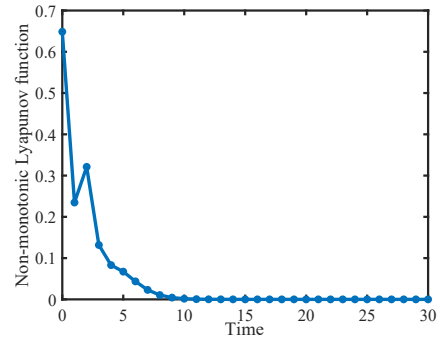


Fig. 5: Time response of non-monotonic Lyapunov function.

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