

# Dynamics-based Feedforward Control of CNC-controlled Robots using Digital Twins from Virtual Commissioning with Extended Dynamical Behavior

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**Abstract**—Conventional CNC controllers, originally developed for machine tools, are increasingly used to control robotic systems. However, their decentralized axis structure and kinematic abstraction limit dynamic performance compared to dedicated robot controllers. To address this limitation, this paper presents a methodology for integrating dynamics-based feedforward control within CNC-controlled robots by reusing digital twins from virtual commissioning. The approach extends existing virtual models with inverse dynamics through standardized interfaces and deploys them directly on the CNC controller as real-time digital twins. These models generate feedforward torque signals that complement conventional feedback control, improving trajectory tracking without modifying the CNC architecture. The main contribution lies in a generalizable workflow that links simulation, commissioning, and operation within a unified framework. The method is validated experimentally on a Delta robot, demonstrating significant reductions in tracking error and showing that the approach can be readily adapted to other CNC-controlled robotic systems.

## I. INTRODUCTION AND STATE-OF-THE-ART

The increasing labor costs and shortage of qualified personnel are drivers for an increasing degree of automation in industrialized countries. Many material handling tasks are carried out by industrial robots.

Instead of opting for an out-of-the-box solution for an industrial robot, machine manufacturers are increasingly choosing to integrate so called controller-independent robots. In this case, manufacturers buy the robot hardware including the gearboxes from an independent manufacture. They then select the industrial drives to power the robot. Lastly, they integrate the robot into their preferred control system, typically a computerized numerical control (CNC). The advantages over using standard industrial robots controlled by a robot controller (RC) is that only one unified control system is used. Hence, there is no need for an interface between the main controller and the RC. Further, the robot can be programmed using the same programming language as the machine, and thus fewer programmers are required to program and service the machine.

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However, the disadvantages are that CNC controllers typically originate from machine tool applications and are therefore less suited for dynamic robot control. Although CNC-based robot control is possible, it lacks the performance of dedicated robot controllers [1]. This limitation arises because CNC systems use purely kinematic abstractions and separate axis controllers rather than a unified, dynamics-based control strategy. While most CNC controllers and industrial drives technically allow the implementation of advanced control schemes, their practical use remains rare, as doing so requires interdisciplinary expertise across control theory, mechanical dynamics, and software engineering.

This paper addresses this gap by introducing a novel methodology that enables dynamics-based feedforward control within CNC-controlled robots through the reuse of digital twins developed during virtual commissioning. The proposed contribution is not a new control law but a workflow that integrates existing dynamic modeling methods into industrial CNC environments in a systematic and deployable manner. In the presented approach, virtual commissioning models are extended with inverse dynamics representations via standardized interfaces. These enhanced digital twins are then deployed to the controller to compute feedforward torque values in real-time, where they complement the conventional axis feedback control.

This integration of dynamics-based feed forward control schemes, digital twin technology, and CNC-based robot control represents the main technical contribution of this work. It provides a practical and scalable method for improving the motion accuracy of CNC-controlled robots without requiring users to manually implement or tune complex dynamic models. The following section presents the detailed methodology and implementation of this workflow.

### A. Virtual commissioning

Virtual commissioning (VC) refers to the initial development and testing of control programs for industrial controllers, such as programmable logic controllers (PLCs) or CNCs, using a simulation model, which avoids the necessity of a physical machine during the early stages of development and commissioning [2], [3]. This approach enhances the efficiency and quality of the control programs at the start of production. Typically, a real-time simulation model of the machine interfaces either with an emulated control system in Software-in-the-Loop Simulation (SiLS) or directly with

the real control system through a fieldbus in Hardware-in-the-Loop Simulation (HiLS) scenarios [4].

Virtual commissioning primarily focuses on validating the basic kinematics of manufacturing systems and testing automation solutions across various functionalities, specific scenarios and performance benchmarks. The primary focus of VC is on PLC validation and optimization rather than dynamic process simulation and energy analysis [2]. However, the simulation models can be expanded to include system dynamics, so that VC can be applied for tasks such as tuning control parameters [5].

### B. Digital Twins and Online Simulation

Digital twins have evolved as a collective term in manufacturing for various applications such as simulation models or decision models which stand in some relation to a physical object. While a standardized definition does not exist, Kritzing et al. [6] performed a comprehensive literature review in the area of manufacturing and classified a digital twin by its level of integration: a digital twin has bidirectional data exchange with the physical object, meaning that it not only mirrors the physical object but also influences it.

Another aspect of digital twins, detailed by Kapteyn and Willcox [7], is that they provide a comprehensive representation of a the physical object and can be used for multiple use cases such as decision-making, predictive maintenance, and advanced control strategies.

Online simulation builds upon this concept by coupling simulation models with live operational data from physical systems [8], [9]. This enables real-time monitoring, virtual sensing, and predictive analysis of future states. In most industrial applications, online simulations are used passively—to provide operators with insights rather than actively influencing the system. Only recently digital twins from virtual commissioning have started to directly influence the real system, such as in Klingel et. al. [10] where the collision detection is used to trigger a feed-stop of a robot or in Pfeifer et. al. [11] where the digital twin is used for online trajectory planning for robots.

Simulation models developed during virtual commissioning can be leveraged as digital twins in online simulation, helping to reduce the time and effort required to create the models. However, since virtual commissioning typically emphasizes only the system's kinematic aspects, these models often require enhancement or extension to meet the specific requirements of online simulation applications.

### C. CNC

A CNC controller is part of an industrial control system consisting of a Human-Machine-Interface (HMI) a PLC and a fieldbus interface. While the PLC performs the logic control, the CNC generates the motion of a machine or robot. Typically, a CNC receives its instructions in form of a NC-program. The CNC consists of an asynchronous part which is executed according to the available computing power, and a synchronous part, which is executed in every control cycle. The architecture of a CNC controller is illustrated in Fig. 1.

During the asynchronous path preparation part, the NC program is converted into a path and the trajectory is planned. Then, the trajectory is interpolated and transformed from TCP-coordinates into joint coordinates in the synchronous path interpolation part, before the interpolated values are sent to the drives via the fieldbus interface [12].

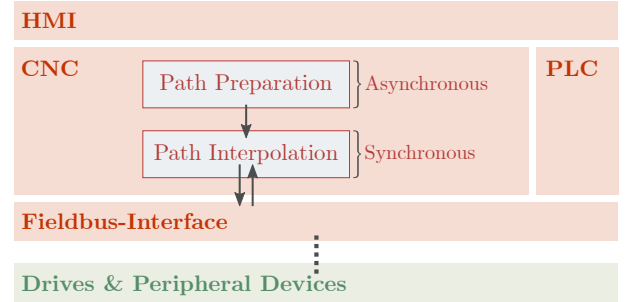


Fig. 1. Schematic components of a CNC controller with asynchronous and synchronous tasks, based on [12]

### D. Dynamics and Feedforward Control

The dynamics of a robot determine its possible performance and need to be taken into account for efficient and performant operation. Most robots have a tree-like structure and for many applications they can be modeled as rigid multibody systems. There are different methods to derive the dynamics of rigid multibody systems, such as the Newton-Euler method or Lagrange method [13], [14]. The dynamics can be written as the nonlinear equations of motion:

$$M(q) \cdot \ddot{q} + k(q, \dot{q}) = h(q, \dot{q}) + \tau_M. \quad (1)$$

$M$  represents the mass matrix,  $k$  the vector of the generalised Coriolis, centrifugal and gyroscopic forces and  $h$  are external forces such as the gravitational force and friction.  $\tau_M$  are the motor torques acting on the joints and  $q$  are the generalized coordinates, typically corresponding to the joint angles of the individual robot links.

A control method which is well established in robotics and considers the nonlinear dynamic behavior of a system is feedforward nonlinear control [15]. The use of a two degrees of freedom control strategy, consisting of a feedforward controller and a feedback controller, allows for better tracking behavior and more robust tuning of the feedback controller. The principle of dynamics-based feedforward control is to calculate the feedforward torques  $\tau_{ff}$  required for a specific motion. Therefore, the equations of motion are converted to an inverse dynamics form:

$$\tau_{ff} = M(q_d)\ddot{q}_d + k(q_d, \dot{q}_d) - h(q_d, \dot{q}_d). \quad (2)$$

The subscript "d" denotes that these are desired values of the planned trajectory. Another advantage of this method is that it can be computed outside of the control loops at a different cycle time than the servo controllers of industrial drives [15]. The block diagram of the control structure is

displayed in Fig. 4. The feedforward control generates the feedforward torques  $\tau_{ff}$  to control for the known behavior of the system described by the dynamics model while the feedback controller compensates disturbances and unmodeled behavior with the feedback torques  $\tau_{fb}$ . Together they comprise the motor torques  $\tau_M$ .

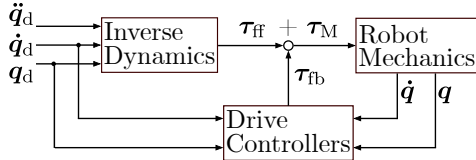


Fig. 2. Schematics of the control of a robot comprised of individual axis controllers and centralized, dynamics-based feedforward control, based on [15]

### E. Paper outline

This paper is structured as follows: In section II the method of integrating the inverse dynamics model into the digital twin and deploying it onto the controller as feedforward control is presented. Then the method is validated by implementing the digital twin-based feedforward control on a real Delta robot in section III. Finally, the results and method are discussed in the conclusion and an outlook on future work is given.

## II. METHOD

A virtual commissioning platform is used as a basis for digital twins for feedforward control as it offers significant advantages. The digital twins can be computed in real-time and offer interfaces to industrial control platforms. Since digital twins from virtual commissioning neglect the dynamic behavior, this section presents a method to integrate dynamics models into the digital twins. Further, a method is presented to deploy the digital twin to an industrial controller to provide feedforward control to complement the robot's axis controllers. The focus is set on a user-friendly implementation considering the users' know-how.

### A. Digital Twins from Virtual Commissioning extended with Dynamics Models

Although the architecture of virtual commissioning platforms satisfies many requirements, the digital twins typically lack the capability to simulate the dynamic behavior necessary for implementing feedforward control. Therefore, to enable this functionality, the feedforward control model must be integrated into the digital twin, as illustrated in Fig. 3.

This integration spans multiple engineering disciplines, including dynamics, software engineering, and industrial control engineering. Dynamics models, often formulated in specialized tools for (elastic) multibody systems, are exported and imported into virtual commissioning software as black box models. To simplify this integration, standard interfaces such as the Functional Mock-up Interface (FMI) [16] are used. This allows the use of discipline-specific

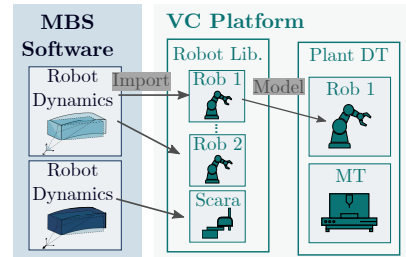


Fig. 3. Schematics of integrating dynamics models into digital twin libraries which then are available for industrial control engineers to model the digital twins of production plants without extensive know-how or effort

design methods to create black-box models which are used in VC so that industrial control engineers can employ digital twins without extensive knowledge of dynamics or software engineering. Black-box models can be parameterized to fit different robots by adapting kinematic and dynamic parameters, such the link lengths and masses. This method allows existing robot libraries in virtual commissioning tools to be extended with dynamics models, enabling feedforward control functionality in the digital twins. This enhances usability as the models can be adapted, exchanged and deployed without requiring extensive effort or know-how.

The approach reuses existing virtual commissioning models and enriches them into functional digital twins. In line with the concept of Kapteyn and Willcox [7], these enhanced models extend beyond single-use simulations and provide multiple capabilities of the physical system depending on the required use case. Digital twins which are used for virtual commissioning can then be re-used for feedforward control by deploying them the controller after the commissioning phase, streamlining modeling efforts and saving development time.

### B. Deployment of the Digital Twin to the Industrial Controller

In order for the digital twin to run on the industrial controller, it is compiled and integrated into the control project. There, it is connected with the CNC control and fieldbus interface via the internally available variables. The inputs to the digital twin are the desired positions  $p_d$ ,  $q_d$  and velocities  $\dot{p}_d$ ,  $\dot{q}_d$  for both the robot's TCP and joints and the digital twin's outputs are the feedforward control signals  $I_{ff}$ . As acceleration values are not available on all controllers, the digital twin performs a numerical derivation of the velocity signals to obtain these signals.

The setup of the digital twin and controller during the robot's operation is depicted in Fig. 4. The digital twin is computed at the same cycle time as the CNC and at each time step it receives the input values from the CNC and writes the output values to via the fieldbus interface to the drives. The drives typically contain a cascade controller with an inner current controller calculating a feedback current  $I_{fb}$  to actuate the drive [12]. The feedforward torques  $\tau_{ff}$  calculated by the digital twin are converted to feedforward currents  $I_{ff}$  by considering the behavior of the motors. The

feedforward currents are added to the feedback currents  $I_{fb}$ , resulting in a total current acting on the motor  $I_M$  which in turn generates the torques actuating the robot's links to perform the desired motion.

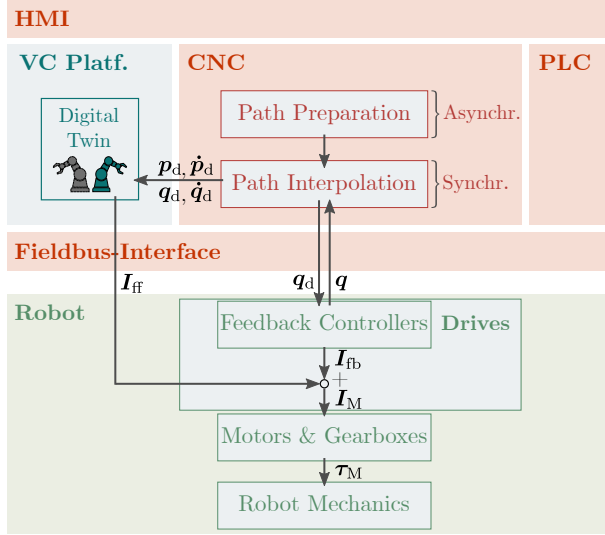


Fig. 4. Schematics of digital twin-based feedforward control

### III. IMPLEMENTATION AND VALIDATION

#### A. Experimental Setup

The methods presented in section II are validated on a delta robot setup provided by a machine manufacturer as depicted in Fig. 5. The delta robot is an Autonox DELTA RL4-1200-3kg robot and it is equipped with Beckhoff AM8042-0F21 motors and AX5000 inverters. The robot is connected to the Beckhoff TwinCAT [17] control platform via EtherCAT fieldbus. The user interacts with the system via the machine builders own HMI. The control cycle is 2 ms.

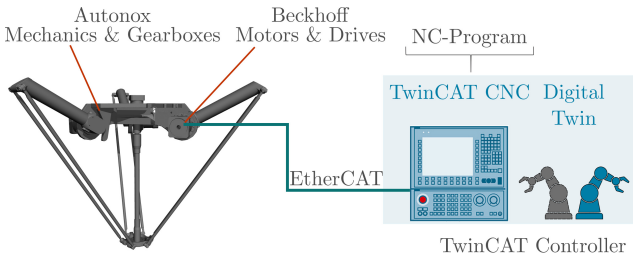


Fig. 5. Experimental setup of the delta robot and the controller with CNC and digital twin

For the validation the robot performs a typical pick-and-place motion, which was specified by the machine manufacturer. The desired trajectory is depicted in Fig. 6 for the cartesian coordinates of the TCP and the joint angles. The robot moves from its starting position to an initial position (ca. 0.3 s), then to the material pick-up point (ca. 0.8 s) and finally to the material drop-off point (ca. 1.4 s). The motion is performed with a weight of 3.2 kg permanently attached to the TCP, symbolizing a gripper and a workpiece.

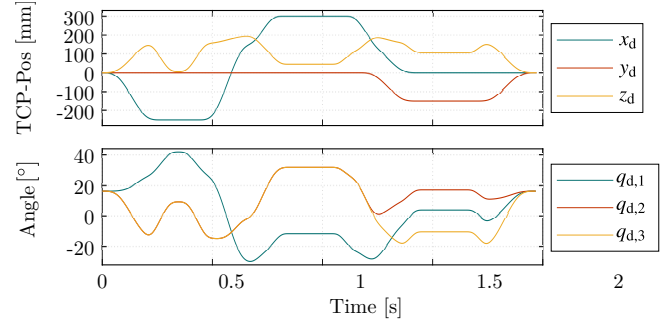


Fig. 6. Desired trajectory of the robot's TCP and joints

#### B. Inverse Dynamics Model and Implementation of the Digital Twin

The dynamics model of the delta robot is modeled according to the method of Asadi et Heydari [18]. This method has the advantage that it generates an explicit analytical dynamics model, similar to Eq. 1, over other methods which rely on additional algebraic equations as loop closure conditions:

$$M(p, q) \ddot{p} + C(p, q, \dot{p}, \dot{q}) \dot{p} + G(p, q) - F_P = J^T(p, q) (\tau). \quad (3)$$

In this notation,  $M$  represents the mass matrix,  $C$  the matrix of velocity dependent forces,  $G$  the matrix of gravitational forces and  $F_P$  other external forces acting on the platform.  $\tau$  corresponds to the motor torques. It is standing out, that the equations of motion are written in terms of the actuated joint angles  $q$  and the cartesian coordinates of the TCP  $p$ , with their respective derivatives. The relation between the joint angle and the TCP coordinates is obtained via the Jacobian  $J$ :

$$\dot{q} = J(p, q) \dot{p}.$$

This modeling technique introduces some simplifications to reduce the time to compute the model, such as neglecting joint friction. Further, the lower arms are comprised of two links which are simplified to three point masses: one at each joint and one in the center of gravity. The fourth axis is modeled as an additional mass on the TCP.

From these equations the inverse dynamics equations can be derived by solving them for the feedforward torques  $\tau_{ff}$ :

$$\tau_{ff} = J(p, q)^{T^{-1}} \cdot (M(p, q) \ddot{p} + C(p, q, \dot{p}, \dot{q}) \dot{p} + G(p, q) - F_P). \quad (4)$$

The dynamics and inverse dynamics equations from Eq. 3 and 4 were first written by a dynamics expert in Matlab [19] from where they were imported into the virtual commissioning software ISG-virtuos [20] as a parameterizable black-box model. It is integrated into the digital twin of the delta robot which includes other disciplines such as the visualization and collision detection. With this method, the VC engineer does

not require extensive know-how about dynamics modeling as the dynamics are part of the digital twin. The kinematics and dynamics parameters are provided by the robot manufacturer.

The digital twin is then compiled for the control platform, where it runs synchronously and is executed every control cycle. The digital twin reads the desired positions and velocities of the actuated joints and the TCP platform from the CNC controller and performs a numerical derivation to calculate the accelerations. Next, the feedforward torques  $\tau_{ff}$  are calculated according to Eq. 4 and then converted to the current values  $I_{ff}$  that the Beckhoff drives expect. Therefore, the torque values are multiplied by the gear ratios  $i$ , converted to current values using a linear motor model with the motor constant given by the Beckhoff drive parameter P-0-0092 and finally scaled to per mille values using the following relationship:

$$I_{ff} = \frac{\tau_{ff} \cdot 1000}{P-0-0092 \cdot i} \quad (5)$$

These feedforward currents are then transmitted to the drives via the fieldbus where they are added to the feedback currents, performed at a lower cycle time of 0.25ms.

### C. Results

The motion is performed without and with active feedforward control and measurements of joint positions and tracking errors are taken using the TwinCAT measurement project function directly in the control platform. The tracking errors for the three actuated joints are plotted in Fig. 7 for the measurement with active feedforward control  $e_{ff,1-3}$  and with solely the feedback controller  $e_{fb,1-3}$ .

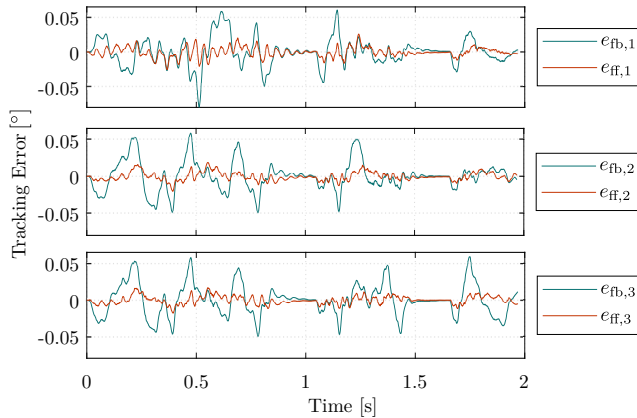


Fig. 7. Tracking error of the robot joints 1-3 using the original feedback controller  $e_{fb,1-3}$  and the digital twin based feedforward control combined with the feedback controller  $e_{ff,1-3}$

For all joints the use of the digital twin-based feedforward control strongly reduces the tracking error. The mean errors and the error reduction in percent is listed in Tab. I.

Since in most applications the TCP-accuracy is of importance the kinematic model of the digital twin is used to calculate the tracking error at the TCP. This method assumes perfect rigidity of the structure of the robot and the robot's links being manufactured exactly to specification.

Joint	$e_{fb,1-3,mean}$ [°]	$e_{ff,1-3,mean}$ [°]	Reduction [%]
M1	0.7917	0.2887	63.52
M2	0.7834	0.2420	69.11
M3	0.9254	0.2355	74.56

TABLE I  
COMPARISON OF THE MEAN TRACKING ERRORS AT THE THREE JOINTS

Fig. 8 illustrates the euclidian distance from the desired TCP-position, also demonstrating an improvement in accuracy. This is also reflected in the mean tracking errors at the TCP shown in Tab. II.

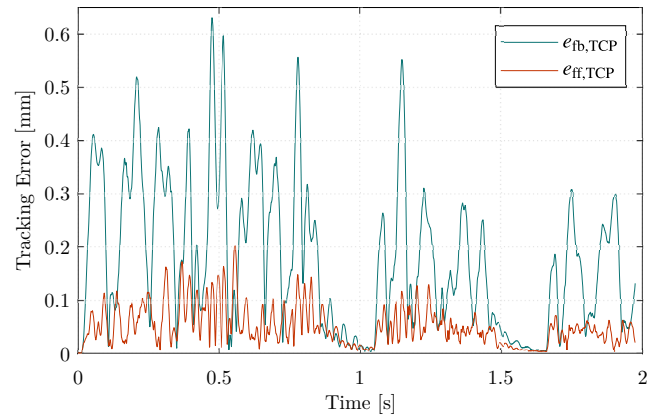


Fig. 8. Tracking error of the TCP using the original feedback controller  $e_{fb,TCP}$  and the digital twin based feedforward control combined with the feedback controller  $e_{ff,TCP}$

	$e_{fb,TCP,mean}$ [mm]	$e_{ff,TCP,mean}$ [mm]	Reduction [%]
TCP Dist.	0.1780	0.0512	71.25

TABLE II  
COMPARISON OF THE MEAN TRACKING ERRORS AT THE TCP

## IV. CONCLUSION AND OUTLOOK

This paper presents a novel approach to extending virtual commissioning models with inverse dynamics models to use them as digital twins for the feedforward control of a CNC-controlled robots. This method was implemented in a virtual commissioning platform to leverage the platform's functions such as real-time capability and to re-use the dynamics-based digital twins in a user-friendly without requiring dynamics know-how from the industrial control engineers.

The proposed method was validated on an experimental setup with a TwinCAT CNC controller and an Autonox Delta robot performing a pick-and-place motion while the digital twin runs on the controller, interacting with the synchronous CNC tasks and sending feedforward torque values to the robot's drives.

The results show that this method helps improve the tracking accuracy of the robot significantly, reducing the

mean tracking error at the joints by over 64 % and at the TCP by over 70 %. The latter number will be further determined with more precision, e.g. by using cameras to track the TCP-motion, as so far the TCP motion was calculated from the axis motion assuming a perfectly stiff structure. Nevertheless, the results demonstrate the significant potential of using digital twins to improve the control accuracy of CNC controlled robots. Increased control accuracy is important for applications such as picking materials from a moving conveyor where the robot has to move as accurately as possible.

This method can not only be applied to the specific robot used in this research but it is scalable to different delta robots by adapting the parameters of the digital twins. Further, it can be extended to different robot types, such as six axis robots or scara robots, by implementing inverse dynamics models for these types.

For future work, this area offers significant potential for research, particularly through the use of automatic model-generation techniques to create digital twins for entire robot libraries. We intend to demonstrate how this approach can simplify the application of advanced control methods across different types of CNC-controlled robots in industrial environments. Further, the model used as inverse dynamics model can be improved by e.g. taking into account friction behavior. Also, other types of model-based control could be explored, such as computed-torque control or adaptive control.

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