

Exploring Perceptual Effects of Phase Spectra in Vibrotactile Rendering

Takumi Kuhara¹, Akifumi Kawai¹, Yuto Inoue¹, Hikari Yukawa¹, and Yoshihiro Tanaka^{1,2}

Abstract—Tactile information is known to be used and integrated into various fields, such as robotics, virtual reality, and healthcare, to improve immersion, telepresence, performance, and substitute for different sensations. Vibrotactile stimulation is a widely used haptic modality, owing to its ease of modification and integration into existing interfaces. Many studies have focused on the characteristics of the magnitude spectrum in the frequency domain. We focus on the phase spectrum as a novel parameter to modify for a wider variety of rendered vibrotactile stimuli. In this study, we evaluated the effect of the phase spectrum on the tactile perception by generating Noise-Texture chimeras, which consists of the magnitude spectrum of colored noise and the phase spectrum of measured skin vibration for tracing certain textures. The results demonstrated that the phase spectrum is crucial to the tactile perception as well as the magnitude spectrum, indicating the possibility of rendering various vibrotactile textures with the usage of the phase spectrum and colored noise.

I. INTRODUCTION

The sense of touch comprises multiple modalities – pressure, thermal, and vibrotactile. Rendered tactile stimuli are widely used in virtual reality, robotics, and medical applications to boost immersion, manipulation performance, and telepresence. As an example for the usage of tactile information in virtual reality, Jayasiri et al. [1] demonstrated a string-based haptic interface, named SPIDAR, to provide the sense of force. Shirota et al. [2] showed that presenting thermal stimuli to the face using a head-mounted display and vibrotactile stimuli to the feet can induce a feeling as if the user is taking a shower. Although multiple interfaces and studies have explored the application of rendered pressure and thermal stimuli, these approaches typically require rigid and bulky hardware, which is often difficult to control and integrate. In contrast, vibrotactile stimuli are widely adopted for conveying tactile information owing to their ease of modulation and integration with existing systems. The applications that present vibrotactile stimuli are broadly expanding and can be categorized into two primary domains. One major area is the enhancement of virtual and remote interactions. For example, O'Malley et al. [3] demonstrated that presenting vibrotactile feedback in a virtual environment can improve not only the user's sense of presence but also their performance in manipulation tasks. Naqash et al. [4] showed that providing vibrotactile feedback for robot manipulation improves the performance in detecting slip motion.

A second significant domain is skill acquisition and sensory augmentation. Research has shown that vibrotactile feedback can accelerate motor learning. For instance, Yukawa et al. reported that presenting skin-propagating vibrations from an expert's fingertip to a novice improved the beginner's learning speed of a particular skill [5]. Wei et al. [6] demonstrated that for visually impaired people, providing vibrotactile stimuli allows them to understand their spatial surroundings.

For both the rendering of vibrotactile stimuli and to elucidate the perceptual principles for our sense of touch, many studies have been conducted focusing on the magnitude spectrum obtained by conducting a Fourier transform. For example, Bensmaïa et al. [7] showed that the perceived intensity is related to the power spectrum weighted by the sensitivity of Pacini bodies. Tozuka et al. [8] discussed that the regression model using the extracted factors from the magnitude spectrum can be linked to physical characteristics. Heravi et al. [9] demonstrated that the time-acceleration waveform can be reconstructed by deep learning using the magnitude spectrum as input.

However, our sense of touch is a spatiotemporal perception. We are capable of perceiving the duration and the transition of magnitude throughout the entire vibrotactile stimulus. Prior studies have shown that the duration can not only present continuous transitions but can also affect our tactile perception. Bocheureau et al. [10] have demonstrated that the duration of Gabor waveforms affects the perceived intensity of the waveform. Kuhara et al. [11] demonstrated that not only the duration, but also the decaying shape of sinusoidal waveforms, also affects the perceived intensity. These studies indicate that there is a potential factor in the temporal component that we can modify for rendering broad vibrotactile stimuli.

We propose using the phase spectrum obtained from conducting a Fourier transform on signals as a novel parameter to modify for rendering vibrotactile stimuli. In the auditory field, a common methodology is to create chimeric stimuli by exchanging the temporal envelope of one sound with the temporal fine structure of another. This technique allows researchers to evaluate the distinct perceptual contributions of each component [12][13][14][15]. Thus, in this study, we created Noise-Texture chimeras (N-T chimeras) to investigate how the phase spectrum influences the perception of a controlled magnitude spectrum. To do this, we generated 30 unique Noise-Texture (N-T) chimeric stimuli by combining the magnitude spectra from five different colored noises (Violet, Blue, White, Pink, and Brown) with the phase spectra from six different skin-propagating vibrations recorded

¹Takumi Kuhara, Akifumi Kawai, Yuto Inoue, Hikari Yukawa, and Yoshihiro Tanaka are with the Department of Electrical and Mechanical Engineering, Graduate School of Engineering, Nagoya Institute of Technology, Japan t.kuhara.538@nitech.jp

²Yoshihiro Tanaka is also with Inamori Research Institute for Science, Japan

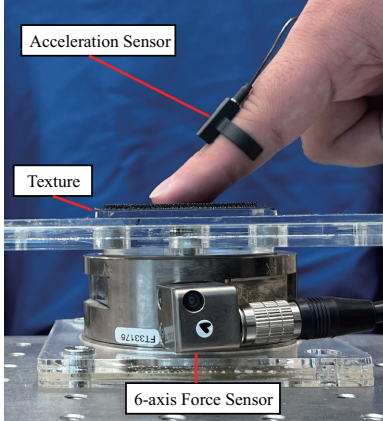


Fig. 1: Setup for measuring the skin-propagating vibration when tracing texture; the texture was mounted on an acrylic plate fixed on the 6-axis force/torque sensor

while tracing textures. We evaluated the perceptual effect of the phase spectrum by measuring the perceived similarity between the N-T chimeras and the original skin-propagating vibrations.

II. EXPERIMENTAL SETUP

A. Stimuli

For generating the stimuli, we first prepared five different colored noises and six skin-propagating vibrations when tracing textures. The colored noise was generated using an open-sourced Python library, `colorednoise`, which generates various noises based on an algorithm by Timmer et al. [16] as shown below

$$S(f) \propto 1/f^\beta, \quad (1)$$

where $S(f)$ represents the power spectral density, which is proportional to the frequency f , and β represents the exponent of the formula. We generated 1 s of violet noise ($\beta = -2$), blue noise ($\beta = -1$), white noise ($\beta = 0$), pink noise ($\beta = 1$), and brown noise ($\beta = 2$) with a sampling rate of 10 kHz. Fig. 2 shows the difference of the magnitude spectrum in the frequency domain as a logarithmic graph for the generated noise. As for the skin-propagating vibration, the textures we used were leather, mesh fabric, satin fabric, suede fabric, and the hook and loop sides of Velcro tape. Skin-propagated vibrations were measured using a ring-mounted accelerometer (Showa Sokki, 2302B) positioned between the first and second joints of the dominant index finger as shown in Fig. 1. The textured sample was mounted to a 6-axis force/torque sensor (ATI Industrial Automation, Gamma). One participant traced it using the index finger for 1 s at 80 mm/s while maintaining a normal force of 0.5 N. Fig. 3 shows the magnitude spectrum and phase spectrum of the measured skin-propagated vibration in the frequency domain for each traced texture. For all of the prepared colored noise and measured vibration, we conducted the Fast Fourier Transform.

Next, to generate the Noise-Texture chimeras (N-T chimera), we integrated the magnitude spectrum obtained

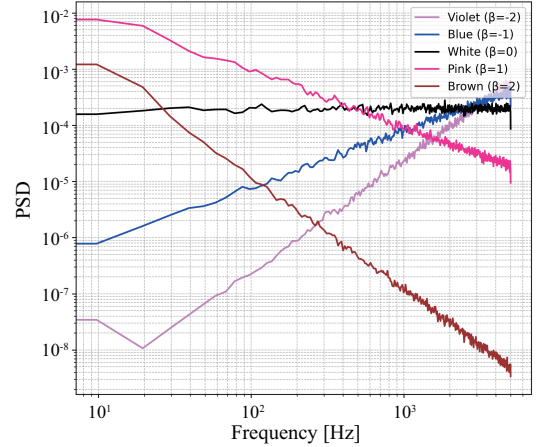


Fig. 2: Magnitude Spectrum of the generated colored noise: The color of the line corresponds to each colored noise, and the β of each color stands for the slope of the relationship between frequency and the PSD

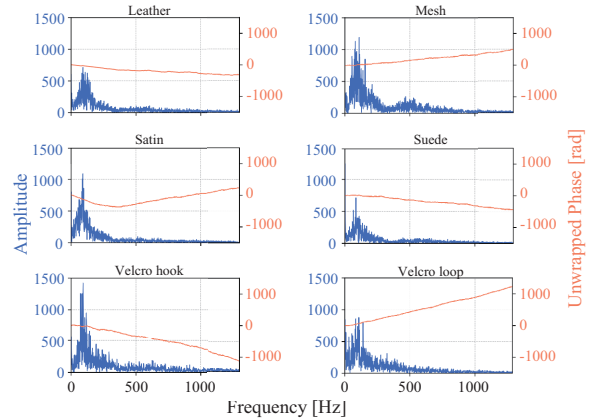


Fig. 3: Magnitude and phase spectrum for measured skin vibration of each texture; the blue line shows the magnitude spectrum and the orange line represents the unwrapped phase spectrum

from the generated colored noise and the phase spectrum of the measured skin vibration for each texture, and conducted an Inverse Fast Fourier Transform to obtain the signal in the time domain, as shown in Fig. 4. All of the combinations of the magnitude spectra from the five color noises and phase spectra from the six textures were used to generate N-T chimeras, totaling 30 stimuli.

B. Presentation System

We prepared a GUI that controlled the presented noise and recorded the answers for each participant throughout the experiment. The vibrotactile stimuli were presented through the PC's audio output using a class D amplifier (Foster, AP05 mkII) with a fixed volume and a vibrotactile actuator (Foster, 639897). The actuator was mounted inside a cylinder case composed of ASA using a 3D printer. In front of the participant, we prepared the vibrotactile actuator and a

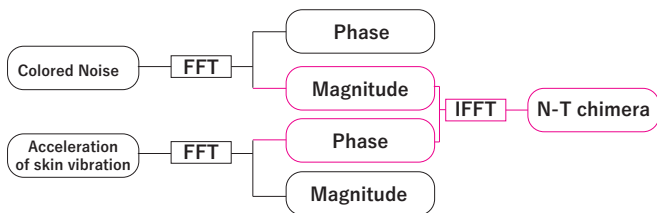


Fig. 4: Procedure of generating N-T chimeras; the generated colored noise and measured skin vibration were separated into phase spectrum and magnitude spectrum using fast Fourier transform, and the N-T chimeras were generated using the magnitude of colored noise and the phase of measured skin vibration

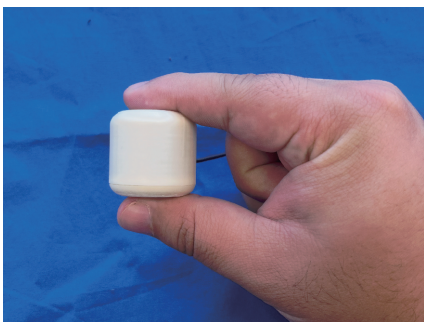


Fig. 5: Posture of holding the vibrotactile actuator instructed to the participants; the participants were asked to hold the actuator with their thumb and index finger of their dominant hand

numeric keypad to control the GUI, so that the participants could proceed with the experiment independently.

III. METHOD

A. Procedure

Nine participants (eight males, one female, aged 21-24, eight right-handed, and one left-handed) took part in the experiment. The dominant hand of each participant was identified using a questionnaire [17]. The participants were asked to hold the vibrotactile actuator in the vibrating direction with the index finger and the thumb of their dominant hand, as shown in Fig. 5. For the experiment, participants were instructed to compare the two stimuli presented with a 0.5 s silent interval and to intuitively rate their similarity on a scale of 0 to 100, with 0 indicating no similarity and 100 indicating complete similarity. A pair of the two stimuli presented for each trial was an N-T chimera and the original skin vibration that was measured against a texture used to generate the corresponding chimeras. The order of the two stimuli and the order of the presented pair were randomized for each participant, and all chimeras were evaluated once. Also, to eliminate any auditory influences, the participants wore noise-cancelling headphones (Sony, WH-1000XM5) and were presented with white noise throughout the experiment.

B. Analysis

To investigate the relationship between the physical properties of the stimuli and their perceptual ratings, we performed two primary analyses. First, we established a measure of physical similarity by calculating Spearman's rank correlation (ρ) between the magnitude spectrum of each original texture and that of each colored noise. The Spearman's rank correlation is calculated using the equation shown below

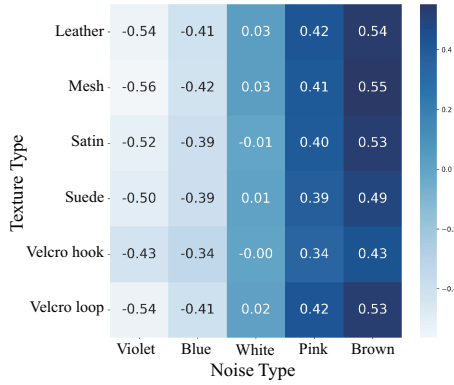
$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}, \quad (2)$$

where ρ is the coefficient of Spearman's rank correlation, d_i represents the difference between the ranks of magnitude for each frequency of the two magnitude spectra (noise and texture), and n represents the number of data points compared. We compared the results of the physical similarity based on the magnitude spectra and the perceived similarity to see whether the magnitude spectrum was dominant for the evaluation of the perceived similarity of N-T chimeras. The physical similarity was calculated for the frequencies of 20 to 1300 Hz, a range selected to align with both the specifications of the acceleration sensor and the primary bandwidth of human vibrotactile perception. Subsequently, we analyzed the perceptual ratings using a two-way repeated measures analysis of variance (ANOVA) to examine the effects and interaction of texture (phase spectrum) and noise type (magnitude spectrum) on perceived similarity. To test whether the collected results followed a normal distribution, we conducted a Shapiro-Wilk test. As the normality assumption was violated ($p < .05$), the Aligned Rank Transform (ART) [18] procedure was applied to the data before conducting the two-way repeated measures ANOVA. The ART enabled us to treat non-parametric data as if it were parametric. For any significant main effects, pairwise comparisons were conducted between levels of noise types or between texture types using the Wilcoxon signed-rank test with Bonferroni correction. The significant interaction was further explored by examining the effect of noise type within each texture, again using Wilcoxon tests with Bonferroni correction.

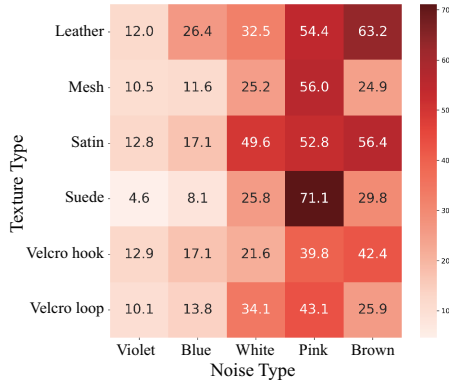
IV. RESULTS

A. Physical and Perceptual Similarity

Fig. 6a shows the results of the physical similarity calculated, and Fig. 6b shows the results of the answer of perceptual similarity. The results of the physical similarity show that the magnitude spectrum of brown noise had the highest correlation among all of the colored noises, and the violet noise was the least similar in terms of the magnitude spectrum for all of the measured skin vibration of each texture. Next, comparing the results between the physical similarity and perceptual similarity, the results for leather, satin, and the hook side of Velcro showed a correspondence in the results for physical similarity with the answers for N-T chimeras generated with brown noise being the highest. However, the results for mesh, suede, and the loop side of



(a) Physical similarity between the magnitude spectrum of the measured skin vibration and generated noise: Spearman's ρ shows the correlation between the two magnitude spectra and the coefficient is output from -1 to 1



(b) Answer of perceptual similarity between each N-T chimera generated from the combinations of the horizontal and vertical axis and the measured skin vibration

Fig. 6: Experiment results for comparing the answered similarity with the physical similarity

Velcro tape showed a different pattern, with the answers for N-T chimeras generated with pink noise being the highest.

B. Effects of Texture and Noise Type on Similarity

The ANOVA revealed a significant main effect for the noise factor ($F(4, 20) = 203.0, p < .001$), indicating that the structure of the magnitude spectrum significantly influenced similarity ratings. A significant main effect for the texture factor ($F(5, 25) = 203.0, p = .002$) was also found, indicating that the phase spectrum from tracing different textures significantly influenced the similarity ratings. Additionally, a significant interaction effect between texture and noise type ($F(20, 100) = 203.0, p = .003$) was observed, suggesting that the effect of noise type on perceived similarity depended on the specific original texture.

To break down the significant effects from the ANOVA, we conducted post-hoc pairwise comparisons using the Wilcoxon signed-rank test with Bonferroni correction for multiple comparisons. Follow-up tests for the main effect of noise type revealed several significant differences after Bonferroni correction. The results categorized by noise type are shown in Fig. 7. In these comparisons, chimeras made

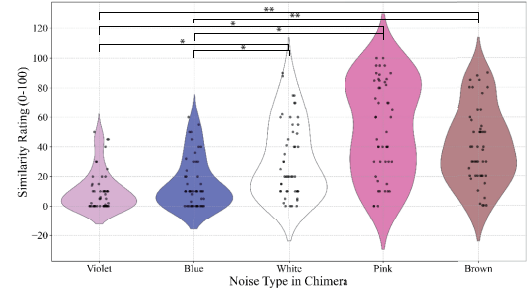


Fig. 7: Experimental results labeled by the adopted colored noise shown as a violin plot; the deviation for each noise type is shown as the width of the plot, and the individual answers are shown as black dots (***) ($p < .001, ** < .01, * < .05$).

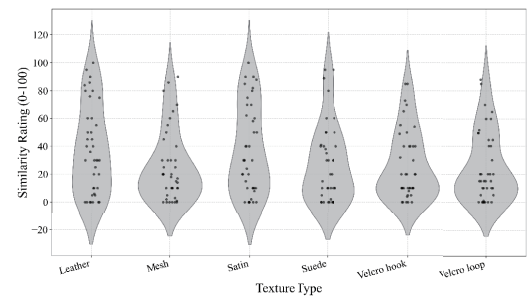


Fig. 8: Experimental results labeled by the adopted texture shown as a violin plot; the deviation for each texture is shown as the width of the plot, and the individual answers are shown as black dots.

with Brown, Pink, and White noise were consistently rated as more similar to the original textures than those made with Blue and Violet noise. Specifically, ratings for Brown noise chimeras were significantly higher than for both Blue ($p_{corr} = .003, d = 1.67, BF10 = 101$) and Violet noise ($p_{corr} = .003, d = 2.11, BF10 = 123$). Similarly, Pink noise ratings were significantly higher than for Blue ($p_{corr} = .021, d = 2.01, BF10 = 22.3$) and Violet noise ($p_{corr} = .015, d = 2.36, BF10 = 29.1$). Ratings for White noise were also significantly higher than for Blue ($p_{corr} = .029, d = 1.05, BF10 = 17.4$) and Violet noise ($p_{corr} = .019, d = 1.47, BF10 = 24.4$). In contrast, post-hoc tests for the main effect of texture revealed no significant differences between any pair of textures after Bonferroni correction. The results categorized by texture type are shown in Fig. 8.

Fig. 9 shows the results for all of the N-T chimeras as a box plot. The significant interaction effect was explored by examining the effect of noise type within each texture. The results that showed significant differences are shown in Table I. The N-T chimeras that were generated using pink and brown noise tended to be significantly different compared to those generated with other color noise, depending on the texture.

V. DISCUSSION

The results demonstrate that the perceptual similarity of a vibrotactile stimulus is not determined by its magnitude

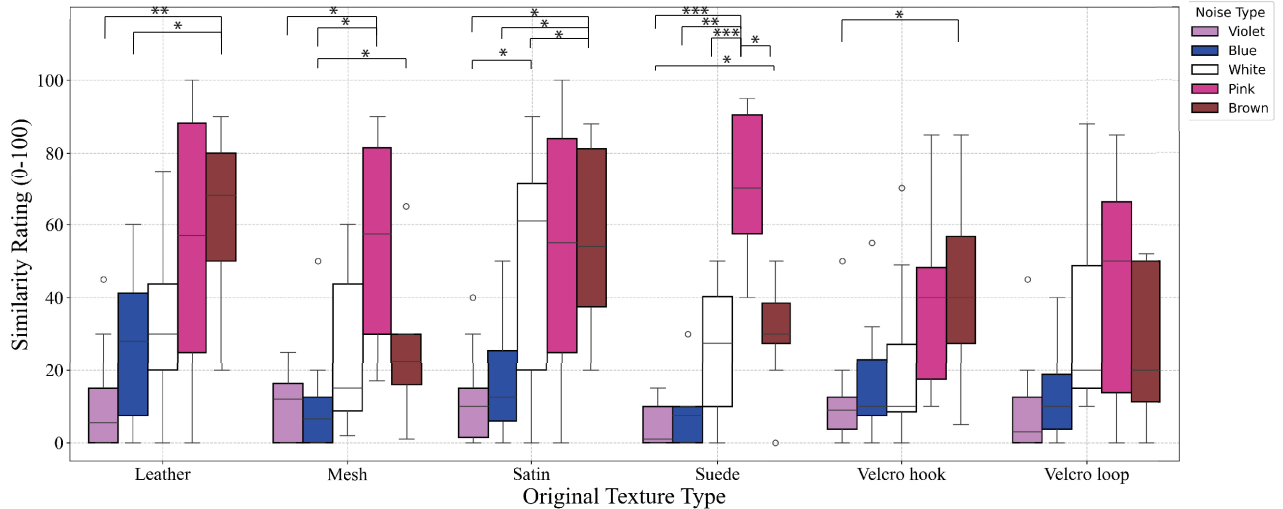


Fig. 9: Experimental results shown as a box plot; the color of the box plot corresponds to the color of the noise (***) $< .001$, ** $< .01$, * $< .05$).

TABLE I: Summary of significant pairwise post-hoc comparisons (Wilcoxon signed-rank test with Bonferroni correction) for the interaction between texture and noise type. Only comparisons with a corrected p-value less than .05 are shown.

Texture	Comparison	p_{corr}	Cohen's d	BF_{10}
Suede	Pink vs. Violet	$< .001$	4.23	277
	Pink vs. Blue	.002	3.78	154.4
	Pink vs. White	$< .001$	2.28	42.1
	Pink vs. Brown	.044	2.24	12.3
	Violet vs. Brown	.014	-2.20	30.8
Velcro (Hook)	Brown vs. Violet	.048	1.38	11.7
Mesh	Pink vs. Violet	.021	2.11	22.9
	Pink vs. Blue	.026	1.87	19.3
	Brown vs. Blue	.013	0.73	32.3
Satin	Brown vs. Blue	.018	1.79	24.8
	Brown vs. White	.014	2.07	31.7
	Brown vs. Violet	.014	2.08	31.7
	White vs. Violet	.036	1.48	14.6
Leather	Brown vs. Violet	.002	2.56	112.4
	Brown vs. Blue	.022	1.63	21.7

spectrum alone but by a complex interplay with its phase spectrum. This was first indicated by the results of Spearman's correlation coefficient. The comparison between the physical similarity based on the magnitude spectrum and the perceived similarity did not show a complete alignment. The results of the perceptual similarity for the textures of suede and mesh fabric were perceived as the most similar to the N-T chimeras generated with pink noise, whereas the results of the hook side of Velcro tape and leather were perceived as the most similar to the N-T chimeras generated with brown noise. This indicates that the magnitude spectrum is not completely dominant for tactile perception, and other factors are affecting the perceived similarity of N-T chimeras.

The results of the two-way repeated measures ANOVA showed a significant difference in the main effect of both noise type and texture type. The main effect of texture

type did not show any significant differences in the post-hoc comparison. On the other hand, the main effect of the noise type confirms that the magnitude spectrum is a crucial factor in tactile perception. The results for N-T chimeras generated with pink and brown noise were significantly different, indicating a distinct tendency. The results of the adopted color noise with a strong PSD for lower frequencies tended to be perceived more similarly than those of the chimeras generated with a small PSD. Similarity ratings for chimeras generated with white noise, which has a flat power spectral density (PSD), tended to be lower than ratings for chimeras with strong low-frequency PSDs (Pink and Brown noise) but higher than those with weak low-frequency PSDs (Blue and Violet noise).

The results of the two-way ANOVA also revealed a significant interaction effect between texture and noise type. This interaction demonstrates that the perceptual impact of a given magnitude spectrum (noise type) is dependent on the phase spectrum (original texture) with which it is paired. Post-hoc analyses of the effects of noise type within each texture confirmed this dependency. The post-hoc analyses suggest that perceptually critical information lies in the interaction between magnitude and phase spectra. Although the Spearman analysis showed that Brown noise was physically the most similar to the adopted skin vibrations, perceptual ratings indicated that Pink noise produced more convincing chimeras stimuli for textures such as suede, mesh fabric, and the loop side of Velcro. This suggests that the temporal structure carried by the phase spectrum becomes effective when supported by an appropriate magnitude distribution. Moreover, all six recorded skin vibrations exhibited a characteristic low-frequency-dominant spectrum, which contributed to the relatively high physical similarity of both Brown and Pink noise to the original signals. Taken together, these findings indicate that as long as magnitude-based similarity meets a sufficient threshold, the phase spectrum shapes the perceptual

reconstruction of texture, enabling the chimera stimulus to be perceived as similar to the original skin vibration. The integration of the magnitude and phase spectra allows to control the envelope of the auditorial vibration as well as the temporal fine structure. The phase spectrum controls mainly the temporal fine structure and the start and end of the vibrotactile stimulus, where the envelope fades in and out. The magnitude spectrum controls mainly the amplitude of the vibrotactile stimulus and the frequency content. Also, as a reason why pink noise was more effective than brown noise for certain textures, may be related to the low-frequency power.

Several limitations of the present study should be noted. First, a limitation of this study is its focus on a single perceptual metric: similarity to a reference. This approach does not capture other critical dimensions of the tactile experience. For instance, we did not independently measure the subjective naturalness or pleasantness of the stimuli, nor did we collect descriptive ratings such as perceived roughness and the ease of identification. Second, the reference we adopted for the experiment implicitly assumed the original recording represented an optimal or "high-quality" tactile experience. The subjective quality of the reference was not considered, which could have affected the evaluation of similarity. Future work could include separate ratings for stimulus quality or realism to provide a more comprehensive understanding of the perceptual space. Finally, the participant pool was not controlled for demographic variables such as age or gender, nor for prior experience with psychophysical experiments involving tactile stimuli. While our within-subjects design helps mitigate some of this variability, these factors could potentially influence perceptual sensitivity and response criteria.

VI. CONCLUSION

In this study, we proposed the usage of phase spectrum as a novel parameter to modify for vibrotactile stimuli and investigated the effect of the phase spectrum by generating N-T chimeras with various colored noise and measured skin vibration. We conducted an experiment that evaluated the similarity in the presentation with a vibrotactile actuator between the generated N-T chimeras and the measured skin vibration. The results insist that the magnitude spectrum is a crucial factor in tactile perception, as prior studies have shown, but also that the phase spectrum is equally critical in terms of similarity involving the interaction with the magnitude, which indicates the possibility of generating various vibrotactile textures with the phase information and colored noise.

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